

Structural Characterization of certain Symmetric n -Sigraphs

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Abstract. In this article, author introduces a common minimal common neighborhood dominating symmetric n -sigraph (or common minimal CN -dominating symmetric n -sigraph) of a given symmetric n -sigraph and obtains a structural characterization of common minimal common neighborhood dominating symmetric n -sigraph. In the sequel, we also obtained switching equivalence characterization: $\overline{\mathfrak{S}}\text{-}CMCN(S_n)$, where $\overline{\mathfrak{S}}$ and $CMCN(S_n)$ are complementary symmetric n -sigraph and common minimal CN -dominating symmetric n -sigraph of a symmetric n -sigraph S_n respectively.

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1. Introduction

Throughout this paper we consider finite simple graphs. Unless mentioned or defined otherwise, the reader is referred to for all terminology and notions in graph theory [3].

Let $n \geq 1$ be an integer. An n -tuple (b_1, b_2, \dots, b_n) is *symmetric*, if $b_j = b_{n-j+1}$, $1 \leq j \leq n$. Let $H_n = \{(b_1, b_2, \dots, b_n) : b_j \in \{+, -\}, b_j = b_{n-j+1}, 1 \leq j \leq n\}$ be the set of all symmetric n -tuples. Note that H_n is a group under coordinate-wise multiplication, and the order of H_n is 2^m ,

where $m = \lfloor \frac{n}{2} \rfloor$.

A *symmetric n -sigraph* (symmetric n -marked graph) is an ordered pair $S_n = (G, \sigma)$ ($S_n = (G, \mu)$), where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \rightarrow H_n$ ($\mu : V \rightarrow H_n$) is a function.

In this paper by an *n -tuple/ n -sigraph/ n -marked graph*, we always mean a symmetric n -tuple/symmetric n -sigraph/symmetric n -marked graph.

An n -tuple (b_1, b_2, \dots, b_n) is the *identity n -tuple*, if $b_j = +$, for $1 \leq j \leq n$, otherwise, it is a *non-identity n -tuple*. In an n -sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n -tuple is called an *identity edge*, otherwise, it is a *non-identity edge*.

Further, in an n -sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the n -tuple $\sigma(A)$ is the product of the n -tuples on the edges of A .

In [10], the authors defined two notions of balance in n -sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P. S. K. Reddy [6]):

Definition 1.1. Let $S_n = (G, \sigma)$ be an n -sigraph. Then,

- (i) S_n is *identity balanced* (or *i -balanced*), if the product of n -tuples on each cycle of S_n is the identity n -tuple, and
- (ii) S_n is *balanced* if every cycle in S_n contains an even number of non-identity edges.

Note 1.1: An i -balanced n -sigraph need not be balanced and conversely. The following characterization of i -balanced n -sigraphs is obtained in [10].

Theorem 1.1. (E. Sampathkumar et al. [10]) An n -sigraph $S_n = (G, \sigma)$ is i -balanced if, and only if, it is possible to assign n -tuples to its vertices such that the n -tuple of each edge uv is equal to the product of the n -tuples of u and v .

Let $S_n = (G, \sigma)$ be an n -sigraph. Consider the n -marking μ on vertices of S_n defined as follows: each vertex $v \in V$, $\mu(v)$ is the n -tuple which is the product of the n -tuples on the edges incident with v . The complement of S_n is an n -sigraph $\overline{S}_n = (\overline{G}, \sigma^c)$, where for any

edge $e = uv \in \overline{G}$, $\sigma^c(e) = \mu(u)\mu(v)$. Clearly, \overline{S}_n is defined here as an i -balanced n -sigraph due to Theorem 1.1.

In [10], the authors also have defined switching and cycle isomorphism of an n -sigraph $S_n = (G, \sigma)$ as follows: (See also [4-9, 11-25])

Let $S_n = (G, \sigma)$ and $S' = (G', \sigma')$ be two n -sigraphs. Then S_n and S' are said to be n -isomorphic if there exists an isomorphism $\phi : G \rightarrow G'$ such that if uv is an edge in S_n with label (a_1, a_2, \dots, a_n) then $\phi(u)\phi(v)$ is an edge in S' with label (a_1, a_2, \dots, a_n) .

Given an n -marking μ of an n -sigraph $S_n = (G, \sigma)$, switching S_n with respect to μ is the operation of changing the n -tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. The n -sigraph obtained in this way is denoted by $S_\mu(S_n)$ and is called the μ -switched n -sigraph or just switched n -sigraph.

Further, an n -sigraph S_n switches to n -sigraph S' (or that they are switching equivalent to each other), written as $S_n \sim S'$, whenever there exists an n -marking of S_n such that

$$S_\mu(S_n) \cong S'$$

Two n -sigraphs $S_n = (G, \sigma)$ and $S' = (G', \sigma')$ are said to be cycle isomorphic, if there exists an isomorphism $\phi : G \rightarrow G'$ such that the n -tuple $\sigma(C)$ of every cycle C in S_n equals to the n -tuple $\sigma'(\phi(C))$ in S' .

