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Pseudo Regular On Direct Sum Of Two Pseudo Regular Fuzzy Graph

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Abstract :

In this paper ,pseudo degree pseudo regular fuzzy graph and totally pseudo regular fuzzy graph are defined.On direct sum between pseudo regular and totally pseudo regular fuzzy graph is done. A necessary and sufficient condition under which they are equivalent is provided.Characterization of pseudo regular fuzzy graph in which crisp graph is a cycle is nvestigated. Also,whether he results hold for totally pseudo regular fuzzy graph is examined .

Keywords :

2-degree,average degree of a vertex in graph ,regular fuzzy graph,totally reguar fuzzy graph, the direct sum of fuzzy graph

AMS Mathematics Subject Classifiction (2010); 05C12,03E72,05C72

1.Introduction :

In this paper,we consider only finite ,simple, connected graph.We denote the vertex set and the edge set of a graph G by V(G) and E(G) respectively.The degree of a vertex v is the number of edges incident at ,and it is denoted by d(v).A graph G is regular if all its vertices have the same degree. The 2-degree of [4] is the sum of the degrees of the vertices adjacent to and it is denoted by t(v). We callt(v)d(v)he average degree[3].

Fuzzy set theory was first introduced by Zadeh in 1965 [17]. The first defination of fuzzy graph was introduced by Haufmann in1973 based on Zadeh's fuzzy relations in 1971 In 1975, A Rosenfled introduced the concept of fuzzy graph [8]. Now ,fuzzy graphs have many applications in branches of engineering and technology.

2.Review of literature :

Nagoorgani and Radha introduced the concept of degree, total degree,regular fuzzy graphs in 2008[7]. Nagoorgani and Latha introduced the concept of irregular fuzzy graphs ,neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008[6]. Mathew,Sunitha and Anjali introduced the some connectivity concept in bipolar fuzzy graphs[15].

Akram and Dudek introduced the notions of regular bipolar fuzzy graphs[1] and also introduced intutionstic fuzzy graph [2]. Samanta and Pal introduced the concept of irregular bipolar fuzzy graph [13]. Maheswari and Sekar introduced (2,k) –regular fuzzy graphs and totally (2,k) –regular fuzzy graph [9].Maheswari and Sekar introduced m-neighbourly irregular fuzzy irregular fuzzy graphs[12]. Maheswari and Sekar neighbourly edge irregular fuzzy graphs[1] . Maheswari and Sekar introduced neighbourly edge irregular bipolar fuzzy graphs [11]. Pal and Hassan introduced irregular interval- valued fuzzy graphs[16] .Sunitha and Mathew discussed about growth of fuzzy graph theory[14].

Regular fuzzy graphs play a central role in combinatorics and theroretical computer science. These motivate us to define pseudo degree in fuzzy graphs and pseudo regular fuzzy graphs and discussed some of its properties.This paper deals with significant properties of pseudo regular fuzzy graphs.

3.Preliminaries :

we present some known definitions and result for ready references to go through the work presented in this paper.

By graph, we mean a pair G*=(V,E),where V is the set and E is a relation on v. The element of V are vertices of G*and the element of E are edges of G*.

Definition 3.1:

2-degree of v is defined as the sum of the degrees of the vertices adjacent to v and it is denoted by t(v) [4].

Definition3.2:

Average degree of v is defined as t(υ)d(υ),where t(υ) is the 2-degree of υ and it is denoted by d_a(v)[3].

Definition 3.3:

A graph is called pseudo-regular if ever vertex of G has equal average-degree [3].

Definition 3.4:

A fuzzy graph denoted by G:(σ,μ) on the graph G*:(V,E) is a pair μ:V×V →0,1 is a symmetric fuzzy relation on σ such that for all u, v in V the relation μ(u, v) = μ(u v)≤σ(u)∧σ(v) is satisfied ,where σ and μ are called membership function[7].

Definition **3.5:**

Let G :(σ,μ) be a fuzzy graph on G*(V,E) . The degree of a vertex u is $d_G(u) = \sum\mu(uv)$, for u v ϵE and μ(u v)=0 for u v not in E, this is equivalent to $d_G(u) = \sum\mu(u\ v)$,u≠ v and uv∈E7.

