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# INVERSE LAPLACE TRANSFORM FORMULA INVOLVING GENERAL POLYNOMIALS AND MULTIVARIABLE H-FUNCTION

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**Abstract**

In this present paper, we obtain the inverse Laplace transform of the product of a general class of multivariable polynomials and the multivariable H-function. The polynomials and the functions involved in our main formula as well as their arguments are quite general in nature. Therefore, the inverse Laplace transform of the product of a large variety of polynomials and numerous simple special functions can be obtained as simple special cases of our main result. The results obtained by Gupta and Soni [7] and Soni and Singh [9] follows as special cases of our main result.

**Keywords:** Inverse Laplace transform, General class of polynomials and Multivariable H-function.

## 1. Introduction

The Laplace transform of the function f(t) is defined in the following usual manner

$$F(s) = L\{f(t);s\} = \int_0^\infty e^{-st} f(t)dt, \mathrm{Re}(s) > 0. \tag{1.1}$$

The function f(t) is called the inverse Laplace transform of F(s) and will be denoted by $L^{-1}\{F(s)\}$ in the paper.

Srivastava [5,p.185,Eq.(7)]introduced the multivariable general class of polynomials:

$$S_{N_1,\ldots,N_R}^{M_1,\ldots,M_R}[x_1,\ldots,x_R] = \sum_{k_1=0}^{[N_1/M_1]} \cdots \sum_{k_R=0}^{[N_R/M_R]} \frac{(-N_1)_{M_1k_1}}{k_1!} \cdots \frac{(-N_R)_{M_Rk_R}}{k_R!} A[N_1,k_1;\ldots;N_R,k_R] x_1^{k_1} \ldots x_R^{k_R} \tag{1.2}$$

where $N_i = 0,1,2,\ldots,M_i \neq 0$ (i=1,2,...,R)
The coefficients $A[N_1,k_1;\ldots;N_R,k_R]$ are arbitrary constant, real or complex and $M_i$ is an arbitrary positive integer.

The H-function of r complex variables $z_1,\ldots,z_r$ was introduced by Srivastava and Panda [2]. We shall define and represent it in the following contracted form [4, p. 251, Eq. (c.1)] :

$$H_{p,q:p_1,q_1;\ldots;p_r,q_r}^{0,n:m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\middle| \begin{matrix} (a_j;\alpha_j',\ldots,\alpha_j^{(r)})_{1,p} : (c_j',\gamma_j')_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta_j',\ldots,\beta_j^{(r)})_{1,q} : (d_j',\delta_j')_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi_1(\xi_1) \cdots \phi_r(\xi_r) \psi(\xi_1 \cdots \xi_r) z_1^{\xi_1} \cdots z_r^{\xi_r} d\xi_1 \cdots d\xi_r \tag{1.3}$$

where $\omega = \sqrt{-1}$

For the convergence, existence conditions and other details of the above multivariable H-function, we refer to the book by Srivastava et al [4, p.251-253, Eq.(C.2)-(C.8)].

**2. Main Result**

$$L^{-1}\left\{ s^{-\rho}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-\sigma_i}\times S_{N_1,\ldots,N_R}^{M_1,\ldots,M_R}\left[e_1 s^{-\lambda_1}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-\eta_i^{'}},\ldots,e_R s^{-\lambda_R}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-\eta_i^{(R)}}\right]\right.$$

$$\left.\times H_{p,q:p_1,q_1;\ldots;p_r,q_r}^{0,n:m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix} z_1 s^{-u_1}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-v_i^{'}} \\ \vdots \\ z_r s^{-u_r}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-v_i^{(r)}}\end{matrix}\left|\begin{matrix}\left(a_j;\alpha_j^{'},\ldots,\alpha_j^{(r)}\right)_{1,p}:\left(c_j^{'},\gamma_j^{'}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r} \\ \left(b_j;\beta_j^{'},\ldots,\beta_j^{(r)}\right)_{1,q}:\left(d_j^{'},\delta_j^{'}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\end{matrix}\right.\right]\right\}$$

$$=x^{\rho+\sigma_1 l_1+\cdots+\sigma_\tau l_\tau-1}\times\sum_{k_1=0}^{[N_1/M_1]}\cdots\sum_{k_R=0}^{[N_R/M_R]}\frac{(-N_1)_{M_1k_1}}{k_1!}\cdots\frac{(-N_R)_{M_Rk_R}}{k_R!}A[N_1,k_1;\ldots;N_R,k_R]$$

