

Discrete Mathematics: The Backbone of Computer Science

Mamta Parmar

Assistant Professor, Department of Applied Sciences,
Chandigarh Engineering College, Landran, Punjab, India

Abstract-This paper aims to emphasize the importance of Computer Science in discrete mathematics teaching. Firstly, stress on the importance of certain mathematical concepts for computer Science and then the relationship between Computational Thinking and the teaching of discrete mathematics is analysed. Secondly, described a comprehensive table of mathematics topics and their computer science applications.

Keywords- Discrete Mathematics, Graph Theory, Set Theory, Trees, etc.

I. Introduction:

Discrete mathematics is the branch of mathematics dealing with objects that can assume only distinct, separated values. Discrete mathematics is the mathematical language of computer science. Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in all branches of computer science, such as computer algorithms, languages cryptography, automated theorem proving, and software development. Conversely, computer implementations are tremendously significant in applying ideas from discrete mathematics to real-world applications, such as in operations research. Other fields of mathematics that are considered to be part of discrete mathematics include graph theory and the theory of computation. Topics in number theory such as congruence's and recurrence relations are also considered part of discrete mathematics. Discrete mathematics is an exciting and appropriate vehicle for working toward and achieving the goal of educating informed citizens who are better able to function in our increasingly technological society; have better reasoning power and problem-solving skills; are aware of the importance of mathematics in our society; and are prepared for future careers which will require new and more sophisticated analytical and technical tools. It is an excellent tool for improving reasoning and problem-solving abilities.

II. IMPORTANCE OF DISCRETE MATHEMATICS IN COMPUTER SCIENCE

There are several important reasons for studying discrete mathematics. First, this course can develop mathematical maturity: that is, the ability to understand and create mathematical arguments. Second, discrete mathematics is the gateway to more advanced courses in all parts of the mathematical sciences. Discrete mathematics provides the mathematical foundations for many computer science courses including data structures, algorithms, database theory, automata theory, formal languages, compiler theory, computer security, and operating systems. Also, discrete mathematics contains the necessary mathematical background for solving problems in operations research (including many discrete optimization techniques), chemistry, engineering, biology, and so on. Discrete mathematics is the mathematical language of computer science, and as such, its importance has increased dramatically in recent decades.

III. APPLICATIONS OF DISCRETE MATHEMATICS

Theoretical Computer Science

Theoretical computer science includes areas of discrete mathematics relevant to computing. It draws heavily on graph theory and logic. Included within theoretical computer science is the study of algorithms for computing mathematical results. Computability studies what can be computed in principle, and has close ties to logic, while complexity studies the time taken by computations. Automata theory and formal language theory are closely related to computability. Petri nets and process algebras are used to model computer systems, and methods from discrete mathematics are used in analyzing VLSI electronic circuits. Computational geometry applies algorithms to geometrical problems, while computer image analysis applies them to representations of images. Theoretical computer science also includes the study of various continuous computational topics. Discrete mathematics is the mathematical language of computer science, and as such, its importance has increased dramatically in recent decades.

Information Theory

Information theory involves the quantification of information. Closely related is coding theory which is used to design efficient and reliable data transmission and storage methods. Information theory also includes continuous topics such as analog signals, analog coding, analog encryption and Mathematical logic.

Mathematical logic

Logic is the study of the principles of valid reasoning and inference, as well as of consistency, soundness, and completeness. For example, in most systems of logic (but not in intuitionist logic) Peirce's law $((P \rightarrow Q) \rightarrow P) \rightarrow P$ is a theorem. For classical logic, it can be easily verified with a truth table. The study of mathematical proof is particularly important in logic, and has applications to automated theorem proving and formal verification of software.

Logical formulas are discrete structures, as are proofs, which form finite trees or, more generally, directed acyclic graph structures (with each inference step combining one or more premise branches to give a single conclusion). The truth values of logical formulas usually form a finite set, generally restricted to two values: true and false, but logic can also be continuous-valued, e.g., fuzzy logic. Concepts such as infinite proof trees or infinite derivation trees have also been studied e.g. infinitely logic.

Set theory

Set theory is the branch of mathematics that studies sets, which are collections of objects, such as {blue, white, and red} or the (infinite) set of all prime numbers. Partially ordered sets and sets with other relations have applications in several areas.

In discrete mathematics, countable sets (including finite sets) are the main focus. The beginning of set theory as a branch of mathematics is usually marked by Georg Cantor's work distinguishing between different kinds of infinite set, motivated by the study of trigonometric series, and further development of the theory of infinite sets is outside the scope of discrete mathematics. Indeed, contemporary work in descriptive set theory makes extensive use of traditional continuous mathematics.