Definition 3.6:

The total degree of a vertex u is defined td(u) = ∑μ(u v)+σ(u) = d(u)+σ(u) ,uvϵE[9].If each vertex of G has the same total degree k, then G is said to be a regular fuzzy graph of degree k or k-totally regular fuzzy graph of degree k or k- totally regular fuzzy graph [7].

4.Pseudo regular fuzzy graph :

In this section , we define pseudo degree , pseudo regular fuzzy graph and discussed about its properties.

Definition 4.1:

Let G:(σ,μ) be a fuzzy graph onG*:(V,E).The 2- degree of a vertex in G is defined as the sum of degrees of the vertices adjacent to v and is denoted by $t_G(v)$.That is $t_G(v)$-$\sum d_G(v)$, where $d_G(u)$is the degree of the vertex u which is adjacent with the vertex v.

Definition 4.2:

Let G:(σ,μ) be a fuzzy graph on G*:(V,E). A pseudo (average) degree of a vertex v in fuzzy graph G is denoted by $d_a(v)$ and is defined by $d_a(v)$ = tG(v)dv) where $d_{G*}($ v)is the number of edges incident at v.

Definition 4.3:

Let G:(σ,μ) be a fuzzy graph on G*:(V,E). If $d_a(v)$ = k for all u in V. the G is called k-pseudo regular fuzzy graph.

Definition 4.4 :

Let G_1:(σ_1, v_1) and $G_{2=}$(σ_2, v_2) denote two fuzzy graphs with two fuzzy graph with underlying crisp graphs G_1*:($V_{1,}E_1$)and respectively , let v=$v_{1\cup}v_2$ and let E={uv/ u,vϵE;uvϵE_1(or) uvϵE_2But not both}define G: (σ,υ).

$$\sigma(\mu)=\begin{cases}\sigma_1(u) & \text{if } u\epsilon v_1\\ \sigma_2(u) & \text{if } u\epsilon v_2\\ \sigma_1(u)\wedge\sigma_2(u) & \text{if } u\epsilon v_{1,}v_2\end{cases} \quad \text{and } \mu(uv)=\begin{cases}\mu_1(uv) & \\ & uv\epsilon E\\ \mu_2(uv) & \end{cases}$$

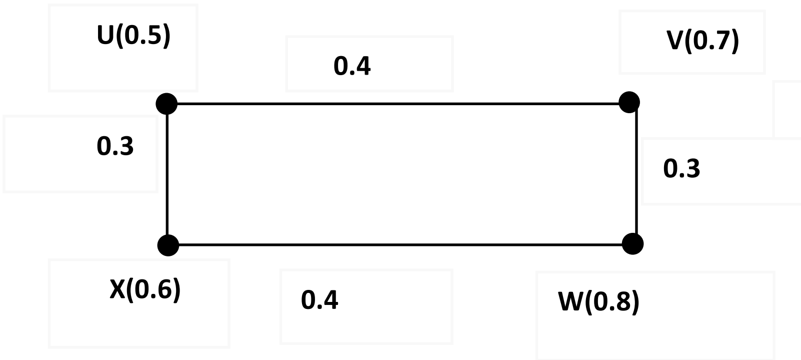
Then if uvϵE_1 ,μ(uv)= $\mu_1(uv)\leq \sigma_1(u)\wedge\sigma_1(v)$,if uvϵE_2 ,μ(uv) = $\mu_2(uv) \leq \sigma_2(u)$

$\wedge\sigma_2(v)$, Therefore (σ,μ) defines a fuzzy graph this is called the direct sum of two