We make use of the following known result (see [10]).

Theorem 1.2. (E. Sampathkumar et al. [10]) Given a graph G , any two n -sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.

2. Common minimal common neighborhood dominating n -sigraph of an n -sigraph Let $G=(V, E)$ be a graph. A set $D \subseteq V$ is a dominating set of G , if every vertex in $V-D$ is adjacent to some vertex in D . A dominating set D of G is minimal, if for any vertex $v \in D, D-\{v\}$ is not a dominating set of G .

Let $G=(V, E)$ be a simple graph with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$. For $i \neq j$, the common neighborhood of the vertices v_i and v_j is the set of vertices different from v_i and v_j which are adjacent to both v_i and v_j and is denoted by $Y(v_i, v_j)$. Further, a subset

D of V is called the common neighborhood dominating set (or CN -dominating set) if every $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|Y(u, v)| \geq 1$, where

$|Y(u, v)|$ is the number of common neighborhoods between u and v . This concept was introduced by Alwardi et al. [1].

A common neighborhood dominating set D is said to be minimal common neighborhood dominating set if no proper subset of D is common neighborhood dominating set (See [1]).

Alwardi and Soner [2] introduced a new class of intersection graphs in the field of domination theory. The commonality minimal CN -dominating graph is denoted by $CMCN(G)$ is the graph which has the same vertex set as G with two vertices are adjacent if and only if there exist minimal CN -dominating in G containing them.

In this paper, we introduce a natural extension of the notion of common minimal CN - dominating graphs to the realm of n -sigraphs.

Motivated by the existing definition of complement of an n -sigraph, we extend the notion of common minimal CN -dominating graphs to n -sigraphs as follows: The common minimal CN -dominating n -sigraph $CMCN(S_n)$ of an n -sigraph $S_n = (G, \sigma)$ is an n -sigraph whose underlying graph is $CMCN(G)$ and the n -tuple of any edge uv is $CMCN(S_n)$ is

$\mu(u)\mu(v)$, where μ is the canonical n -marking of S_n . Further, an n -sigraph $S_n = (G, \sigma)$ is called common minimal CN -dominating n -sigraph, if $S_n \cong CMCN(S')$ for some n -sigraph

S' . The purpose of this paper is to initiate a study of this notion.

The following result indicates the limitations of the notion $CMCN(S_n)$ as introduced above, since the entire class of i -unbalanced n -sigraphs is forbidden to be common minimal CN -dominating n -sigraphs.

Theorem 2.1. For any n -sigraph $S_n = (G, \sigma)$, its common minimal CN -dominating n -sigraph $CMCN(S_n)$ is i -balanced.

Proof: Since the n -tuple of any edge uv in $CMCN(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n -marking of S_n , by Theorem 1.1, $CMCN(S_n)$ is i -balanced.

For any positive integer k , the k^{th} iterated common minimal CN -dominating n - sigraph, $CMCN^k(S_n)$ of S_n is defined as follows:

$$CMCN^0(S_n) = S_n, CMCN^k(S_n) = CMCN(CMCN^{k-1}(S_n)).$$

Corollary 2.2. For any n -sigraph $S_n = (G, \sigma)$ and for any positive integer k , $CMCN^k(S_n)$ is i -balanced.

The following result characterizes n -sigraphs which are common minimal CN - dominating n -sigraphs.

Theorem 2.3. An n -sigraph $S_n = (G, \sigma)$ is a common minimal CN-dominating n -sigraph if, and only if, S_n is i -balanced n -sigraph and its underlying graph G is a common minimal CN-dominating graph.

Proof: Suppose that S_n is i -balanced and G is a common minimal CN-dominating graph. Then there exists a graph H such that $CMCN(H) \cong G$. Since S_n is i -balanced, by Theorem 1.1, there exists a marking ζ of G such that each edge $e = uv$ in S_n satisfies $\sigma(uv) = \zeta(u)\zeta(v)$. Now consider the n -sigraph $S_n' = (H, \sigma')$, where for any edge e in H , $\sigma'(e)$ is the

marking of the corresponding vertex in G . Then clearly, $CMCN(S_n') \cong S_n$. Hence S_n is a common minimal CN-dominating n -sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a common minimal CN-dominating n -sigraph. Then there exists an n -sigraph $S_n' = (H, \sigma')$ such that $CMCN(S_n') \cong S_n$. Hence G is the common minimal CN-dominating graph of H and by Theorem 2.1, S_n is i -balanced.

In [2], the authors characterized graphs for which $CMCN(G) \cong G$.

Theorem 2.4. (Anwar Alwardi et al. [2])

For any graph $G=(V, E)$, $CMCN(G) \cong G$ if and only if every minimal CN-dominating set of G is independent.

