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Three Warehouse Production Inventory Model for Deteriorating Items with Advertisement and Price Dependent Demand

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This paper attempts to develop an inventory model for deteriorating items when demand depends on both, frequency of the advertisement and selling price of the items. The manufacturer uses three warehouses to store the items. They can be categorized as own warehouse (OW) and rental warehouse (RW). Rental warehouse considered in the present study are of two types; one having low rent with high deterioration and the other having high rent but low deterioration.

The present model is mainly applicable for dairy products, bakery items, floral industry etc. The main objective is to optimize the total inventory cost. The theoretical developments are numerically justified by an example. In order to study the effect of changes in the parameters on total cycle time and total cost, a sensitivity analysis is also carried out.

KEYWORDS: Three warehouse, production inventory, deterioration, advertisement dependent demand.

1. INTRODUCTION

Two warehouse inventory models for deteriorating items have been studied by many researchers in past years. This paper is about the development of production inventory model consisting of three warehouses. Traditional inventory models for deteriorating items are developed mainly for a single warehouse. But in reality, due to limited space capacity of own warehouse and for the betterment of the condition of the products, an additional warehouse is required to hold a large stock which can be considered as rental warehouse. The single warehouse inventory models are not suitable for the storage of large stock. Then for storage of large units more additional warehouses are hired on rent. The concept of two warehouses was first introduced by **Hartley (1976)**. **Patel and Parekh (2014)** have discussed an inventory model for deteriorating items having two warehouses with linear demand, inflation and permissible delay in payment. **Yadavetal. (2016)** also have developed two warehouse inventory model for deteriorating items with variable holding cost, time dependent demand and shortages.

A three warehouse inventory model to store partial inventory have been proposed by **Shuklaetal. (2009).** In this paper we have considered an own warehouse and two rental warehouses which may be located near the OW or a little away from OW. One rental warehouse (RW_1) has low rent, high deterioration and the other rental warehouse (RW_2) has high rent and low deterioration.

In most of the inventory models demand rate is assumed to be constant, time dependent, stock dependent etc. Selling price also plays an important role in inventory system. According to recent trend, selling price of product and advertisement on social media play a vital role in attracting the consumers or the buyers. An inventory system of ameliorating items for price dependent demand rate has been proposed by **Mondal et al. (2003).Chang etal. (2010)** developed an inventory models with stock and price dependent demand for deteriorating items based on limited shelf space. **Hesham et al. (2016)** proposed an inventory policy with price dependent demand and deterioration under partial backlogging, inflation and time value of money. **Rastogi et al. (2017)**have developed two warehouse inventory policies with price dependent demand and deterioration under partial backlogging. **Srivastava and Gupta (2011)** proposed a three warehouse inventory model for deteriorating items with price and advertisement dependent demand.

This paper is about the development of production inventory model consisting of three warehouses. This model is useful for the inventory management for deteriorating items where demand depends upon the frequency of the advertisement and selling price of the product.
In this paper, we have considered an own warehouse and two rental warehouses: one with low rent, high deterioration and another with high rent, low deterioration, the demand is dependent on selling price of the product and also on the advertisement.The concept of advertisement in the development of the present model is particularly introduced, since the electronic media, newspapers and internet are the best sources for the promotion of products, which in turn is directly involved in increasing sale of the product resulting in increased profit to the management system.

Objectives

- The main objective of the suggested model is to determine the optimum cycle time which minimizes the total system cost resulting in increased sale and profit to the system.
- This model is basically applicable for dairy products, bakery items floral industries where number of warehouses are required to maintain the stock.

The notations and assumptions to derive the model is given in section 2. The mathematical development of model is done in section 3. In section 4, a numerical illustration is given and to check the effectiveness of the derived model, sensitivity analysis is done in section 5. At last, interpretations are given for the suggested model in section 6 while, policy implications are suggested to the management in section 7.

2. NOTATIONS AND ASSUMPTIONS

2.1. Notations:

$Z(t)$ = Inventory level at any instant of time 't' in OW
$Y(t)$ = Inventory level at any instant of time 't' in RW_1
$X(t)$ = Inventory level at any instant of time 't' in RW_2
T = Cycle time at which inventory reaches to zero
t_1 = Time period at which inventory level reaches to maximum capacity in OW
t_2= Time period at which inventory level reaches to maximum capacity in RW_1
t_3= Time period at which inventory level reaches to maximum capacity in RW_2
t_4= Time period at which inventory level falls to zero in RW_2

t_5= Time period at which inventory level falls to zero in RW_1
α = Maximum capacity of OW
β = Maximum capacity of RW_1
γ = Maximum capacity of RW_2
A = Frequency of the advertisement in the cycle
λ = Rate of change of frequency of advertisement
p = Selling price of product
C_S = Set- up cost
P = Production cost of a unit
TC = Total system cost
$\theta_1, \theta_2, \theta_3$ = Deterioration rate in OW, RW_1, RW_2respectively
C_1, C_2, C_3 = Holding cost of inventory stored in OW, RW_1, RW_2 respectively
D_1, D_2, D_3 = Deterioration cost of product in OW, RW_1, RW_2 respectively