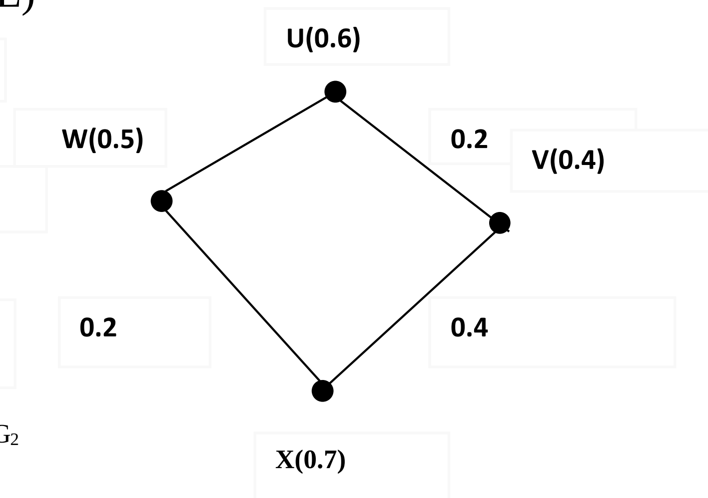
fuzzy graph.

Example 4.5 :

Consider a fuzzy graph on G*(V,E)

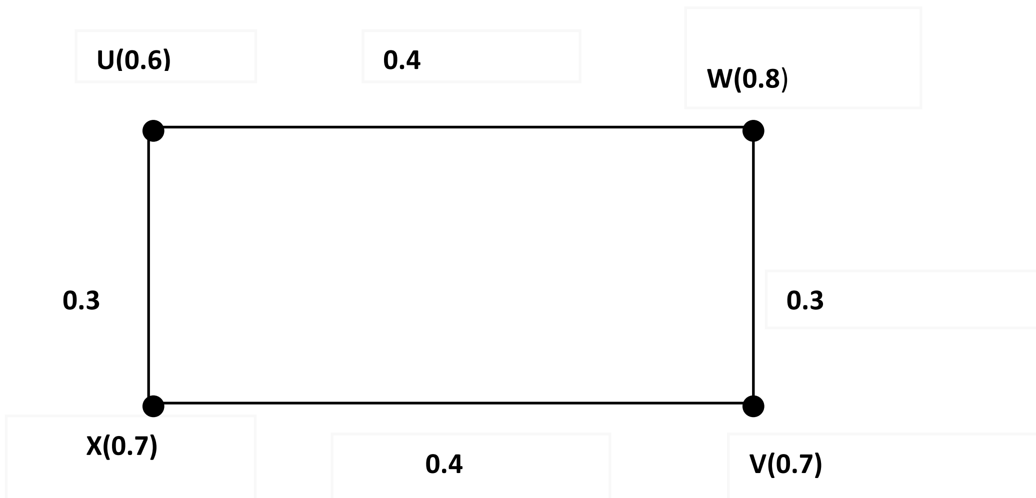


G_1



G_2

$G_1 \oplus G_2$



Here d(u) = 0.7,d(v) = 0.7,d(w) = 0.7,d(x) = 0.7 [illegible] = 0.7 for all uϵv now u adjacent w and x .d_a(u) = d(w)+d(x)2=0.7+0.72=0.7.so d_a(w) = 0.7 d_a(v) = 0.7,d_a(x) = 0.7 for all w,x,vϵV .

5.Totally Pseudo Regular Fuzzy Graphs

Definition 5.1:

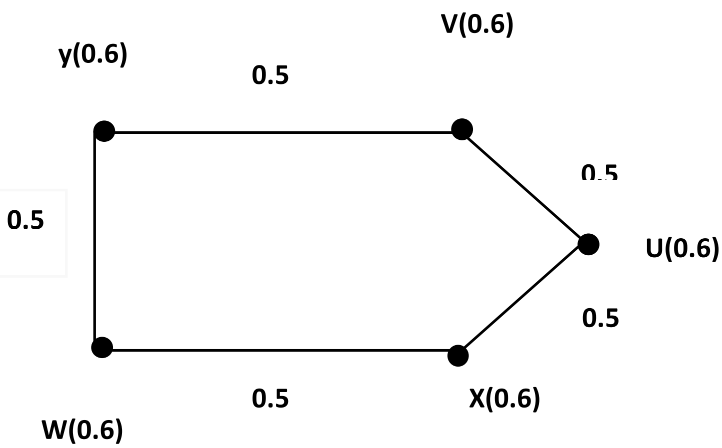
Let G: (σ,µ) be a fuzzy graph on G*(V,E). The total Pseudo degree of a vertex V is G is denoted by td_a(V) and is defined as td_a(V) = da(V) + σ (V), for all V ϵ V.

