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TOTALLY REGULAR PROPERTY OF THE JOIN & UNION OF THREE FUZZY GRAPHS

A.T.ASLENA AND Dr.N.ARUL PANDIAN AND A.RASATHI

PG Department of Mathematics, Naina Mohamed College Of Arts And Science, Rajendrapuram

ABSTRACT:

In general, the join & union of three totally regular fuzzy graphs need not be a totally regular fuzzy graph. In this paper, necessary and sufficient conditions for the join & union of three totally regular fuzzy graphs to be totally regular under some restrictions are obtained.

KEYWORDS:

Total degree of a vertex, regular fuzzy graph, join,union

1.INTRODUCTION

A Fuzzy subset of a set V is a mapping σ from V to [0,1]. A fuzzy graph G is a pair of functions G:(σ,μ)where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ, satisfying $\mu(uvw)\leq \sigma(u)\wedge\sigma(v)\wedge\sigma(w)$. The underlying crisp graph of G:(σ,μ) is denoted by G*:(V,E) where E V×V. Fuzzy graph theory was introduced by Rosenfeld in 1975. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various concepts in connectedness in fuzzy graphs. Mordeson and peng introduced the concept of operations on fuzzy graphs.

The operations of union,join on three fuzzy graphs were defined by Mordeson and Peng. Sunitha and Vijayakumar discussed about the complement of the operations of union, join on three fuzzy graphs. The Regular property of fuzzy graphs which are obtained from three given fuzzy graphs using the operations union,join was discussed by Nagoorgani and Radha. In this paper we study about the totally regular property of the join & union of three fuzzy graphs and the number of vertices in the fuzzy graphs G_1,G_2 and G_3 are denoted by p_1,p_2 and p_3 respectively.

2. BASIC DEFINITIONS:

DEFINITION 2.1.

The order of a fuzzy graph G is defined by

$$O(G) = \sum_{u\epsilon v} \sigma(u)$$

DEFINITION 2.2.

Let G:(σ,μ) be a fuzzy graph. The degree of a vertex u in G is defined by

$$d_G(u) = \sum_{u\neq v} \mu(uv) = \sum_{uv\in E} \mu(uv)$$

DEFINITION 2.3.

Let G:(σ,μ) be a fuzzy graph on G*. The total degree of a vertex u∈v is defined by

$$td_G(u) = \sum_{u\neq v} \mu(uv) + \sigma(u) = d_G(u)+\sigma(u)$$

If each vertex of G has the same total degree K, then G is said to be a totally regular fuzzy graph of total degree K.

Or a K-totally regular fuzzy graph.

JOIN 2.4.

Assume that $V_1 \cap V_2 \cap V_3 = \varphi$. The join (sum) of G_1,$G_2$and G_3 is ∪G = G1+G2+G3 : $(\sigma_1+\sigma_2+\sigma_3, \mu_1+\mu_2+\mu_3)$ on G*:(V,E) where $V = V_1 \cup V_2 \cup V_3$ and $E = E_1 \cup E_2 \cup E_3 \cup E'$ where E' is the set of all edges joining vertices 0f V_1 with vertices of V_2 with vertices of V_3 with

$(\sigma_1+\sigma_2+\sigma_3)(u) = (\sigma_1 \cup \sigma_2 \cup \sigma_3)(u)$ for all $u \in V_1 \cup \mathbf{V_2} \cup \mathbf{V_3}$

$=\sigma_1(u)$ if $u \in V_1$

$=\sigma_2(u)$ if $u \in V_2$

$=\sigma_3(u)$ if $u \in V_3$

$$(\mu_1+\mu_2+\mu_3)(uvw) = \begin{cases} (\mu_1 \cup \mu_2 \cup \mu_3)(uvw) & \text{if } uvw \in E1 \cup E2 \cup E3 \\ \sigma_1(u) \wedge \sigma_2(v) \wedge \sigma_3(w) & \text{if } uvw \in E' \end{cases}$$

$= \mu_1(uvw)$ if $u \in V_1$ (i.e) if $uvw \in E_1$

$= \mu_2(uvw)$ if $u \in V_2$ (i.e) if $uvw \in E_2$

$= \mu_3(uvw)$ if $u \in V_3$ (i.e) if $uvw \in E_3$

UNION 2.5.

The union of three fuzzy graphs G_1,G_2 and G_3 is defined as a fuzzy graph $G = G_1 \cup G_2 \cup G_3 : (\sigma_1 \cup \sigma_2 \cup \sigma_3, \mu_1 \cup \mu_2 \cup \mu_3)$ on $G^* : (V,E)$ where $V= V_1 \cup V_2 \cup V_3$ and $E= E_1 \cup E_2 \cup E_3$ with

$$(\sigma_1 \cup \sigma_2 \cup \sigma_3)(u) = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 - V_2 \\ \sigma_2(u) & \text{if } u \in V_2 - V_3 \\ \sigma_3(u) & \text{if } u \in V_3 - V_1 \\ \sigma_1(u) \vee \sigma_2(u) \vee \sigma_3(u) & \text{if } u \in V1 \cap V2 \cap V3 \end{cases}$$

and
$$(\mu_1 \cup \mu_2 \cup \mu_3)(e) = \begin{cases} \mu_1(e) & \text{if } e \in E_1 - E_2 \\ \mu_2(e) & \text{if } e \in E_2 - E_3 \\ \mu_3(e) & \text{if } e \in E_3 - E_1 \\ \mu_1(e) \vee \mu_2(e) \vee \mu_3(e) & \text{if } e \in E1 \cap E2 \cap E3 \end{cases}$$

3.TOTAL DEGREE OF A VERTEX IN JOIN :

Here $V_1 \cap V_2 \cap V_3 = \varphi$. Hence $E_1 \cap E_2 \cap E_3 = \varphi$

By definition,

$td_{G1+G2+G3}(u) = \sum_{uvw\in E1\cup E2\cup E3}(\mu 1 \cup \mu 2 \cup \mu 3)(uvw)$