Combinatorics

Combinatorics studies the way in which discrete structures can be combined or arranged. Enumerative combinatorics concentrates on counting the number of certain combinatorial objects - e.g. the twelvefold way provides a unified framework for counting permutations, combinations and partitions. Analytic combinatorics concerns the enumeration (i.e., determining the number) of combinatorial structures using tools from complex analysis and probability theory. In contrast with enumerative combinatorics which uses explicit combinatorial formula and generating functions to describe the results, analytic combinatorics aims at obtaining asymptotic formula. Design theory is a study of combinatorial designs, which are collections of subsets with certain intersection properties. Partition theory studies various enumeration and asymptotic problems related to integer partitions, and is closely related to q-series, special functions and orthogonal polynomials. Originally a part of number theory and analysis, partition theory is now considered a part of combinatorics or an independent field. Order theory is the study of partially ordered sets, both finite and infinite.

Graph theory

Graph theory, the study of graphs and networks, is often considered part of combinatorics, but has grown large enough and distinct enough, with its own kind of problems, to be regarded as a subject in its own right. Graphs are one of the prime objects of study in discrete mathematics. They are among the most ubiquitous models of both natural and human-made structures. They can model many types of relations and process dynamics in physical, biological and social systems. In computer science, they can represent networks of communication, data organization, computational devices, the flow of computation, etc. In mathematics, they are useful in geometry and certain parts of topology, e.g. knot theory. Algebraic graph theory has close links with group theory. There are also continuous graphs, however for the most part research in graph theory falls within the domain of discrete mathematics.

Discrete probability theory

Discrete probability theory deals with events that occur in countable sample spaces. For example, count observations such as the numbers of birds in flocks comprise only natural number values $\{0, 1, 2, \dots\}$. On the other hand, continuous observations such as the weights of birds comprise real number values and would typically be modeled by a continuous probability distribution such as the normal. Discrete probability distributions can be used to approximate continuous ones and vice versa. For highly constrained situations such as throwing dice or experiments with decks of cards, calculating the probability of events is basically enumerative.

Number theory

Number theory is concerned with the properties of numbers in general, particularly integers. It has applications to cryptography, cryptanalysis, and cryptology, particularly with regard to modular arithmetic, Diophantine equations, linear and quadratic congruence's, prime numbers and primarily testing. Other discrete aspects of number theory include geometry of numbers. In analytic number theory, techniques from continuous mathematics are also used. Topics that go beyond discrete objects include transcendental numbers, Diophantine approximation, and analysis and function fields.

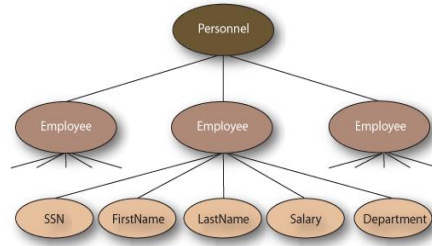
Algebraic structures occur as both discrete examples and continuous examples. Discrete algebras include: Boolean algebra used in logic gates and programming; relational algebra used in databases; discrete and finite versions of groups, rings and fields are important in algebraic coding theory; discrete semi groups and monodies appear in the theory of formal languages.

Discrete geometry and computational geometry

Discrete geometry and combinatorial geometry are about combinatorial properties of discrete collections of geometrical objects. A long-standing topic in discrete geometry is tiling of the plane. Computational geometry applies algorithms to geometrical problems.

Trees

Trees are used to represent data that has some hierarchical relationship among the data elements.



Topology

Although topology is the field of mathematics that formalizes and generalizes the intuitive notion of "continuous deformation" of objects, it gives rise to many discrete topics; this can be attributed in part to the focus on topological invariants, which themselves usually take discrete values. See combinatorial topology, topological graph theory, topological combinatorics, computational topology, discrete topological space, finite topological space, topology (chemistry).

CONCLUSIONS

Conclusion can be drawn from the above study that Discrete Mathematics plays the essential role in the development of computer science both for the particular knowledge and for the reasoning skills associated with mathematical maturity. Discrete mathematics has become popular in recent decades because of its applications to computer science. Discrete mathematics is the mathematical language of computer science. Discrete mathematics is the mathematical language of data science, and as such, its importance has increased dramatically in recent decades.

References

- [1] <http://cybercomputing.blogspot.com/2012/06/discrete-mathematics-applications-and.html>
- [2] <https://ivyleaguecenter.wordpress.com/2015/03/17/why-discrete-math-is-very-important/>
- [3] Discrete Mathematics with Applications (4 ed.). Boston: Wadsworth Publishing Co Inc. Gottlieb, P. (2007).
- [4] <https://www.ukessays.com/essays/computer-science/the-importance-of-discrete-mathematics.php>
- [5] <https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=6499853>
- [6] Bardley, James, "The Role of Mathematics in the Computer Science Curriculum", SIGCSE Bulletin, Vol.20, No. 1 (February 1988), p.100-103.

International Research Journal
IJNRD
 Research Through Innovation