Definition 5.2:

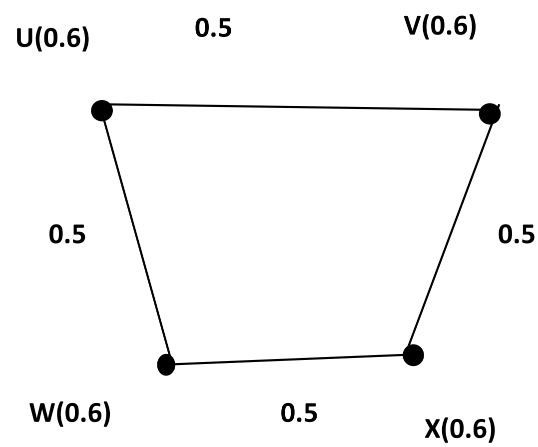
Let G be a fuzzy graph on G*(V, E). if all the vertices of G have the same total pseudo degree K. then G said to be a totally K- pseudo regular fuzzy graph.

Example 5.3:

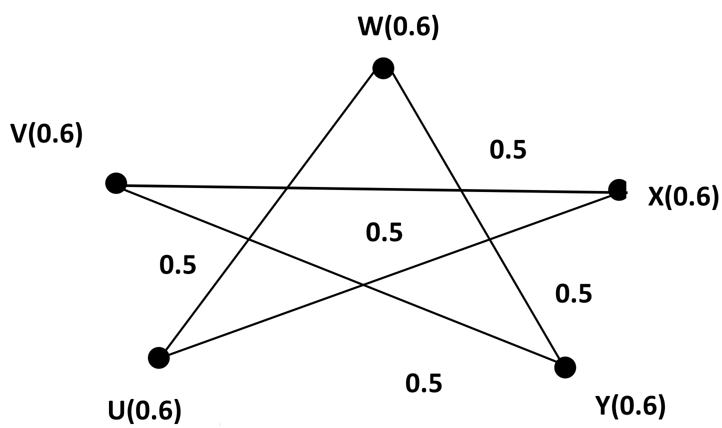
Consider a fuzzy graph on G*(V,E)



G_1



G_2



$G_1 \oplus G_2$

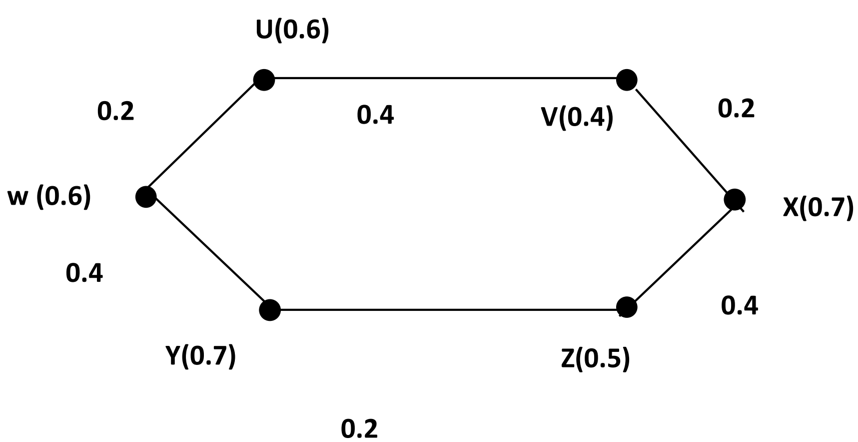
Here $d_a(U) = 1.6$ $d_a(V) = 1.6$ $d_a(X) = 1.61$ $d_a(Y) = 1.6$ $d_a(Y) = 1.6$, for all u ϵ v so $G_1 \oplus G_2$ is a 2.2- totally pseudo regular fuzzy graph.and also $G_1$and G_2 is 2.2- totally pseudo fuzzy graph.

