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SOFT BITOPOLOGICAL SPACES

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ABSTRACT

In this paper introduce and study the concept of soft bitopological spaces which are defined over an initial universe with a fixed set of Parameters .Also introduce and investigate some new separation axioms called pairwise soft T_0,pairwise soft T_1 and pairwise soft T_2 spaces and study some of their basic properties in soft bitopological spaces.

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Index Terms:
Soft sets ,Soft topology,Soft bitopology.

1.INTRODUCTION

In this year 1999,Russian researcher Molodtsov (6),initiated the concept of soft sets as a new mathematical tool
to deal With uncertainities while modeling problems in engineering physics ,computer science ,economics,social siecnces and medical sciences.In 2003,Maji , Biswas and Roy(5),studied the theory of soft sets,subset and super set of a soft set with examples.Soft binary operations like AND ,OR and also the operations of union and intersection were also defined .In 2005 ,D.Chen(2),presented a new definition of soft set parametrization reduction in rough set theory.

Topological structures of soft set have been studied by some authors in recent years.In 2011,M.Shabir et al. and Naim Cagman et al .initiated the study of soft topology and soft topological spaces independently.M.Sahbir and M.Naz (7),introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters and showed that a soft topological gives a parameterized family of topological space.They introduced the definition of soft separation axioms.Also they obtained some interesting results for soft separation axiomsvaluable for which are really researchin this field.N.Cagman ,S.Karatas and S.Enginoglu (1),defined the soft topology on a soft set and presented the related properties and foundation s of the theory of soft topologicalspaces.In 1963,J C Kelly (4),first initiated the concept of bitopological spaces.He defined a bitopological space
(X,τ_1,τ_2,E) to a set X equipped with two topologies τ_1 and τ_2 on X and initiated the systematic study of bitopological space .Later work done by C.W.Patty (8),I.L.Reilly (9)and others.Reilly discussed separation axioms properties in bitopological spaces.

The following definitions which are prerequisties for present study.

1.1 Definition(6): Let U be an initial universe and E be a set of parameters.Let P(U) denote the power set of U and A be a non empty subset of E.A pair (F,A)is called a soft set over U,where F is a mapping given by $F:A\rightarrow P(U)$.

In other words ,a soft set over U is a parametrized family of subsets of the universe U.For eεA,F(e) may be considered as the e-approximate elements of the soft set (F,A).Clearly,a soft set not a set.

1.2 Definition (5): The complement of a soft (F,A) is denoted by $(F,A)^c$ and is defined by $(F^c,\urcorner A)$ where $F^c:\urcorner A\rightarrow P(U)$ is a mapping given by $F^c(\alpha)=U-F(\alpha)$ for all $\alpha\varepsilon\urcorner A$.

Let us call F^c to be the soft complement function of F.clearly($F^c)^c$ is the same as F and($(F,A)^c)^c$ =(F,A)

1.3 Definition (5):A soft set (F,A) over U is said to be a NULL soft set denoted by Φif for all e εA, F(e)=Φ(null set).

1.4 Definition (5):The union of two soft sets of (F,A) and (G,B) over the common universe U,is the soft set (H,C),where C=A∪B and for all eεE,

$$H(e)=\begin{cases} F(e) if e\varepsilon A-B \\ G(e) if\ e\varepsilon B-A \\ F(e)\cup G(e) if e\varepsilon A\cap B \end{cases}$$

And is written as (F,A) ∪(G,B)=(H,C).

1.5 Definition (3): The intersection (H,C)of two soft sets (F,A) and (G,B) over a common universe U,denoted by (F,A)∩(G,B),is defined as C=A∩B, and H(e) = F(e)∩ G(e) for all eεC.

1.6 Definition (7): Let (F,E)be a soft set over X and xεX..we say that xε(F,E) read as x belongs to the soft set (F,E) whenever xεF(α) for all $\alpha\varepsilon$E.

Note that for any xεX,x∉(F,E) if x∉ F(α) for some $\alpha\varepsilon$E .

1.7 Definition(7): Let Y be a non empty subset of X ,then Y denotes the soft set (Y,E) over X for which Y(α) = Y for all $\alpha\varepsilon$E

1.8 Definition (7): Let x εX ,then (x,E) denotes the soft set over X for which x(α)={x},for all $\alpha\varepsilon$E.

1.9 Definition(7): Let (F,E) be a soft set over X and Y be a non empty subset of (F,E) over Y denoted by $({}^YF,E)=Y\cap(F,E)$.