$$\left(e_1x^{\lambda_1+\eta_1^{'}l_1+\cdots+\eta_\tau^{'}l_\tau}\right)^{k_1}\cdots\left(e_Rx^{\lambda_R+\eta_1^{(R)}l_1+\cdots+\eta_\tau^{(R)}l_\tau}\right)^{k_R}\times H_{p+\tau,q:+\tau+1\,p_1,q_1;\ldots;p_r,q_r;0,1;\ldots;0,1}^{0,n+\tau:m_1,n_1;\ldots;m_r,n_r;1,0;\ldots;1,0}\left[\begin{matrix} z_1x^{u_1+v_1^{'}l_1+\cdots+v_\tau^{'}l_\tau} \\ \vdots \\ z_rx^{u_r+v_1^{(r)}l_1+\cdots+v_\tau^{(r)}l_\tau} \\ \alpha_1x^{l_1} \\ \vdots \\ \alpha_\tau x^{l_\tau}\end{matrix}\right|$$

$$\left(a_j;\alpha_j^{'},\ldots,\alpha_j^{(r)},0,\ldots,0\right)_{1,p},(1-\sigma_1-\eta_1^{'}k_1-\cdots-\eta_1^{(R)}k_R;v_1^{'},\ldots,v_1^{(r)},1,0,\ldots,0),\ldots,$$

$$\left(b_j;\beta_j^{'},\ldots,\beta_j^{(r)},0,\ldots,0\right)_{1,q},(1-\rho-l_1\sigma_1-l_\tau\sigma_\tau-\left(\lambda_1+\eta_1^{'}l_1+\cdots+\eta_\tau^{'}l_\tau\right)k_1-\cdots-\left(\lambda_R+\eta_1^{(R)}l_1+\cdots+\eta_\tau^{(R)}l_\tau\right)k_R;$$

$$\left(1-\sigma_\tau-\eta_\tau^{'}k_1-\cdots-\eta_\tau^{(R)}k_R;v_1^{'},\ldots,v_1^{(r)},0,\ldots,0,1\right),\qquad\text{———}\qquad,$$

$$\left(u_1+v_1^{'}l_1+\cdots+v_\tau^{'}l_\tau\right),\ldots,\left(u_r+v_1^{(r)}l_1+\cdots+v_\tau^{(r)}l_\tau\right),l_1,\ldots,l_\tau);\left(1-\sigma_1-\eta_1^{'}k_1-\cdots-\eta_1^{(R)}k_R;v_1^{'},\ldots,v_1^{(r)},0,\ldots,0\right),\ldots,$$

$$\left.\begin{matrix}\text{———}\qquad :\left(c_j^{'},\gamma_j^{'}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r};\text{———};\ldots;\text{———} \\ \left(1-\sigma_\tau-\eta_\tau^{'}k_1-\cdots-\eta_\tau^{(R)}k_R;v_1^{'},\ldots,v_1^{(r)},0,\ldots,0\right):\left(d_j^{'},\delta_j^{'}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r};(0,1);\ldots;(0,1)\end{matrix}\right] \tag{2.1}$$

The result (2.1) is valid under the following set of conditions:

(i) The quantities $\lambda_1,\ldots,\lambda_R,\eta_1',\ldots,\eta_\iota',\eta_1^{(R)},\ldots,\eta_\iota^{(R)}$ and $u_1,\ldots,u_r,v_1',\ldots,v_\iota',v_1^{(r)},\ldots,v_\iota^{(r)}$ are all positive and Re(s)>0.

(ii) $0<l_i<1$, $|\operatorname{Arg}\alpha_i|<(1-l_i)(\pi/2)$, $i=1,2,\ldots,\iota$ or $l_1=l_\iota=1$ and $\alpha_1>0,\ldots,\alpha_\iota>0$.