Remark 5.4:

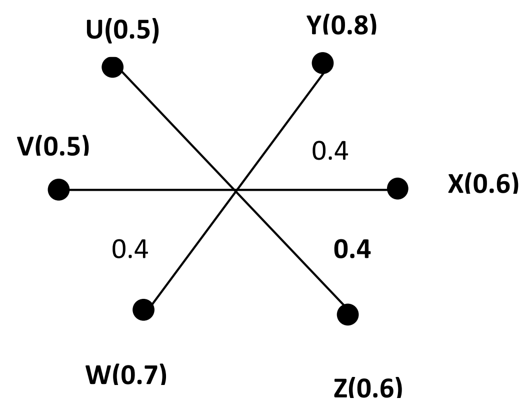
A pseudo regular fuzzy graph need not be a totally pseudo regular fuzzy graph.

Example 5.5

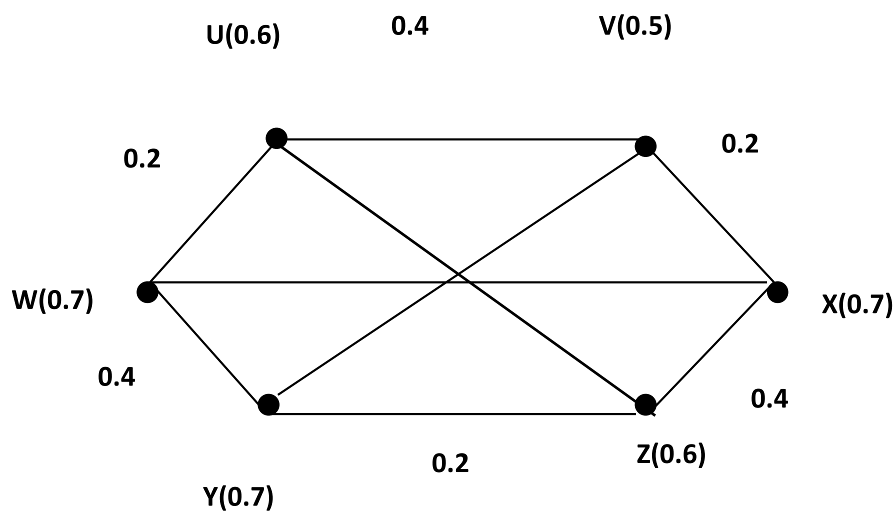
Consider a fuzzy graph on G*(V,E)



G_1



G_2

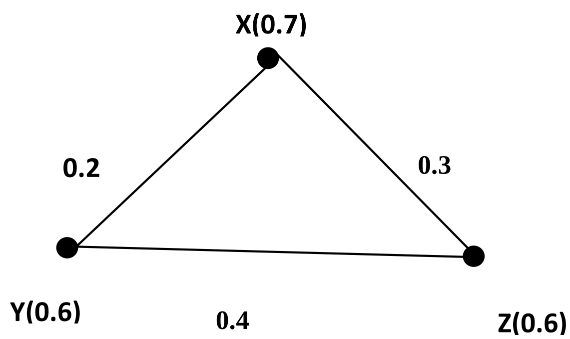


$G_1 \oplus G_2$

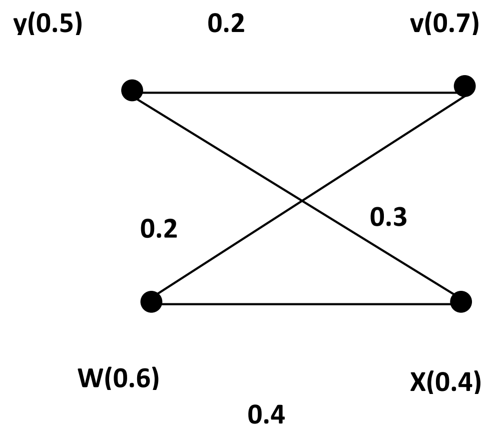
The graph is G_1 is 0.6-peseudo regular fuzzy graph and G_2 is a 0.4 – pseudo regular fuzzy graph $G_1 \oplus G_2$ 0.6 –pseudo regular fuzzy graph .but $td_a(u) \neq td_a(v)$ $G_1 \oplus G_2$ is not a totally pseudo regular fuzzy graph.