We now characterize n -sigraphs whose common minimal CN-dominating n -sigraphs and complementary n -sigraphs are switching equivalent.

Theorem 2.5. For any n -sigraph $S_n = (G, \sigma)$, $\overline{S_n} \sim CMCN(S_n)$ if, and only if, every minimal CN-dominating set of G is independent.

Proof: Suppose $\overline{S_n} \sim CMCN(S_n)$. This implies, $CMCN(G) \cong \overline{G}$ and hence by Theorem 2.4, every minimal CN-dominating set of G is independent.

Conversely, suppose that every minimal CN-dominating set of G is independent. Then

$CMCN(G) \cong \overline{G}$ by Theorem 2.4. Now, if S_n is an n -sigraph with underlying graph G satisfies

the conditions of Theorem 2.4, by the definition of complementary n -sigraph and Theorem 2.1, $\overline{S_n}$ and $CMCN(S_n)$ are i -balanced and hence, the result follows from Theorem 1.2.

Theorem 2.6. For any two n -sigraphs S_n and S_n' with the same underlying graph, their common minimal CN-dominating n -sigraphs are switching equivalent.

Proof. Suppose $S_n = (G, \sigma)$ and $S_n' = (G', \sigma')$ be two n -sigraphs with $G \cong G'$. By Theorem 2.1, $CMCN(S_n)$ and $CMCN(S_n')$ are i -balanced and hence, the result follows from Theorem 1.2.

For any $m \in H_n$, the m -complement of $a = (a_1, a_2, \dots, a_n)$ is: $a^m = am$. For any $M \subseteq$

H_n , and $m \in H_n$, the m -complement of M is $M^m = \{a^m : a \in M\}$.

For any $m \in H_n$, the m -complement of an n -sigraph $S_n = (G, \sigma)$, written $(S_n)^m$, is the same graph but with each edge label $a = (a_1, a_2, \dots, a_n)$ replaced by a^m .

For an n -sigraph $S_n = (G, \sigma)$, the $CMCN(S_n)$ is i -balanced. We now examine, the condition under which m -complement of $CMCN(S_n)$ is i -balanced, where for any $m \in H_n$.

Theorem 2.7. Let $S_n = (G, \sigma)$ be an n -sigraph. Then, for any $m \in H_n$, if $CMCN(G)$ is bipartite then $(CMCN(S_n))^m$ is i -balanced.

Proof: Since, by Theorem 2.1, $CMCN(S_n)$ is i -balanced, for each k , $1 \leq k \leq n$, the number of n -tuples on any cycle C in $CMCN(S_n)$ whose k^{th} co-ordinate are $-$ is even. Also, since $CMCN(G)$ is bipartite, all cycles have even length; thus, for each k , $1 \leq k \leq n$, the number

of n -tuples on any cycle C in $CMCN(S_n)$ whose k^{th} co-ordinate are $+$ is also even. This implies that the same thing is true in any m -complement, where for any $m \in H_n$. Hence $(CMCN(S_n))^m$ is i -balanced.

Theorem 2.6 provides easy solutions to other n -sigraph switching equivalence relations, which are given in the following results.

Corollary 2.8. For any two n -sigraphs S_n and S_n' with the same underlying graph, $CMCN(S_n)$ and $CMCN((S_n)^m)$ are switching equivalent.

Corollary 2.9. For any two n -sigraphs S_n and S_n' with the same underlying graph, $CMCN((S_n)^m)$ and $CMCN(S_n)$ are switching equivalent.

Corollary 2.10. For any two n -sigraphs S_n and S_n' with the same underlying graph, $CMCN((S_n)^m)$ and $CMCN((S_n')^m)$ are switching equivalent.

Corollary 2.11. For any two n -sigraphs $S_n = (G, \sigma)$ and $S_n' = (G', \sigma')$ with the $G \cong G'$ and G, G' are bipartite, $(CMCN(S_n))^m$ and $CMCN(S_n')$ are switching equivalent.

Corollary 2.12. For any two n -sigraphs $S_n = (G, \sigma)$ and $S_n' = (G', \sigma')$ with the $G \cong G'$ and G, G' are bipartite, $CMCN(S_n)$ and $CMCN((S_n')^m)$ are switching equivalent.

Corollary 2.13. For any two n -sigraphs $S_n = (G, \sigma)$ and $S_n' = (G', \sigma')$ with the $G \cong G'$ and G, G' are bipartite, $(CMCN(S_1))^m$ and $(CMCN(S_2))^m$ are switching equivalent.

3. Conclusion

A new notion for n -signed graphs called common minimal CN - dominating n -sigraph of an n -signed graph was introduced. Some results were proved and the structural characterization of a common minimal CN -dominating n -signed graph was presented. There is no structural characterization of a common minimal CN -dominating graph, but we have obtained the structural characterization of a common minimal CN -dominating n - signed graph.

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