(iii)

$$\operatorname{Re}\left(\rho+l_1\sigma_1+\cdots+l_\tau\sigma_\tau\right)+\sum_{i=1}^{r}\min_{1\le j\le m_i}\operatorname{Re}\left[\left(u_i+v_1^{(i)}l_1+\cdots+v_\tau^{(i)}l_\tau\right)\left(d_j^{(i)}/\delta_j^{(i)}\right)\right]>0,$$

$$\Delta_i\equiv\sum_{j=1}^{p}\alpha_j^{(i)}+\sum_{j=1}^{p_i}\gamma_j^{(i)}-\sum_{j=1}^{q}\beta_j^{(i)}-\sum_{j=1}^{q_i}\delta_j^{(i)}-\left[u_i+v_1^{(i)}l_1+\cdots+v_\tau^{(i)}l_\tau\right]\le 0$$

$$\textit{and}\left|Arg\left(z_i\right)\right|<\left(\tfrac{1}{2}\right)A_i\pi,A_i>0\,\forall i\in\{1,\ldots,r\}\ \textit{where}$$

$$A_i\equiv-\sum_{j=n+1}^{p}\alpha_j^{(i)}+\sum_{j=1}^{n_i}\gamma_j^{(i)}-\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)}-\sum_{j=1}^{q}\beta_j^{(i)}+\sum_{j=1}^{m_i}\delta_j^{(i)}-\sum_{j=m_i+1}^{q_i}\delta_j^{(i)}-\left[u_i+v_1^{(i)}(l_1+1)+\cdots+v_\tau^{(i)}(l_\tau+1)\right]\le 0$$

(iv) $N_i=0,1,2,\ldots,M_i\neq 0\,(i=1,2,\ldots,R)$, the coefficients $A[N_1,s_1;\ldots;N_R,s_R]$ are arbitrary constant, real or complex and $M_i$ is an arbitrary positive integer.

**Proof.** We first express the general class of polynomial occurring on the left hand side of (2.1) in series form given by (1.2), replace the multivariable H-function occurring therein by its well known Mellin-Bernes contour integral with the help of (1.3). Now, we interchange the orders of summations and integration (which is permissible under the conditions stated with (2.1) ), find the inverse Laplace transform of the result thus obtained by making use of the following known formula[8, p.12, Eq.(12)]

$$L^{-1}\left\{ s^{\sum_{i=1}^{\tau} l_i a_i - \lambda} \prod_{i=1}^{\tau} \left(s^{l_i} + \lambda_i\right)^{-a_i} \right\} = \frac{x^{\lambda-1}}{\prod_{i=1}^{\tau} \Gamma(a_i)} H^{0,0:1,1;\ldots;1,1}_{0,1:1,1;\ldots;1,1}\left[\begin{matrix} \lambda_1 x^{l_1} \\ \vdots \\ \lambda_\tau x^{l_\tau} \end{matrix}\left| \begin{matrix} \text{——} : (1-a_1,1);\ldots;(1-a_\tau,1) \\ (1-\lambda; l_1,\ldots,l_\tau) : (0,1);\ldots;(0,1) \end{matrix}\right.\right]$$

provided that $\mathrm{Re}(s)>0, \mathrm{Re}(\lambda)>0, 0<l_i<1, |\mathrm{Arg}(\lambda_i)|<(1-l_i)(\pi/2)$ or $l_i=1$ and $\lambda_i>0, \forall i=1,2,\ldots,\iota$.

Express the multivariable H-function thus obtained in terms of Mellin-Barnes contour integral with the help of (1.3), and now, on simplification and interpreting in terms of multivariable H-function, we arrive at the desired result (2.1).

**3. Special cases**

On account of general nature of our main result, several known and new results follow as its special cases. The inverse Laplace transform formula (2.1) established here is unified in nature and acts as a key formula. Thus the general class of multivariable polynomials involved in the formula (2.1) reduce to large variety of polynomials listed by Srivastava and Singh[3], and so from the formula (2.1), we can further obtain various inverse Laplace transform involving a number of simpler polynomials.

Now, we give new and interesting inverse Laplace transform that follow as special cases of (2.1)

**(i)** If we take $A[N_1,k_1;\ldots;N_R,k_R]=1$ in (2.1), then the following result involving the product of R-Gould-Hopper polynomials[6, p.58, Eq. (5.2)]

$$L^{-1}\left\{ s^{-\rho} \prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-\sigma_i} \times \prod_{j=1}^{R}\left\{(-1)^{N_j}\left[e_j s^{-\lambda_j}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-\eta_i^{(j)}} \Big/ h_j\right]^{N_j/M_j} \times g_{N_i}^{M_i}\left[-\left\{h_j / e_j s^{-\lambda_j}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-\eta_i^{(j)}}\right\}^{1/M_j}, h_j\right]\right\}\right.$$