Example5.6:

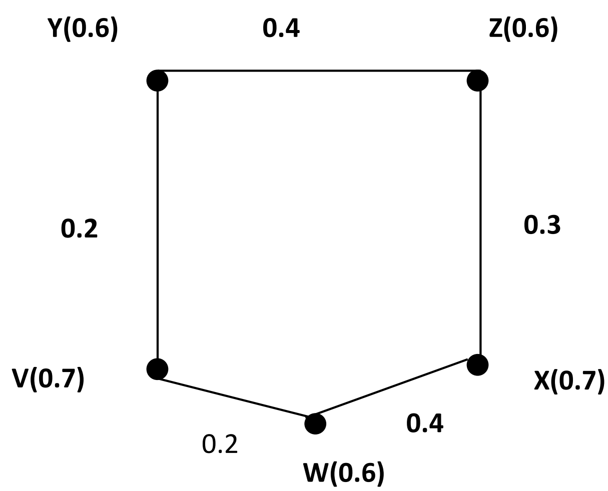
Consider a fuzzy graph on G*(V,E)



G_1



G_2



$G_1 \oplus G_2$

$G_1 \oplus G_2$

Here $G_1 \oplus G_2$ is a neither pseudo regular fuzzy graph nor a totally pseudo regular fuzzy graph.

Theorem :5.7:

Let $G_1 \oplus G_2 : (\sigma,\mu)$ be a fuzzy graph on G*(V,E) an even cycle of length n. if the alternate edges have same membership values then $G_1 \oplus G_2$ is a pseudo fuzzy graph.

Proof:

If the alternate edges have the same membership values

$$\text{Then } \mu\,(ei) = \begin{cases} C_1 & \text{if is odd} \\ C_2 & \text{if is even} \end{cases}$$

If $C_1=C_2$ then μ is a constant function. $G_1 \oplus G_2$ is a Pseudo regular fuzzy graph.

If $C_1 \neq C_2$ then $d_G\,(V) = C_1 + C_2$ for all u £ v. So $t_G(V) = 2\,C_1 + 2C_2$ and $d_G{}^*(V) = 2$

Hence $G_1 \oplus G_2$ is a $(C_1 + C_2)$ pseudo regular fuzzy graph.

Theorem 5.8:

Let G:(σ,μ) be a fuzzy graph on G*(V,E).Then σ is a constant fuction iff the following are equivalent

i)$G_1 \oplus G_2$ is a pseudo regular fuzzy graph.

ii)$G_1 \oplus G_2$is a totally pseudo regular fuzzy graph.

Proof:

Assume that σ is a constant fuction . let σ(u)=c for all u∊V .suppose $G_1 \oplus G_2$ pseudo regular fuzzy graph. Then $d_a(u)=k$ for all u∊V.Now,$td_a(u)=d_a(u)+\sigma(u)=k+c$ hence $G_1 \oplus G_2$ is a totally pseudo regular fuzzy graph thus i)⟹ii) is proved

Suppose $G_1 \oplus G_2$ is a totally pseudo regular fuzzy graph. Then $td_a(u) = K$ for all u ϵ V. hence $G_1 \oplus G_2$ be a pseudo regular fuzzy graph.

Then ii) = i)is proved. Hence i) and ii) are equivalent

Conversely suppose i) and ii) are equivalent let $G_1 \oplus G_2$ be a pseudo regular fuzzy graph and a totally pseudo regular fuzzy graph.

Then $d_a(u) = K_1$ and $tda(u) = K_2$ for all u ϵv. Now $td_a(u) = K_2$ for all u ϵ v $K_1 + \sigma(u) = K_1$ for all u ϵ v $d_a\,(u) + \sigma(u) = K_2$ for all u ϵ v$\sigma(u) = K_2 - K_1$for all u ϵv.

Hence σ is a constant function.