2.2. Assumptions:

1. The inventory model is developed for the single item.
2. Replenishment rate is infinite.
3. Lead time is zero.
4. Shortages are not allowed.
5. The own warehouse and rental warehouses have finite capacity.
6. Firstly the inventory from RW_2 will be exhausted according to the demand followed by RW_1 and lastly from OW.
7. During the cycle time the deteriorating items are neither repaired nor replaced.
8. Demand rate is dependent both on the frequency of the advertisement and selling price of the product.

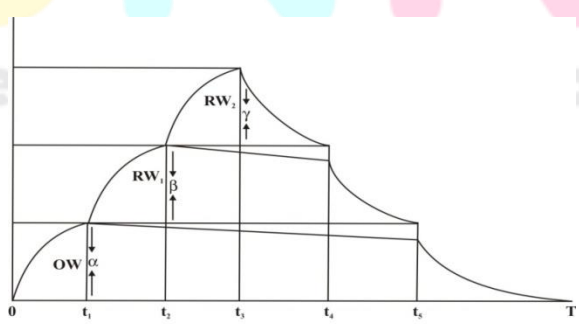
$D(A, p) = A^{\lambda} f(p) \quad ; \quad 0 \le t \le T$

where $f(p) = (b - p) > 0$ and 'b' is a constant.

9. Production rate is greater than demand rate. It is a linear form of inventory level and demand rate.
   $P(t) = [\, I(t) + a\, D(A, p) \,]$,
   where 'a' is constant.

3. MATHEMATICAL MODEL: FORMULATION AND SOLUTION

According to the notations and assumptions mentioned earlier, the behavior of inventory system at time't' can be depicted in the following inventory time diagram.



Inventory-time diagram

The inventory level of the product at time't' over period (0, T) can be represented by the following differential equations:

$$\frac{dZ(t)}{dt} + \theta_1 Z(t) = P(t) - D(A, p) \qquad 0 < t \le t_1$$

$$\frac{dZ(t)}{dt} + \theta_1 Z(t) = Z(t) + (a-1)A^{\lambda} f(p) \qquad 0 < t \le t_1 \quad .....(1)$$

$$Z(t) = M[(\theta_1 - 1)t - \frac{(\theta_1 - 1)^2 t^2}{2}] \qquad 0 < t \le t_1$$

$$where\ M = \frac{(a-1)A^{\lambda} f(p)}{(\theta_1 - 1)}$$

$$\frac{dY(t)}{dt} + \theta_2 Y(t) = P(t) - D(A, p) \qquad t_1 < t \le t_2$$

$$\frac{dY(t)}{dt} + \theta_2 Y(t) = Y(t) + (a-1)A^{\lambda} f(p) \qquad t_1 < t \le t_2 \quad .....(2)$$

$$Y(t) = N[(\theta_2 - 1)(t - t_1) - \frac{(\theta_2 - 1)^2 (t - t_1)^2}{2}] \qquad t_1 < t \le t_2$$

$$where\ N = \frac{(a-1)A^{\lambda} f(p)}{(\theta_2 - 1)}$$

$$\frac{dX(t)}{dt} + \theta_3 X(t) = P(t) - D(A, p) \qquad t_2 < t \le t_3$$

$$\frac{dX(t)}{dt} + \theta_3 X(t) = X(t) + (a-1)A^{\lambda} f(p) \qquad t_2 < t \le t_3 \quad .....(3)$$

$$X(t) = R[(\theta_3 - 1)(t - t_2) - \frac{(\theta_3 - 1)^2 (t - t_2)^2}{2}] \qquad t_2 < t \le t_3$$

$$where\ R = \frac{(a-1)A^{\lambda} f(p)}{(\theta_3 - 1)}$$

$$\frac{dX(t)}{dt} + \theta_3 X(t) = -A^{\lambda} f(p) \qquad t_3 < t \le t_4 \quad .....(4)$$

$$X(t) = \frac{A^{\lambda} f(p)}{\theta_3}[-\theta_3 (t - t_4) + \frac{\theta_3^2 (t - t_4)^2}{2}] \qquad t_3 < t \le t_4$$

$$\frac{dY(t)}{dt} + \theta_2 Y(t) = -A^{\lambda} f(p) \qquad t_4 < t \le t_5 \quad .....(5)$$