1.10 Definition(7): The relative complement of a soft set (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c=(F^c,A)$ where $F^c:A\longrightarrow P(U)$ is a mapping gives by $F^c(\alpha)=U-F(\alpha)$ for all $\alpha\varepsilon A$.

1.11Definition (7): Let τ be the collection of a soft sets over X ,then τ is said to be a soft topology on X,if

i.ϕ,X belongs to τ

ii)the union of any number of soft sets in τ belongs to τ.

iii)the intersection of any two soft sets in τ belongs to τ.

1.12 Definition(7): Let (X, τ,E) be a soft topological space over X,then the number τ are said to be soft open sets in X.

1.13 Definition(7): Let (X, τ,E) be a soft topological space over X.A soft open set (F,E) over X .A soft open set (F,E) over X .A soft open set (F,E) over X is said to be a soft closed set in X,if its relative complement $(F,E)^c$ belongs to τ .

1.14 Definition(7): Let (X ,τ,E) be a soft topological space over X and Y be a non empty subset of X .Then $\tau_y=\{({}^YF,E) : (F,E)\in\tau\}$ is said to be the soft relative topology on Y and (Y, τ_{Y},,E) is called a soft subspace of

(X ,τ,E).

1.15 Definition(7): Let (X ,τ,E) be a soft topological space over X and x,y∈X such that x≠y.

i)If there exists soft open sets (F,E) and (G,E) such that x∈ $(F,E)$and y∉ (F,E) or y∈(G,E) and x∉(G,E) then (X, τ,E) is called a soft T_0 space.

ii)If there exists soft open sets (F,E) and (G,E) such that x∈ (F,E) and y∉ (F,E) and y∈(G,E) and x∉(G,E) then (X, τ,E) is called a soft T_1 space.

iii)If there exists soft open sets (F,E) and (G,E) such that x∈(F,E) ,y∈(G,E) and (F,E)∩(G,E) =∅ , then (X, τ,E) is called a soft T_2 space.

1.16 Definition(4): Let X be a non-empty set and τ_1 ,τ_2 be two different topologies on X .Then (X,τ_1,τ_2) is called a bitopological space.

2.SOFT BITOPOLOGICAL SPACES

Let X be an initial universe set and E be the non-empty set of parameters.

2.1Definition :Let (X,τ_1,E) and (X,τ_2,E) be the two different soft topologies on X .Then

(X,τ_1, τ_2,E)is called a soft bitopological space.

The two soft topologies(X,τ_1,E) and (X,τ_2,E) are independently satisfy the axioms of soft topology.The members of τ_1 are called τ_1 soft open sets and the complements of τ_1 soft closed sets.

Similarly ,The members of τ_2 are called τ_2 soft open sets and the complements of τ_2 soft open sets are called τ_2 soft closed sets.

Throughout this paper (X,τ_1,τ_2,E) denote soft bitopological space over X on which no separation axioms are assumed unless explicitly stated.

2.2 Example:

Let $X=\{h_1,h_2,h_3,h_4\}$,$E=\{e_1,e_2,e_3\}$, $\tau_1=\{X, \emptyset,(F_1,E),(F_2,E),(F_3,E)\}$,

$\tau_2 = \{X, \emptyset,(G_1,E),(G_2,E),(G_3,E),(G_4,E)\}$

where

$(F_1,E),(F_2,E),(F_3,E)$, $(G_1,E),(G_2,E),(G_3,E),(G_4,E)$ are soft sets over X .Defined by

$F_1(e_1)= \{h_4\},F_1(e_2)=\{h_3,h_4\},F_1(e_3)=X$

$F_2(e_1)=\{h_1,h_2\},F_2(e_2)=\{h_1,h_2,h_3\}$ $F_2(e_3)=\{h_4,h_1\}$

$F_3(e_1)=\{h_2,h_3,h_4\}$ $F_3(e_2)=\{h_1,h_2\}$

$G_1(e_1)= \{h_3,h_4,h_1\}$ $G_1(e_2)=\{h_2,h_3,h_1\}$

$G_2(e_1)=\{h_3\},G_2(e_2)=\{h_4\},G_2(e_3)=\{h_1,h_2\},G_3(e_1)=\{h_2,h_3,h_4\}$ $,G_3(e_2)=\{h_2,h_3\},G_3(e_3)=\{h_4,h_1\}$

$G_4(e_1)=\{h_1,h_2,h_3\},G_4(e_1)=\{h_1,h_2,h_3\}$

$G_4(e_2)=\{h_3\},G_4(e_3)=\{h_1,h_2\}$

Then (X,τ_1, τ_2,E) is soft bitopology over X.