Theorem 5.9:

Let G (σ, μ) be a fuzzy graph on G* (V,E) a cycle of length n. if μ is a constant function. Then $G_1 \oplus G_2$ is a pseudo regular fuzzy graph.

proof

if μ is a constant function

say μ(uv) = C for all uv ϵ E then

da(u) = 2C for all u ϵ v

hence $G_1 \oplus G_2$ is C pseudo regular fuzzy graph.

Theorem 5.10:

If G is a regular fuzzy graph on G* (V,E) an r-regular fuzzy graph then $d_a(V) = d_G(V)$, for all VϵG.

Proof

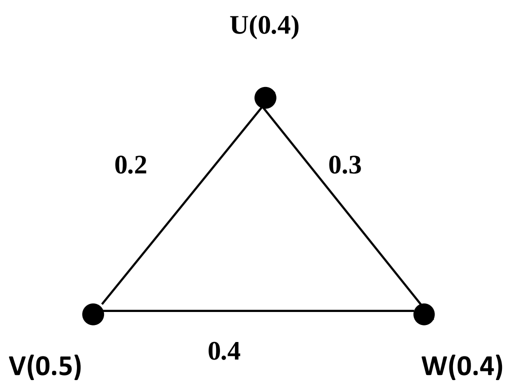
Let G is a K – regular fuzzy graph on G* (V,E) an r-regular graph.

Then d_G (V) = K for all VϵG

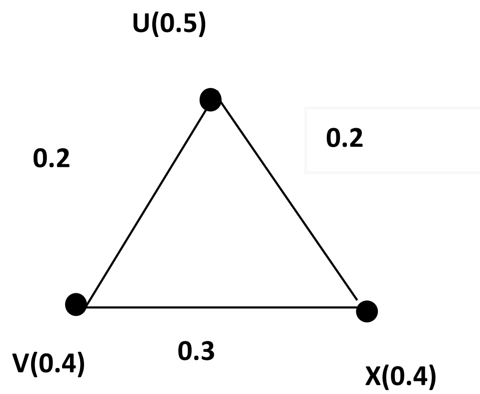
And $d_G*(v)$ = k for allvϵG so $t_G(v) = \sum d_G(v_i)$. where each v_I (for all i = 1,2….) is adjacent with vertex v $\Rightarrow t_G(v) = \sum d_G(v_i)$ = rk. also

$d_a(v) = t_G(v)d_{G^*}(V)$, $\Rightarrow d_a(v) = t_G(v)r$, $\Rightarrow d_a(v) = krr$, $\Rightarrow d_a(v) = k \Rightarrow d_a(v) = d_G(v)$

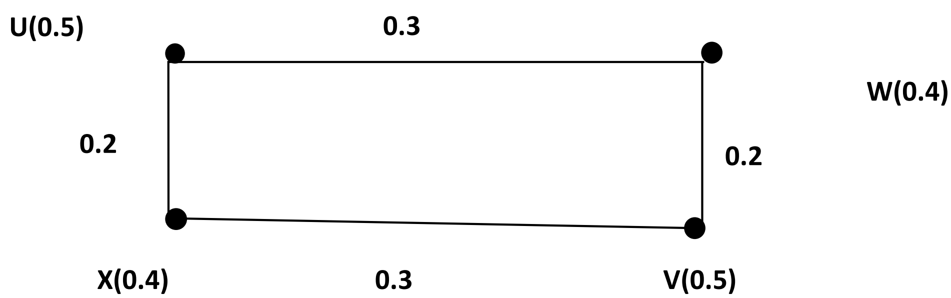
Example5.11:



G_1



G_2



$G_1 \oplus G_2$

Here G_1 and G_2 is a totally 0.9- regular fuzzy graph.$G_1 \oplus G_2$ is0.5pseudo regular fuzzy graph

Theorem 5.12:

A pseudo regular fuzzy graph on any cycle does not fuzzy bridge.

proof

case i)

let G be a fuzzy graph on G* (V,E) an odd cycle of length n. G is pseudo regular fuzzy graph only if μ is constant. So, the removal of any edge does not reduce the strength of connected ness between any pair of vertices. Hence G has no fuzzy bridge.