$+ \sum_{uvw\in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w) + (\sigma_1 \wedge \sigma_2 \wedge \sigma_3)(u)$

For any $u \in V_1$,

$td_{G1+G2+G3}(u) = \sum_{uvw\in E1} \mu 1(uvw) + \sum_{uvw\in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w) + \sigma_1(u)$

$= d_{G1}(u) + \sigma_1(u) + \sum_{uvw\in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$

$= td_{G1}(u) + \sum_{uvw\in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$ (3.1)

For any $u \in V_2$,

$td_{G1+G2+G3}(u)$

$= \sum_{uvw\in E2} \mu 2(uvw) + \sum_{uvw\in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w) + \sigma_2(u)$

$= d_{G2}(u) + \sigma_2(u) + \sum_{uvw\in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$

$= td_{G2}(u) + \sum_{uvw\in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$ (3.2)

Similarly,

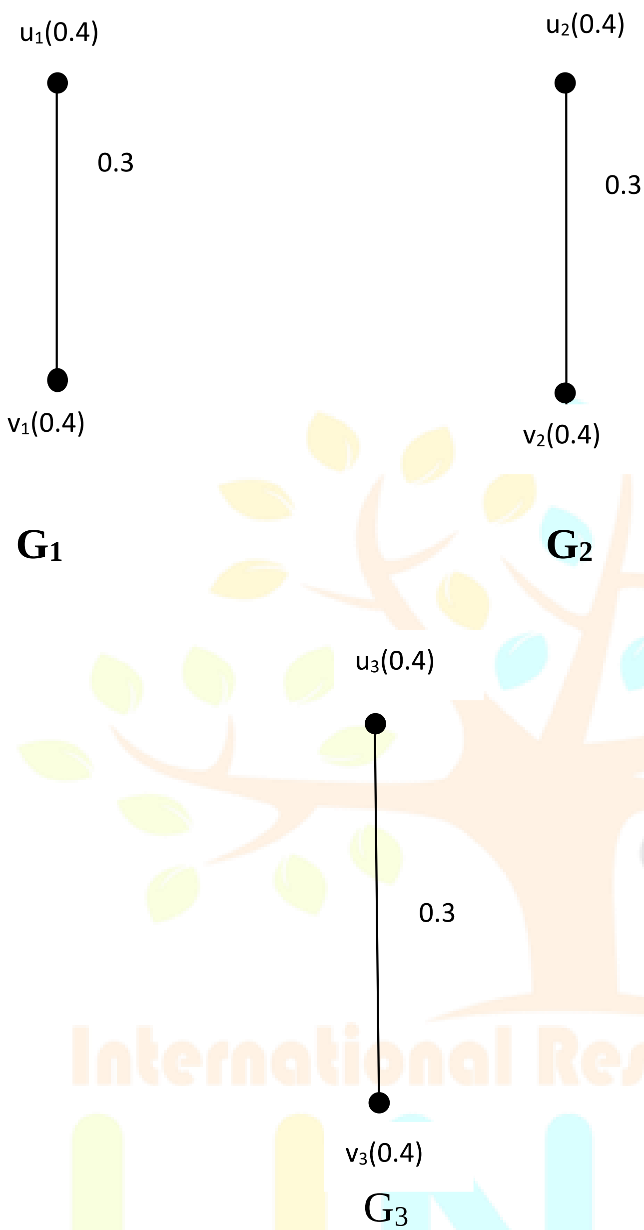
For any $u \in V_3$,

$td_{G1+G2+G3}(u) = td_{G3}(u) + \sum_{uvw\in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$ (3.3)

3.1.TOTALLY REGULAR PROPERTY OF THE JOIN :

If $G_1+G_2+G_3$ is totally regular fuzzy graph then $G_1$is a totally regular fuzzy graph, $G_2$is a totally regular fuzzy graph and $G_3$is a totally regular fuzzy graph in fig

1.