$$Y(t) = \frac{A^{\lambda} f(p)}{\theta_2}[-\theta_2 (t - t_5) + \frac{\theta_2^2 (t - t_5)^2}{2}] \qquad t_4 < t \le t_5$$

$$\frac{dZ(t)}{dt}+\theta_1 Z(t)=-A^{\lambda} f(p) \qquad t_5 < t \le T \qquad \text{.....(6)}$$

$$Z(t)=\frac{A^{\lambda} f(p)}{\theta_1}[-\theta_1 (t-T) + \frac{\theta_1^{\ 2} (t-T)^{\ 2}}{2}] \qquad t_5 < t \le T$$

$$\frac{dZ(t)}{dt}+\theta_1 Z(t)=0 \qquad t_1 < t \le t_5 \qquad \text{.....(7)}$$

$$Z(t)=\alpha[1-\theta_1 (t-t_1) + \frac{\theta_1^{\ 2} (t-t_1)^{\ 2}}{2}] \qquad t_1 < t \le t_5$$

$$\frac{dY(t)}{dt}+\theta_2 Y(t)=0 \qquad t_2 < t \le t_4 \qquad \text{.....(8)}$$

$$Y(t)=\beta\,[1-\theta_2 (t-t_2) + \frac{\theta_2^{\ 2} (t-t_2)^{\ 2}}{2}] \qquad t_2 < t \le t_4$$

Let us assume formathematical simplicity,

$t_1 = U t_2$; $t_2 = V t_3$; $t_3 = W t_4$; $t_4 = X t_5$; $t_5 = YT$
also, $XY = E$; $EW = F$; $VF = G$; $UG = H$.

3.1. Total System Cost:

3.1.1. Holding Cost

Holding cost for OW

$$H_1 = I T^3 + B T^2 + CT$$

$$where\ I = C_1 \left[-\frac{(\theta_1 - 1)^2 H^3}{6} + \frac{\alpha \theta_1^{\ 2} (Y-H)^3}{6} - \frac{A^{\lambda} f(p) \theta_1 (Y-1)^3}{6} \right]$$

$$B = C_1 \left[\frac{M(\theta_1 - 1) H^2}{2} - \frac{\alpha \theta_1 (Y-H)^2}{2} + \frac{A^{\lambda} f(p) (Y-1)^2}{2} \right]$$

$$C = C_1 \left[\alpha (Y-H) \right]$$

Holding cost for RW_1

$$H_2 = J T^3 + K T^2 + LT$$

$$where\ J = C_2 \left[-\frac{(\theta_2 - 1)^2 (G-H)^3}{6} + \frac{\beta \theta_2^{\ 2} (E-G)^3}{6} - \frac{A^{\lambda} f(p) \theta_2 (E-Y)^3}{6} \right]$$

$$K = C_2 \left[\frac{N(\theta_2 - 1)(G-H)^2}{2} - \beta \theta_2 (E-G)^2 + \frac{A^{\lambda} f(p) (E-Y)^2}{2} \right]$$

$$L = C_1 \beta E$$

Holding cost for RW_2

$$H_2 = mT^3 + nT^2$$

$$where\ m = C_3\left[-\frac{(\theta_3-1)^2(F-G)^3}{6} - \frac{A^\lambda\ f(p)\theta_3(F-E)^3}{6}\right]$$

$$n = C_3\left[\frac{R(\theta_3-1)(F-G)^2}{2} + \frac{A^\lambda\ f(p)\ (F-E)^2}{2}\right]$$

Total holding cost

$$HC = (I+J+m)T^3 + (B+K+n)T^2 + (C+L)T$$

3.1.2. Deterioration cost

Deterioration cost for OW

$$DC_1 = D_1[\alpha - A^\lambda\ f(p)\{(H-Y+1)\}T]$$

Deterioration cost for RW_1

$$DC_2 = D_2[\beta - A^\lambda\ f(p)\{(Y-E+G-H)\}T]$$

Deterioration cost for RW_3

$$DC_3 = D_3[\gamma - A^\lambda\ f(p)\{(E-G)\}T]$$

Total deterioration cost

$$DC = D_1\alpha + D_2\beta + D_3\gamma - qT$$

$$where\ \ q = A^\lambda\ f(p)[D_1(H-Y+1) + D_2(Y-E+G-H) + D_3(E-G)]$$

3.1.3. Production Cost

$$PC = P\ (\alpha+\beta+\gamma)$$

Total Inventory Cost PerUnit Time

$$TC = \frac{1}{T}[(I+J+m)T^3 + (B+K+n)T^2 + (C+L)T + D_1\alpha + D_2\beta + D_3\gamma - qT + P\ (\alpha+\beta+\gamma) + C_S]$$