2.3 Theorem

Every soft T_0 – space is T_1 – space .

Proof

Let (X, τ,E) be a soft topological space over X and $x,y\in X$ such that $x\neq y$. (X, τ,E) is soft T_0-space.

If (X, τ,E) is soft T_0 – space there exists soft open sets (F,E) and (G,E) such that $x\in(F,E)$ and $y\notin(F,E)$ or $y\in(G,E)$ and $x\notin(G,E)$

Obviously, (X, τ,E) is soft T_1 – space.

2.4 Theorem

Every soft T_1 space is soft T_2 space .

Proof

Let (X, τ,E) be a soft topological space over X and $x,y\in X$ such that $x\neq y$.

(X, τ,E) is a soft T_1-space.

If (X, τ,E) is soft T_1 – space there exists soft open sets (F,E) and (G,E) such that $x \in(F,E)$ and $y\notin(F,E)$ and $y\in(G,E)$ and $x\notin(G,E)$

Here $(F,E)\cap(G,E)= \emptyset$.

(X, τ,E) is soft T_2- space.

2.5 Theorem

Every soft T_0 – space is soft T_1-space.

Proof

Let (X, τ,E) be a soft topology space over X and x,y∈X and x≠y.

Since (X, τ,E) is a soft T_0 – space there exists soft open sets (F,E) and (G,E) such that x ∈(F,E) and y∉(F,E) and y∈(G,E) and x∉(G,E)

Here x∈(F,E) and y∉(F,E) and y∈(G,E) and x∉(G,E)

Hence (X, τ,E) is a soft T_1- space.

2.6 Theorem

Every soft T_0 – space is soft T_2-space.

Proof

Let (X, τ,E) be a soft topology space over X and x,y∈X and x≠y.

Since (X, τ,E) be a soft T_0-space (G,E) such that that x ∈(F,E) and y∉(F,E) or y∈(G,E) and x∉(G,E)

(F,E)∩(G,E)=∅

(X, τ,E) is a soft T_2 – space.

3.SEPARATION AXIOMS OF SOFT BITOPOLOGICAL SPACES

3.1 Definition : In a soft bitopological space (X,τ_1, τ_2,E)

i) τ_1 is said to be soft T_0 space w.r.to τ_2 if for each distinct points x,y of X then there exist a τ_1 soft open set (F,E) and a τ_2 soft open set (G,E) such that x∈(F,E) and y∉(F,E) or y∈(G,E) and x∉(G,E).

Similarly τ_2 is said to be soft T_0 space w.r.to τ_1 if for each distinct points x,y of X then there exists a τ_2 soft open sets (F,E) and a τ_2 soft open set (G,E) such that x∈(F,E) and y∉(F,E) or y∈(G,E) and x∉(G,E).

(X,τ_1, τ_2,E) is said to be pairwise soft T_0 space if τ_1 is soft T_0 space w.r.to τ_2 and τ_2 is soft T_0 space w.r.to τ_1 .

ii) τ_1 is said to be soft T_1 space w.r.to τ_2 if for each distinct points x,y of X then there exist a τ_1 soft open set (F,E) and a τ_2 soft open set (G,E) such that x∈(F,E) and y∉(F,E) and y∈(G,E) and x∉(G,E).

Similarly τ_2 is said to be soft T1 space w.r.to τ_1 if for each distinct points x,y of X then there exists a τ_2 soft open sets (F,E) and a τ_1 soft open set (G,E) such that x∈(F,E) and y∉(F,E) and y∈(G,E) and x∉(G,E).

(X,τ_1, τ_2,E) is said to be pairwise soft T_1 space if τ_1 is soft T_1 space w.r.to τ_2 and τ_2 is soft T_1 space w.r.to τ_1 .

iii) τ_1 is said to be soft T2 space w.r.to τ_2 if for each distinct points x,y of X then there exist a τ_1 soft open set (F,E) and a τ_2 soft open set (G,E) such that x∈(F,E) y∈(G,E) and (F,E)∩(G,E)=∅

Similarly τ_2 is said to be soft T2 space w.r.to τ_1 if for each distinct points x,y of X then there exists a τ_2 soft open sets (F,E) and a τ_1 soft open set (G,E) such that x∈(F,E) , y∈(G,E) and (F,E)∩(G,E)=∅

(X,τ_1, τ_2,E) is said to be pairwise soft T_2 space if τ_1 is soft T_2 space w.r.to τ_2 and τ_2 is soft T_2 space w.r.to τ_1 .