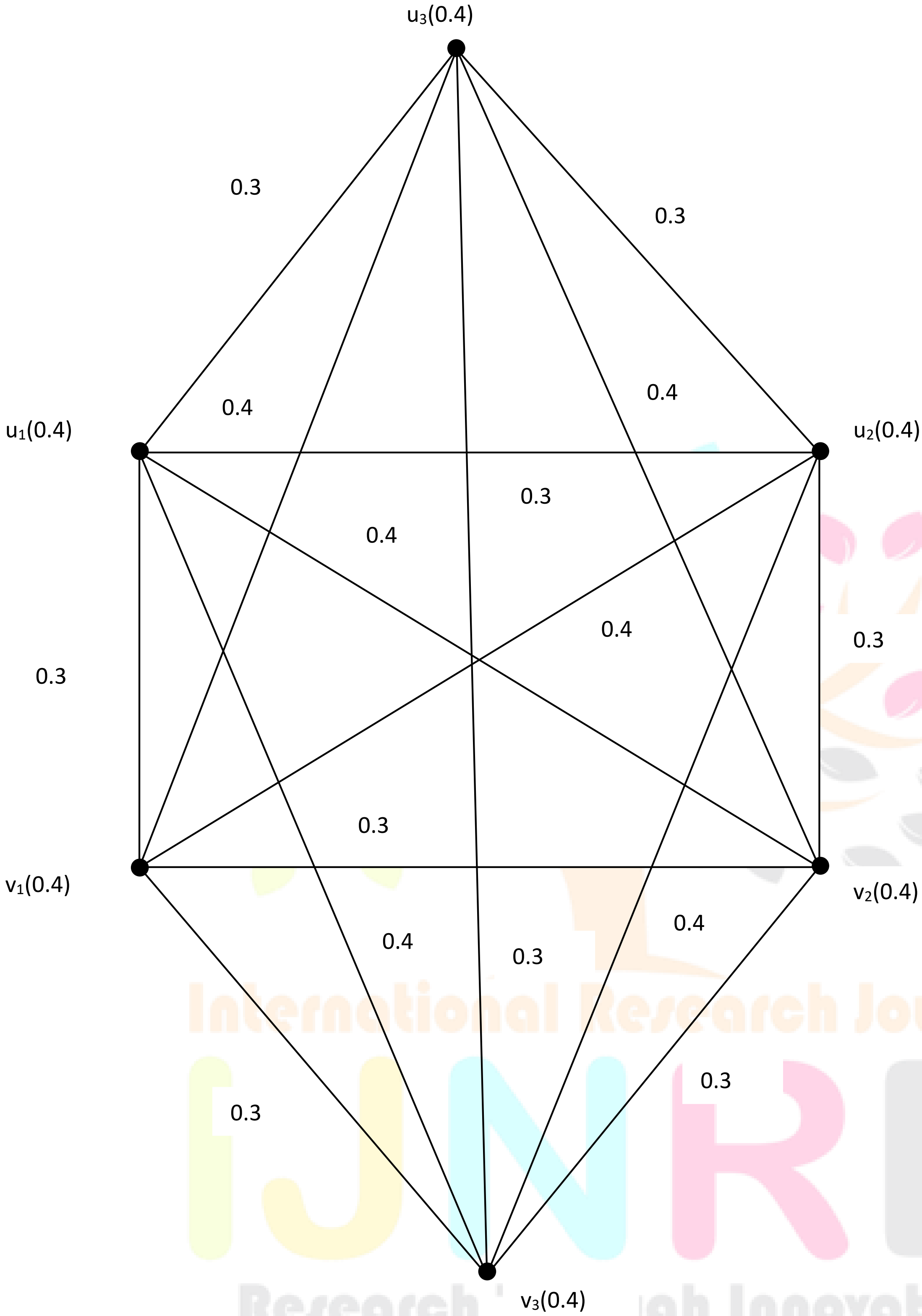


Figure 1 : $G_1+G_2+G_3$

Similarly if $G_1$is not a totally regular fuzzy graph, $G_2$is a totally regular fuzzy graph and $G_3$is a totally regular fuzzy graph, then $G_1+G_2+G_3$ is a totally regular fuzzy graph in fig 2.