$$\left.\times H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 s^{-u_1}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-v_i'} \\ \vdots \\ z_r s^{-u_r}\prod_{i=1}^{\tau}\left(s^{l_i}+\alpha_i\right)^{-v_i^{(r)}} \end{matrix}\left| \begin{matrix} \left(a_j;\alpha_j',\ldots,\alpha_j^{(r)}\right)_{1,p} : \left(c_j',\gamma_j'\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r} \\ \left(b_j;\beta_j',\ldots,\beta_j^{(r)}\right)_{1,q} : \left(d_j',\delta_j'\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r} \end{matrix}\right.\right]\right\}$$

$$= x^{\rho+\sigma_1 l_1+\cdots+\sigma_\tau l_\tau - 1} \times \sum_{k_1=0}^{[N_1/M_1]}\cdots\sum_{k_R=0}^{[N_R/M_R]} \frac{(-N_1)_{M_1 k_1}}{k_1!}\cdots\frac{(-N_R)_{M_R k_R}}{k_R!}$$

$$\left(e_1 x^{\lambda_1+\eta_1' l_1+\cdots+\eta_\tau' l_\tau}\right)^{k_1}\cdots\left(e_R x^{\lambda_R+\eta_1^{(R)} l_1+\cdots+\eta_\tau^{(R)} l_\tau}\right)^{k_R} \times H^{0,n+\tau:m_1,n_1;\ldots;m_r,n_r;1,0;\ldots;1,0}_{p+\tau,q:+\tau+1\,p_1,q_1;\ldots;p_r,q_r;0,1;\ldots;0,1}\left[\begin{matrix} z_1 x^{u_1+v_1' l_1+\cdots+v_\tau' l_\tau} \\ \vdots \\ z_r x^{u_r+v_1^{(r)} l_1+\cdots+v_\tau^{(r)} l_\tau} \\ \alpha_1 x^{l_1} \\ \vdots \\ \alpha_\tau x^{l_\tau} \end{matrix}\right.$$

$$\left(a_j;\alpha_j^{'},\ldots,\alpha_j^{(r)},0,\ldots,0\right)_{1,p},\left(1-\sigma_1-\eta_1^{'}k_1-\cdots-\eta_1^{(R)}k_R;v_1^{'},\ldots,v_1^{(r)},1,0,\ldots,0\right),\ldots,$$

$$\left(b_j;\beta_j^{'},\ldots,\beta_j^{(r)},0,\ldots,0\right)_{1,q},\left(1-\rho-l_1\sigma_1-l_\tau\sigma_\tau-\left(\lambda_1+\eta_1^{'}l_1+\cdots+\eta_\tau^{'}l_\tau\right)k_1-\cdots-\left(\lambda_R+\eta_1^{(R)}l_1+\cdots+\eta_\tau^{(R)}l_\tau\right)k_R;\right.$$

$$\left(1-\sigma_\tau-\eta_\tau^{'}k_1-\cdots-\eta_\tau^{(R)}k_R;v_1^{'},\ldots,v_1^{(r)},0,\ldots,0,1\right),\qquad \text{———} \qquad ,$$

$$\left(u_1+v_1^{'}l_1+\cdots+v_\tau^{'}l_\tau\right),\ldots,\left(u_r+v_1^{(r)}l_1+\cdots+v_\tau^{(r)}l_\tau\right),l_1,\ldots,l_\tau);\left(1-\sigma_1-\eta_1^{'}k_1-\cdots-\eta_1^{(R)}k_R;v_1^{'},\ldots,v_1^{(r)},0,\ldots,0\right),\ldots,$$

$$\left.\begin{matrix} \text{———} & :\left(c_j^{'},\gamma_j^{'}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r};\text{——};\ldots;\text{——} \\ \left(1-\sigma_\tau-\eta_\tau^{'}k_1-\cdots-\eta_\tau^{(R)}k_R;v_1^{'},\ldots,v_1^{(r)},0,\ldots,0\right) & :\left(d_j^{'},\delta_j^{'}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r};(0,1);\ldots;(0,1) \end{matrix}\right] \tag{3.1}$$

**(ii)** If we take R = 2 and r = 1 in (2.1), we get a known result obtained by Soni and Singh [9].
**(iii)** If we R = 2 , r = 1 and ι = 2 in (2.1), we get the result obtained by Gupta and soni [7].

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