3.2 Theorem

Every pairwise soft T_1 space is pairwise soft T_2 space.

Proof

Let (X,τ_1,τ_2,E) be a soft bitopological space over X and $x,y\in X, x\neq y$.

If (X,τ_1,τ_2,E) is a pairwise soft T_1 space.

τ_1 is soft T_1 space w.r.to τ_2 and τ_2 is soft T_1 space w.r.to τ_1 .

If τ_1 is soft T_1 space w.r.to τ_2 then there exists a τ_1 soft open set (F,E) and a τ_2 soft open set (G,E)such that $x\in(F,E)$ and $y\notin(F,E)$ and $y\in(G,E)$ and $x\notin(G,E)$.

$(F,E)\cap(G,E)=\emptyset$

τ_1 is soft T_2 space w.r.to τ_2.

Similarly,If τ_2 is soft T_1 space w.r.to τ_1.

Then there exits τ_2 is soft open set (F,E) and τ_1 soft open set (G,E).

$x\in(F,E)$ and $y\notin(F,E)$ and $y\in(G,E)$ and $x\notin(G,E)$

$(F,E)\cap(G,E)=\emptyset$

Hence τ_2 is soft T_2 space w.r.to τ_1 .

Therefore(X,τ_1,τ_2,E) is a soft T_2 space.

3.3 Theorem

Let (X,τ_1,τ_2,E) be a soft bitopological space over X.If (x,E) is a soft closed set in τ_2 for each $x\in X$ (y,E) is a soft closed set in τ_1 for each $y\in X$ then (X,τ_1,τ_2,E) is pairwise soft T_2 space.

Proof

Suppose that for each $x\in X$.

(X,E) is a soft closed set in τ_2 for each $x\in X$.

$(x,E)^c$ is a soft open set in T_2.

Let $x,y\in X$ such that $x\neq y$.For each $x\in X$,$(x,E)^c$ is a soft open set in τ_2 suchthat $y\in(x,E)^c$ and $x\notin(x,E)^c$.

Similarly ,(y,E) is a soft closed set in τ_1 for each $y\in X$.

$(y,E)^c$ is a soft open set in τ_1 such that $x\in(y,E)^c$ and $y\notin(y,E)^c$

Therefore ,$(x,E)^c\cap(y,E)^c=\emptyset$

Hence (X,τ_1,τ_2,E) is pairwise soft T_2 – space.

3.4 Theorem

Let (X,τ_1,τ_2,E) be a soft bitopological space over X.If (x,E) is a soft closed set in τ_2 for each $x\in X$ and (y,E) is a soft closed set in τ_1 for each $y\in X$ then (X,τ_1,τ_2,E) is pairwise soft T_0 space.

Proof

Let (X,τ_1,τ_2,E) be a soft bitopological space over X.

(x,E) is a soft closed set in τ_2 for each $x\in X$.

$(x,E)^c$ is a soft open set in τ_2 .

Similarly ,(y,E) is a soft closed set in τ_1 for each y∈X.

(y,E)^c is a soft open set in τ_1 for each y∈X.

Such that x∈(y,E)^c and y∉(y,E)^c

Therefore (x,E)^c∩(y,E)^c =∅

Hence (X,τ_1, τ_2,E) is a pairwise T_0 space.

3.5Theorem

Every pairwise soft T_0 space is pairwise soft T_2 space.

Proof

Let (X,τ_1, τ_2,E) be a soft bitopological space over X and x,y∈X such that x≠y.

If (X,τ_1, τ_2,E) is pairwise soft T_0 space if τ_1 is soft T_0 space w.r.to τ_2 and τ_2 is soft T_0 space w.r.to τ_1.

If τ_1 is soft T_0 space w.r.to τ_2 then there is a τ_1 soft open set (F,E) and a τ_2 soft open set (G,E) suchthat x∈(F,E) and y∉(F,E) or y∈(G,E) and x∉(G,E).

Hence (F,E) ∩(G,E)= ∅

Similarly , if τ_2 is said to be soft T_0 space w.r.to τ_1 then there exists a τ_2 soft openset (F,E) and a τ_1 soft openset (G,E) suchthat x∈(F,E) and y∉(F,E) or y∈(G,E) and x∉(G,E).

Obviously (F,E)∩(G,E)=∅

Hence (X,τ_1, τ_2,E) is pairwise soft T_2 space.

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