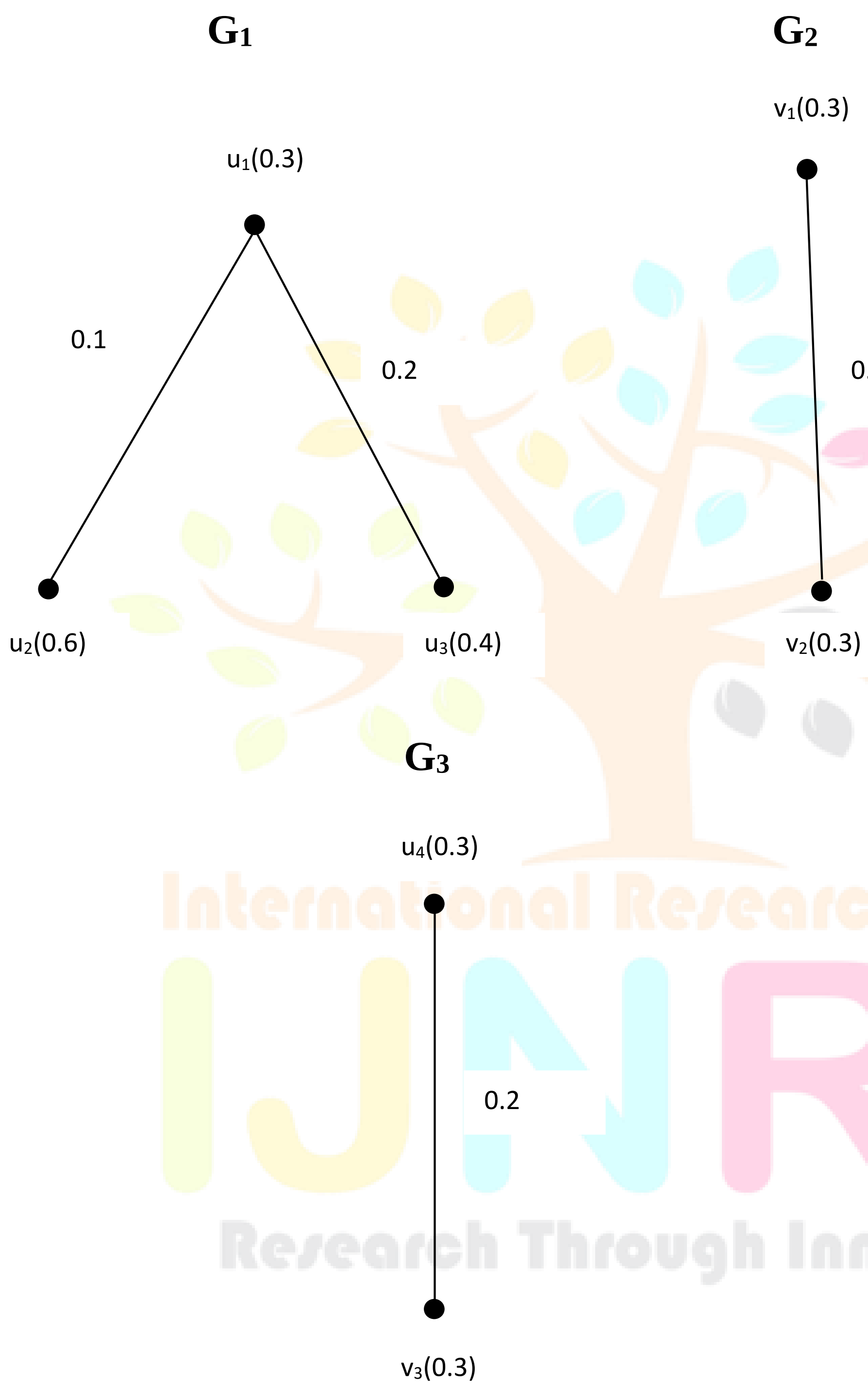
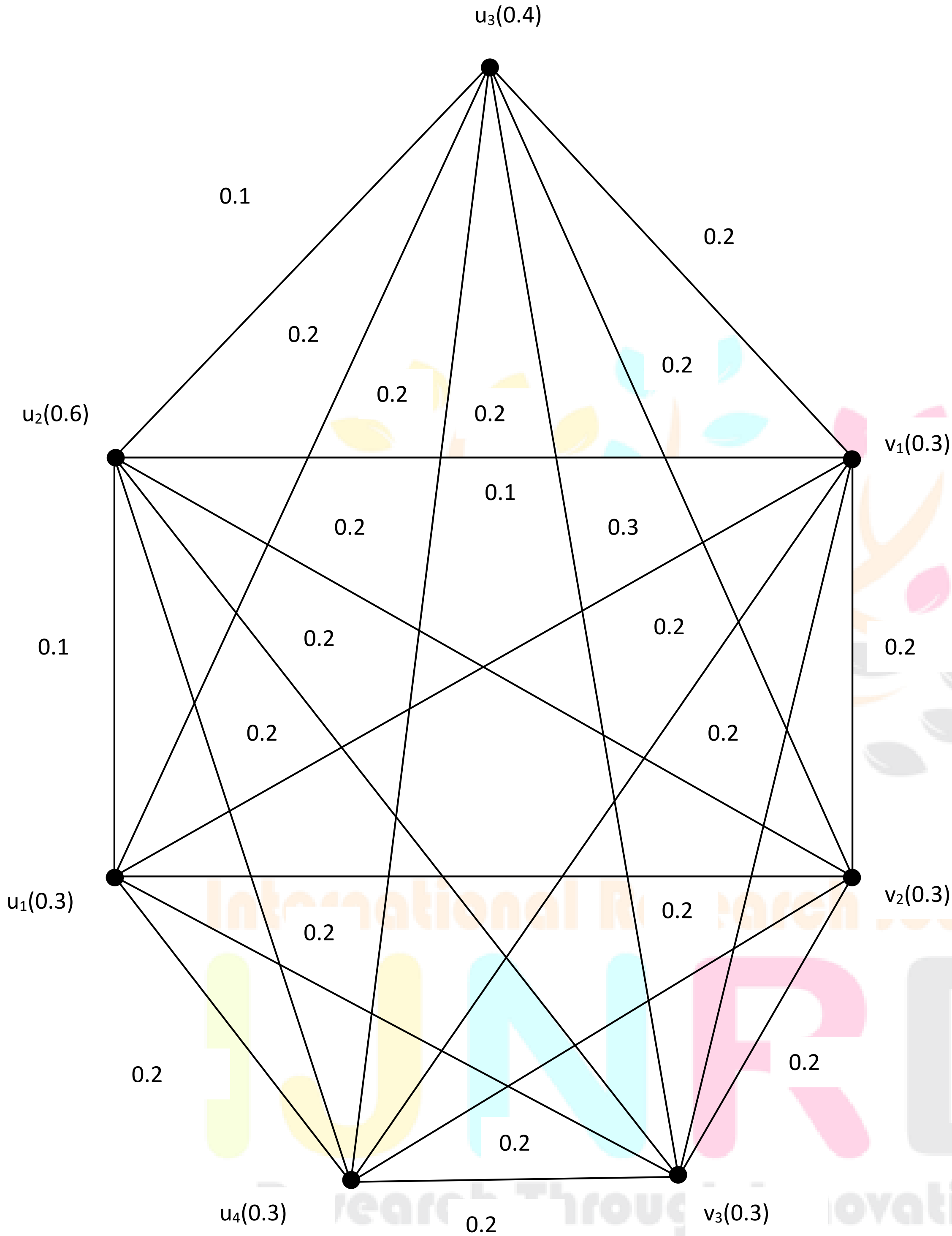


Figure 2 : $G_1+G_2+G_3$



In the following theorems, we obtain necessary and sufficient conditions for the join of three fuzzy graphs to be totally regular in some particular cases.

THEOREM 3.2 :

Let G_1, G_2 and G_3 be totally regular fuzzy graphs of the same degree such that $\sigma_1 \wedge \sigma_2 \wedge \sigma_3$ is constant function. Then $G_1+G_2+G_3$ is totally regular fuzzy graph if and only if $p_1 = p_2 = p_3$.

Proof :

Let G_1, G_2 and G_3 be K- totally regular fuzzy graphs. Let $(\sigma_1(u) \wedge \sigma_2(v) \wedge \sigma_3(w)) = C$ for all $u \in V_1$, $v \in V_2$ and $w \in V_3$, where C is a constant. For any $u \in V_1$, from (3.1)

$$td_{G1+G2+G3}(u) = td_{G1}(u) + \sum_{uvw \in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$$

$$= td_{G1}(u) + \sum_{v \in V3} C = k + cp3 \quad (3.4)$$

For any $u \in V_2$, from (3.2)

$$td_{G1+G2+G3}(u) = td_{G2}(u) + \sum_{uvw \in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$$

$$= td_{G2}(u) + \sum_{v \in V2} C = k + cp2 \quad (3.5)$$

For any $u \in V_3$, from (3.3)

$$td_{G1+G2+G3}(u) = td_{G3}(u) + \sum_{uvw \in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$$

$$= td_{G3}(u) + \sum_{v \in V1} C = k + cp1 \quad (3.6)$$

From (3.4), (3.5) and (3.6), $G_1+G_2+G_3$ is a totally regular fuzzy graph.

$$\Leftrightarrow k + cp3 = k + cp2 = k + cp1 \Leftrightarrow p_3 = p_2 = p_1$$

Hence the theorem.

