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Revisiting The Collatz Conjecture

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Abstract:

Lother Collatz introduced a Conjecture considered an extraordinary difficult problem. In this Conjecture, a series of numbers get generated starting from any natural number ‘n’ called hailstone numbers and it was claimed that all number eventually terminated with 1. In 2021 it was proved that the Conjecture is true and all numbers eventually ended with 1, this paper is the revisiting of the Collatz Conjecture.

Keywords: Collatz Conjecture, Theorem, hailstone numbers, Euclid’s division

Historical Overview

Lothar Collatz was a mathematician from Germany, born in Arnsberg, Westphalia (July 6, 1910 – September 26, 1990). Lother Collatz studied at Universities in Germany including University of Greifswald, University of Berlin, where Alfred Klose supervised him and received his doctorate in 1935 for “ The Finit Difference Method with Higher Approximation for Linear Differential Equations. Working in University of Berlin as assistant and then moved to Technical University of Karlsrue. Collatz held a chair at the Technical university of Hanover from 1943 to 1952 and at the University of Hamburg he get his superannuation and was honored as Professor emeritus. He died from heart attack in Varna, Bulgaria, while attending a mathematics conference.

In 1931 Lother Collatz, proposed a Conjecture namely Collatz Conjecture also called as (3n + 1) Conjecture [1]. This Conjecture states that, if we take any number ‘n’, and it is odd then proceed as (3n + 1) otherwise divide it by 2, till the sequence ended by number 1, a series of numbers get generated called hailstone numbers. The longest progression for initial starting number is less than 10 billion and 100 quadrillion are calculated [2, 3]. In spite of today’s hypothetical inability to prove Collatz Conjecture, several papers have outlined discoveries related towards the actual impracticable proofs. Conceivably the most famous recent development was made by Terence Tao, who showed that most orbits of the Collatz map attain almost bounded values [4]. An magnificent reconsideration of his paper was published in the College Mathematics Journal [5].

Several research papers have outlined various uncreative paths that mathematicians have taken to solve the Conjecture [7-9]. In 2021 [6], an approach was given how to proceed and shows that Collatz Conjecture is true for all number. Here in this paper a proper profile with the help of a theorem is explained.

METHOD

Let N be the set of all positive integers and n Ɛ N, then Collatz Conjecture is defined as:

$$T(n) = \{ n/2 \ \text{(if ‘n’ is even) or} \quad 3n+1 \ \text{(if ‘n’ is odd)}\}$$

T(n) – Terms in the sequence.

Starting from any number ‘n’ (even or odd) moving as per the procedure, hailstorm number gets generated and will eventually end with 1.

Theorem: Prove that Collatz Conjecture is true.

Proof:

We have to prove that, if we take any number (even or odd), and proceed as Collatz suggested, all numbers eventually ended to number 1 and there is no possibility of infinite loop for any number.

Let "N" be the set of all positive integers and 'n' be any number in 'N' (n Ɛ N)

Three cases are possible.

1. 'n' be an even number and is in the form of (2^k, kƐN), successive division of this number by 2 ended to 1. (Euclid's division lemma).
2. 'n' be an even number and is not in the form of (2^k), successive division of this number by 2 reaches to an odd number.
3. 'n' be an odd number.

There is no problem in case (1) but case (2) and (3) ends/starts with an odd number then the next number will be (3n +1) which is an even number, if this even number is in the form of (2^k , k Ɛ N) then case (1) otherwise continue same process.

Let j = 1,2,3,…. for all 'n' (where 'n' is odd numbers 'n' > 1)

This can be written as

$$j = (n-1)/2$$

Now for all odd numbers, new hailstone numbers are

(a) (3n +1)
(b) (3n+1)/2

We can see that

3n +1 > 2n …….. **I** (it is always true for all 'n')

and **(3n+1)/2 = (4n – n +1)/2**

= 2n – j < 2n ……. II (for all 'n')

From I and II, the new number formed never be equal to '2n'

Hence number never be repeated.

This proved that the Collatz Conjecture never enters into looping and it always ends to 1.

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