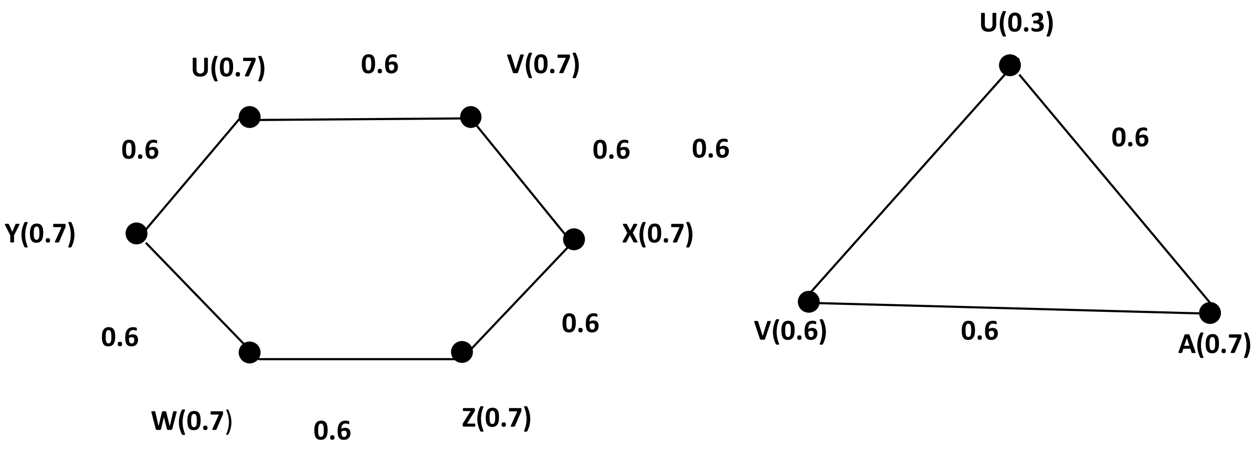
case ii)

let G be a fuzzy graph on G* (V,E) an even cycle of length n. G is pseudo regular fuzzy graph only if μ is constant or alternate edges have some membership values.

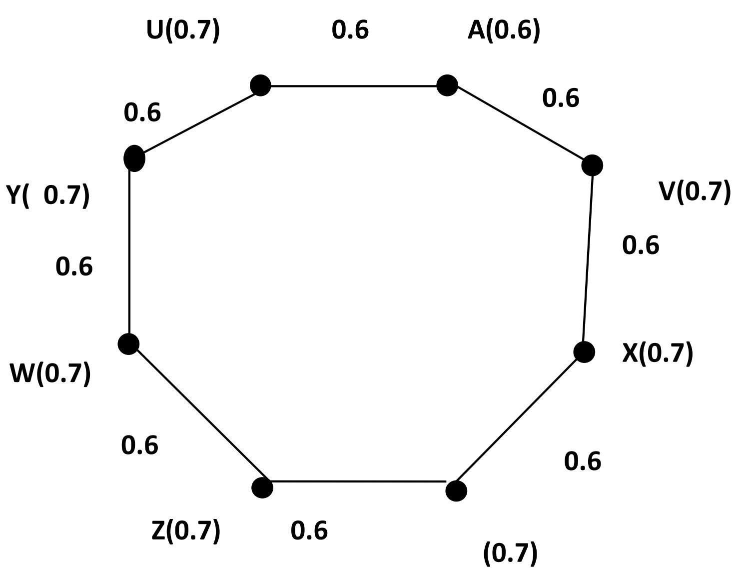
So, the removal of any edge does not reduce the strength of connected ness between any pair of vertices. Hence G has no fuzzy bridge

Example5.13:

Consider a fuzzy graph on G*(V,E)



G_1 G_2



$G_1 \oplus G_2$

Hence G_1 is totally 1.3- Pseudo regular fuzzy graph G_2 is 1.2 Pseudo regular fuzzy graph $G_1 \oplus G_2$ is totally 1.1 Pseudo regular fuzzy graph.

Conclusion

In this paper some new properties of direct sum on pseudo regular fuzzy graph and totally pseudo regular fuzzy graph are discussed it will be more useful for doing further research in the fluid of regular if G and totally pseudo regular fuzzy graph based on direct sum property.

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