THEOREM 3.3 :

Let G_1, G_2 and G_3 be three fuzzy graphs such that $p_1 = p_2 = p_3$ and $\sigma 1 \wedge \sigma_2 \wedge \sigma_3$ is constant function. Then $G_1+G_2+G_3$ is totally regular fuzzy graph, if and only if G_1, G_2 and G_3 are both totally regular fuzzy graphs of the same degree.

Proof :

Let G_1, G_2 and G_3 be three fuzzy graphs, such that $p_1 = p_2 = p_3 = p$ (say) and for any $u \in V_1$, from definition (3.1)

$$td_{G1+G2+G3}(u) = td_{G1}(u) + \sum_{uvw \in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$$

$$= td_{G1}(u) + \sum_{v \in V3} C = tdG1(u) + cp \quad (3.7)$$

For any $v \in V_2$, from definition (3.2)

$$td_{G1+G2+G3}(v) = td_{G2}(v) + \sum_{uvw \in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$$

$$= td_{G2}(v) + \sum_{v \in V2} C = tdG2(v) + cp \quad (3.8)$$

For any $w \in V_3$, from definition (3.3)

$$td_{G1+G2+G3}(w) = td_{G3}(w) + \sum_{uvw \in E'} \sigma 1(u) \wedge \sigma 2(v) \wedge \sigma 3(w)$$

$$= td_{G3}(w) + \sum_{v \in V1} C = tdG3(w) + cp \quad (3.9)$$

From (3.7), (3.8) and (3.9)

$G_1+G_2+G_3$ is a totally regular fuzzy graph

$\Leftrightarrow tdG1(u) + cp = tdG2(v) + cp = tdG3(w) + cp$

$\Leftrightarrow tdG1(u) = tdG2(v) = tdG3(w)$

Where $u \in V_1$, $v \in V_2$ and $w \in V_3$ are arbitrary. Hence $G_1+G_2+G_3$ is totally regular fuzzy graph, if and only if G_1, G_2 and G_3 are totally regular fuzzy graphs of the same degree.

THEOREM 3.4 :

Let $G_1:(\sigma_1,\mu_1)$, $G_2:(\sigma_2,\mu_2)$ and $G_3:(\sigma_3,\mu_3)$ be three fuzzy graphs of degrees K_1,K_2 and K_3 such that $\sigma 1 \wedge \sigma_2 \wedge \sigma_3$ is constant function. Then $G_1+G_2+G_3$ is totally regular fuzzy graph, if and only if K_1-K_2-K_3 = C (p_1-p_2-p_3) where C is the constant value of $\sigma1 \wedge \sigma_2 \wedge \sigma_3$.

Proof :

From (3.7), for any $u \in V_1$

$$td_{G1+G2+G3}(u) = td_{G1}(u) + \sum_{uvw \in E'} \sigma1(u) \wedge \sigma2(v) \wedge \sigma3(w)$$

$$= td_{G1}(u) + \sum_{v \in V3} C = K1 + cp3$$

From (3.8), for any $u \in V_2$

$$td_{G1+G2+G3}(u) = td_{G2}(u) + \sum_{uvw \in E'} \sigma1(u) \wedge \sigma2(v) \wedge \sigma3(w)$$

$$= td_{G2}(u) + \sum_{v \in V2} C = K2 + cp2$$

From (3.9), for any $u \in V_3$

$$td_{G1+G2+G3}(u) = td_{G3}(u) + \sum_{uvw \in E'} \sigma1(u) \wedge \sigma2(v) \wedge \sigma3(w)$$

$$= td_{G3}(u) + \sum_{v \in V1} C = K3 + cp1$$

Hence $G_1+G_2+G_3$ is totally regular fuzzy graph

$\Leftrightarrow K1 + cp3 = K2 + cp2 = K3 + cp1$

$\Leftrightarrow K_1 - K_2 - K_3 = cp1 - cp2 - cp3$

$\Leftrightarrow K_1 - K_2 - K_3 = C(p_1 - p_2 - p_3)$

4. TOTALLY REGULAR PROPERTY OF THE UNION :

For any $u \in V1 \cup V2 \cup V3$, we have three cases to consider.

Case 1 :

Either $u \in V_1$ or $u \in V_2$ or $u \in V_3$ but not both. Then no edge incident at u lies in $E1 \cap E2 \cap E3$

So $(\mu_1 \cup \mu_2 \cup \mu_3)(uvw) = \mu_1(uvw)$ if $u \in V_1$ (i.e) if $uvw \in E_1$

$= \mu_2(uvw)$ if $u \in V_2$ (i.e) if $uvw \in E_2$

$= \mu_3(uvw)$ if $u \in V_3$ (i.e) if $uvw \in E_3$

Hence if $u \in V_1$ then

$$td\, G1 \cup G2 \cup G3\, (u) = \sum_{uvw \in E1} \mu 1(uvw) + \sigma_1(u) = d_{G1}(u) + \sigma_1(u)$$

$$= td_{G1}(u)$$

If $u \in V_2$ then

$$td\, G1 \cup G2 \cup G3\, (u) = \sum_{uvw \in E2} \mu 2(uvw) + \sigma_2(u) = d_{G2}(u) + \sigma_2(u)$$

$$= td_{G2}(u)$$

If $u \in V_3$ then

$$td\, G1 \cup G2 \cup G3\, (u) = \sum_{uvw \in E3} \mu 3(uvw) + \sigma_3(u) = d_{G3}(u) + \sigma_3(u)$$

$$= td_{G3}(u)$$

Case 2 :

$u \in V_1 \cap V_2 \cap V_3$ but no edge incident at u lies in $E_1 \cap E_2 \cap E_3$. Then any edge incident at u is either in E_1, E_2 or E_3 but not both. Also all these edges will be included in $G_1 \cap G_2 \cap G_3$.