Now, the main objective is to minimize the total cost. The necessary condition for TC(T, p) to be minimum is obtained by differentiating TC(T, p) with respect to 'T' and 'p' and equating to zero. The optimum T and p should satisfy the following conditions:

$$\frac{\partial TC^2(T,p)}{\partial T^2} > 0 \; ; \; \frac{\partial TC^2(T,p)}{\partial p^2} > 0$$

$$\frac{\partial TC(T,p)}{\partial T}\frac{\partial TC(T,p)}{\partial p} - \left(\frac{\partial^2 TC(T,p)}{\partial T \partial p}\right)^2 < 0$$

$$eT^2 + fT + g = 0$$

$$where \quad e = \frac{A^\lambda}{6}[\theta_1(Y-1)^3 C_1 + \theta_2(E-Y)^3 C_2 + \theta_3(F-E)^3 C_3]$$

$$f = \frac{A^\lambda}{2}[-(Y-1)^2 C_1 - (F-E)^2 C_3]$$

$$g = A^\lambda [D_1(H-Y+1) + D_2(Y-E+G-H) + D_3(E-G)]$$

4. NUMERICAL ILLUSTRATION

To illustrate the result obtained for the suggested model, a numerical example with the following parameter values in considered;

$C_1 = 1, \; C_2 = 1.5, \; C_3 = 2.0$

$\theta_1 = 0.40, \; \theta_2 = 0.20, \; \theta_3 = 0.10,$

$D_1 = 2.0, \; D_2 = 1.5, \; D_3 = 1.0,$

$A = 3, \; \lambda = 1$

$C_S = 200$

$\alpha = 250, \; \beta = 150, \; \gamma = 100,$

$a = 0.5, \; b = 500,$

$P = 2$

$U = 0.1, \; V = 0.2, \; W = 0.3, \; X = 0.4, \; Y = 0.5,$

$E = 0.2, \; F = 0.06, \; G = 0.012, \; H = 0.0012,$

5. SENSITIVITY ANALYSIS

A sensitivity analysis is done to know the effect of changes in the system parameters by increasing or decreasing the parameters by 50% to 100% .

Table 1:

Effect of change of parameters on optimum cycle time, optimum system cost and optimum selling price

Changing parameters	% change	T*	TC*	p*	profit
λ	-50%	7.5	373.79	425.89	52.10
	+50%	7.6	375.90	475.77	99.87
	+100%	7.5	378.19	485.76	107.57
U,V,W	-50%	7.2	379.14	451.50	72.36
	+50%	7.7	367.60	458.45	90.85
	+100%	8.0	337.32	458.18	120.86
W	+50%	7.7	364.23	457.90	93.67
V	+50%	7.6	376.83	459.20	82.37
U	+50%	7.6	372.33	457.50	85.17

6. INTERPRETATIONS

1. As advertisement rate 'λ' decreases, the total cycle time and total system cost remains approximately same, but there is a significant decrease in selling price.

   And as we increase the rate of advertisement 'λ', the total cycle time and total system cost remain invariably same but there is increment in the selling price.

   The suggested model is sensitive to the advertisement rate λ. The increment in 'λ' results in increment of the profit whereas total system cost and total cycle time remains almost the same.

2. If the proportion of time for retaining the items in all the three warehouses U, V, Wincreases simultaneously, the total system cost decreases significantly but effect on selling price and total cycle time is insignificant.

3. The analysis of the effect of parameter W (proportion of time retaining the products in RW_2) keeping V and U (i.e. proportion of time spent on holding the product in RW_1 and OW respectively) fixed separately, indicates that the total system cost decreases whereas the total cycle time and selling price remains almost the same.

   If we fix two parameters among U,V,W and change the remaining third one, there is only marginal change in all the optimum values. Thus the model is insensitive to the parameters U,V, W separately.

   From the data set and assumptions taken, it can be concluded that the rate of advertisement can be increased to gain more profit to the system, without affecting the total system cost, unlikely the real situation where the increased rate of advertisement directly increase the total cost of the system.

7. POLICY IMPLICATIONS

The above proposed model is applicable for those industries/organizations which require multiple warehouses to store their inventory. The model is useful for bakery products, dairy items and floral industry etc. which decay rapidly with time.

In order to increase profit, the management should increase the rate of advertisement.

If the stock is held in RW_2 for a longer period, the resultant total system cost is reduced. Management can retain inventory in high rental and low deterioration warehouse for a long period, without increasing total system cost; which is the most striking feature of the suggested model.

The present model is based on the assumption that demand is satisfied from all the three warehouses during the production process but as soon as the capacity of the respective warehouses reaches to its maximum level, then the inventory is firstly depleted from RW_2. Since the stock can be held as RW_2 therefore the inventory can be supplied from the other warehouses before RW_2, without appreciably affecting the total system cost.

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