Hence

$$td\, G1 \cup G2 \cup G3\, (u) = \sum_{uvw \in E} (\mu 1 \cup \mu 2 \mu 3)(uvw) + (\sigma_1 \cup \sigma_2 \cup \sigma_3)(u)$$

$$= \sum_{uvw \in E1} \mu 1(uvw) + \sum_{uvw \in E2} \mu 2(uvw) + \sum_{uvw \in E3} \mu 3(uvw) + \sigma_1(u) + \sigma_2(u) + \sigma_3(u)$$

$$= d_{G1}(u) + d_{G2}(u) + d_{G3}(u) + \sigma_1(u) + \sigma_2(u) + \sigma_3(u)$$

$$= td_{G1}(u) + td_{G2}(u) + td_{G3}(u)$$

Case 3 :

$u \in V_1 \cap V_2 \cap V_3$ and some edges incident at u are in $E_1 \cap E_2 \cap E_3$. Any edge uvw which is in $E_1 \cap E_2 \cap E_3$ appear only once in $G_1 \cap G_2 \cap G_3$ and for this uvw,

$$(\mu_1 \cup \mu_2 \cup \mu_3)(uvw) = \mu_1(uvw) \vee \mu_2(uvw) \vee \mu_3(uvw)$$

By definition,

$$td\, G1 \cup G2 \cup G3\, (u) = \sum_{uvw\in E}(\mu1 \cup \mu2\mu3)(uvw) + (\sigma_1\cup\sigma_2\cup\sigma_3)(u)$$

$$= \sum_{uvw\in E1-E2} \mu1(uvw) + \sum_{uvw\in E2-E3} \mu2(uvw) + \sum_{uvw\in E3-E1} \mu3(uvw) + \sum_{uvw\in E1\cap E2\cap E3} \mu1(uvw) \vee\mu2(uvw)\vee\mu3(uvw) + \sigma_1(u) + \sigma_2(u) + \sigma_3(u)$$

$$=[\sum_{uvw\in E1-E2} \mu1(uvw) + \sum_{uvw\in E2-E3} \mu2(uvw) + \sum_{uvw\in E3-E1} \mu3(uvw) + \sum_{uvw\in E1\cap E2\cap E3} \mu1(uvw) \vee\mu2(uvw)\vee\mu3(uvw) + \sum_{uvw\in E1\cap E2\cap E3} \mu1(uvw) \wedge\mu2(uvw)\wedge\mu3(uvw)] - \sum_{uvw\in E1\cap E2\cap E3} \mu1(uvw) \wedge\mu2(uvw)\wedge\mu3(uvw) + \sigma_1(u) + \sigma_2(u) + \sigma_3(u)$$

$$= \sum_{uvw\in E1} \mu1(uvw) + \sum_{uvw\in E2} \mu2(uvw) + \sum_{uvw\in E3} \mu3(uvw) - \sum_{uvw\in E1\cap E2\cap E3} \mu1(uvw) \wedge\mu2(uvw)\wedge\mu3(uvw) + \sigma_1(u) + \sigma_2(u) + \sigma_3(u)$$

$$= d_{G1}(u)+d_{G2}(u)+d_{G3}(u) - \sum_{uvw\in E1\cap E2\cap E3} \mu1(uvw) \wedge\mu2(uvw)\wedge\mu3(uvw) + \sigma_1(u) + \sigma_2(u) + \sigma_3(u)$$

$$td\, G1 \cup G2 \cup G3\, (u) = td_{G1}(u)+td_{G2}(u)+td_{G3}(u) - \sum_{uvw\in E1\cap E2\cap E3} \mu1(uvw) \wedge\mu2(uvw)\wedge\mu3(uvw)$$

EXAMPLE 4.1 :

Consider the fuzzy graphs $G_1:(\sigma_1,\mu_1)$, $G_2:(\sigma_2,\mu_2)$ and $G_3:(\sigma_3,\mu_3)$ in fig 3.

Consider x :

Here $x \in V_1$. So by case 1,

$$td\, G1 \cup G2 \cup G3\, (x) = d_{G1}(x) + \sigma_1(u)$$
$$= td_{G1}(x)$$
$$= (0.1+0.1) + 0.1 + 0.6$$
$$= 0.9$$

Consider v :

We have $v \in V1 \cap V2 \cap V3$ but no edge incident at V lies in $E1 \cap E2 \cap E3$.

So by case 2,

td $G1 \cup G2 \cup G3$ (v) = $td_{G1}(v) + td_{G2}(v) + td_{G3}(v)$ = (0.3+0.1) + 0.3 + 0.8

td $G1 \cup G2 \cup G3$ (v) = 1.5

Consider u :

We have $u \in V1 \cap V2 \cap V3$ and $uvw \in E1 \cap E2 \cap E3$

So by case 3,

td $G1 \cup G2 \cup G3$ (u) $= td_{G1}(u)+td_{G2}(u)+td_{G3}(u)-\sum_{uvw \in E1 \cap E2 \cap E3} \mu1(uvw) \wedge \mu2(uvw) \wedge \mu3(uvw)$

= (0.2+0.3+0.1)+0.4+0.1-(0.3∧0.4)+(0.3∨0.7∨0.1)

= 0.6+0.5-0.3+0.7

= 1.8-0.3

td $G1 \cup G2 \cup G3$ (u) = 1.5

All these degrees can be verified in the figure of $G1 \cup G2 \cup G3$ in fig 3.

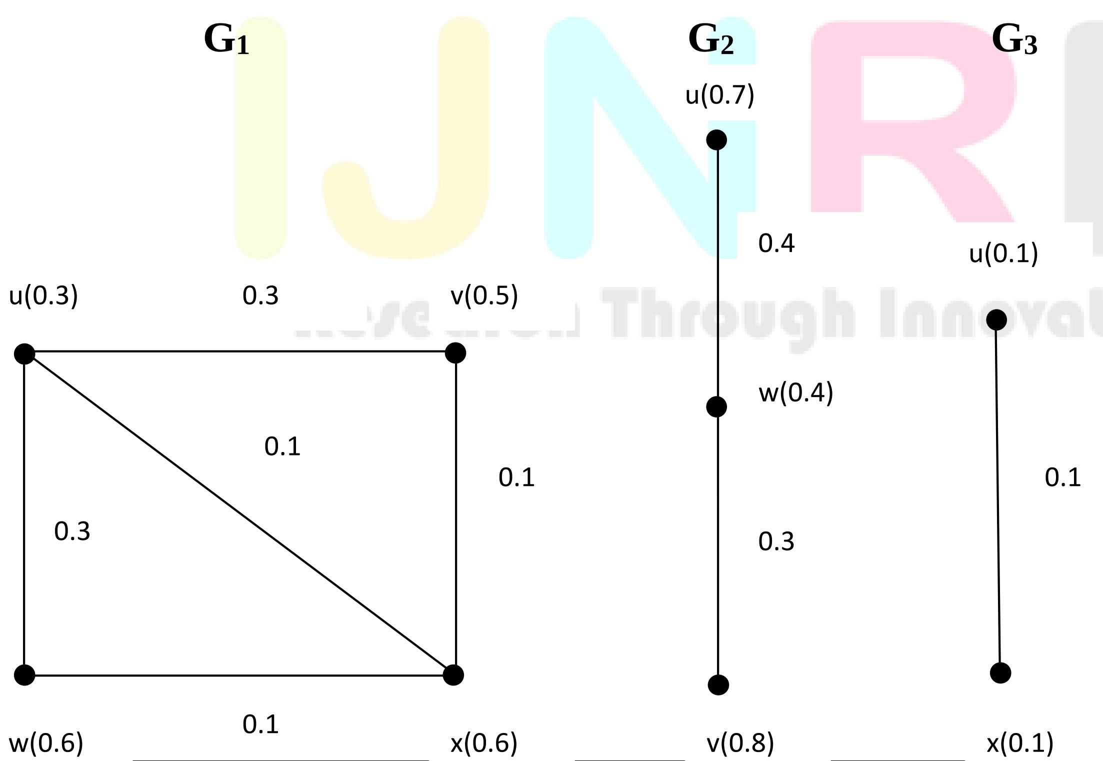
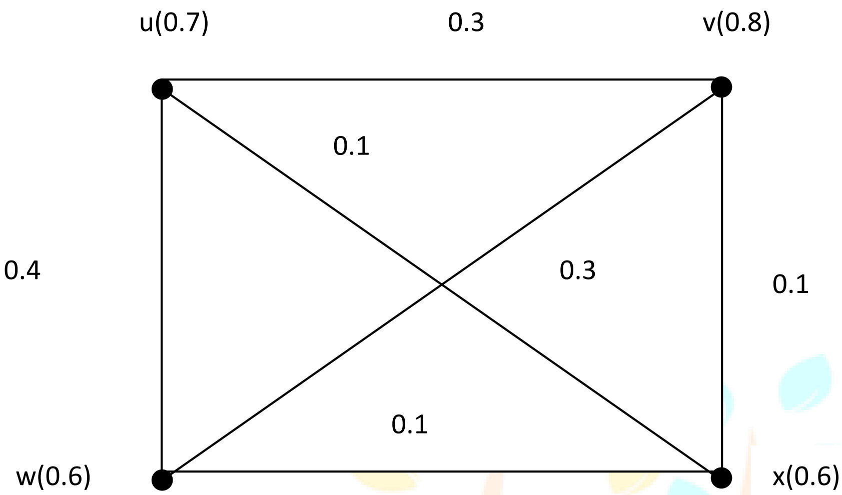


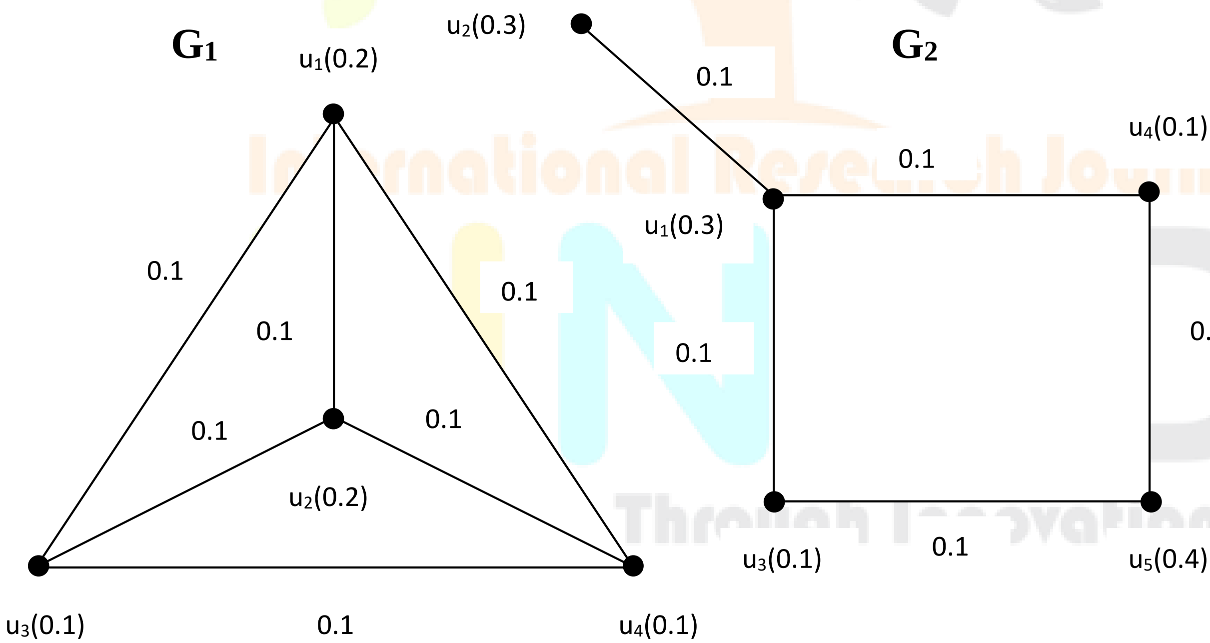
Figure 3 : G1∪G2∪G3

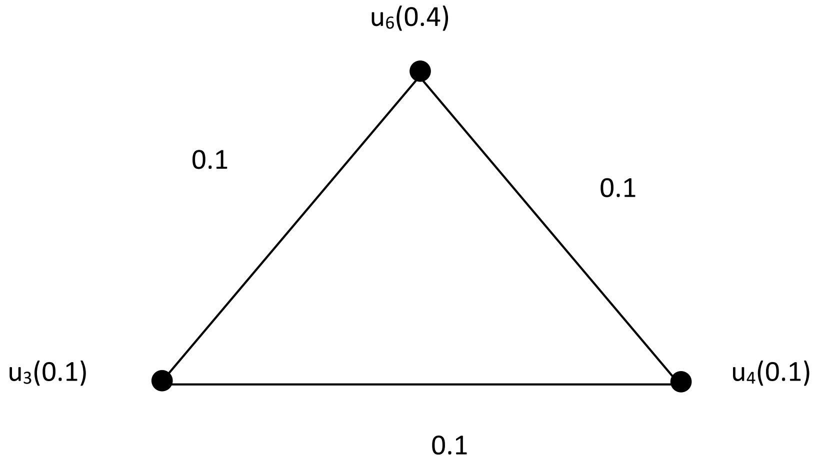


EXAMPLE 4.2 :

If G_1 is not a totally regular fuzzy graph, G_2 is not a totally regular fuzzy graph and G_3 is not a totally regular fuzzy graph, then G1∪G2∪G3

is a totally regular fuzzy graph in fig 4.





G_3

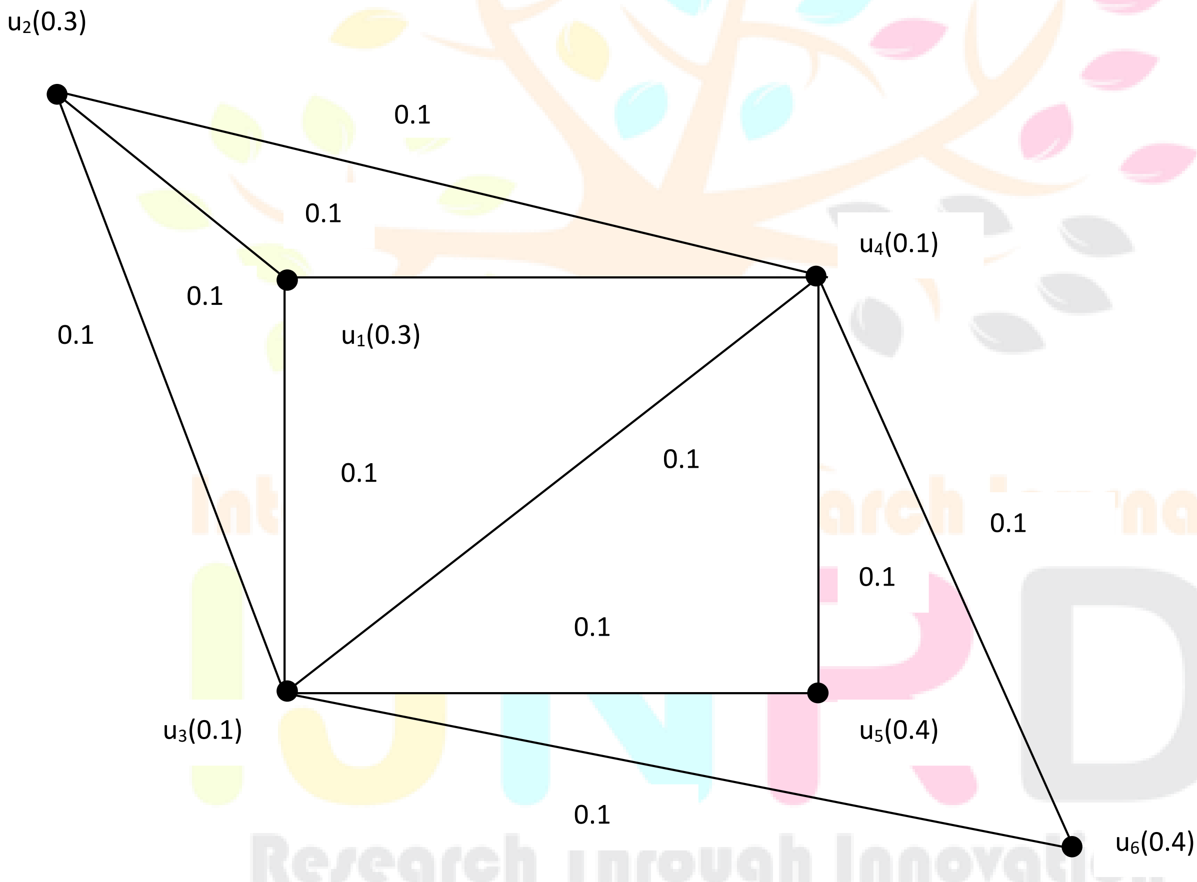


Figure 4 : G1∪G2∪G3

5.CONCLUSION :

In this paper, we have showed that the join & union of three totally regular fuzzy graphs need not be a totally regular fuzzy graph. We have obtained necessary and sufficient condition for the join & union of three fuzzy graphs to be totally regular in some particular cases.

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