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article 1 about theory distribution of prime number

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ABSTRACT

In this article, Theorem (3) impose a rule on p and q for which 36p+6q+1 a composite number. and violation of this rule implies 36p+6q+1 a prime number. similarly, Theorem (4) impose a rule on p and q for which 36p+6q-1 a composite number. and violation of this rule implies 36p+6q-1 a prime number.

INTRODUCTION

Numbers are wonderful, marvelous creature of human. Numbers are classified into many types. They are Natural numbers, Whole numbers, Integers, Real numbers, Complex numbers, Rational numbers, Irrational numbers.

Natural numbers are classified into two categories

1) Prime numbers,
2) Composite numbers.

Prime numbers are Natural numbers which cannot be expressed in the form of product of two Natural numbers both greater than 1.

Composite numbers are other than Prime numbers. i.e. Which can be expressed in the form of product of two Natural numbers both greater than 1.

From the definition of Prime numbers, 2 and 3 are Prime numbers, but 2x3=6 is a composite number. Multiples 6 are also composite numbers. Numbers in the form 6k±2 are even numbers i.e multiples of 2, hence composite numbers, Numbers in the form 6k±3 are odd multiples of 3 hence composite numbers. Where k is any natural number.

Therefore, Prime numbers except 2 and 3 are in the form of 6k±1 where k is natural number, but not for all natural numbers. For some Natural number k, 6k+1is a Prime number but 6k-1 is a Composite number. For some Natural number k, 6k-1 is a Prime number but 6k+1 is a Composite numbers. For some Natural number k, 6k+1 and 6k-1 both are Prime numbers (twin Prime numbers). For some Natural number k, 6k+1 and 6k-1 both are Composite numbers. Hence this k is the key factor that determines Prime numbers and Composite numbers.

Before 2500 years ago Euclid proved that Prime numbers are infinite, Composite numbers are generated by their prime factors, but Prime numbers are not generated. They are distributed among the gaps left by Composite numbers. This article is about Theory of distribution of Prime number. Distributive rule of Prime numbers is nothing but violation of generating rule of Composite numbers. And since all Prime numbers except 2 and 3 are in the form 6k±1, this article is an analysis about natural number k, which determines numbers in the form 6k±1.

NUMBERS IN THE FORM 6k±1

Here after Prime number means Prime number in the form 6k±1, Composite number means Composite number in the form 6k±1 unless explicitly stated.

When we see the numbers in the form 6k±1, It is seen like a binomial expression. We can say it as binomial in 6 or binomial in k. In this article, for analysis number in the form 6k±1 is treated as binomial in 6, with constant term ±1. First let us analyze nature of this form.

Let

$P=6p\pm1$ and $Q=6q\pm1$ are two Natural numbers in the form 6k±1.

$$P \times Q = (6p\pm1)(6q\pm1) = 36pq \pm (p \pm q)6 \pm 1$$

$$= [6pq \pm (p \pm q)]\, 6 \pm 1$$

$$= 6K\pm1 \quad \text{Where } K = 6pq \pm (p \pm q)$$

Implies product of two numbers in the form 6k±1 gets the form 6k±1 again.

Let

$P = (6p\pm1)$, $Q = (6q\pm1)$ and $R = (6r\pm1)$ are three Natural numbers in the form 6k±1.

$$P \times Q \times R = (6p \pm 1)(6q \pm 1)(6r \pm 1)$$

$$= 216pqr \pm (pq \pm qr \pm pr)36 \pm (p \pm q \pm r)6 \pm 1$$

$$= [36pqr \pm (pq \pm qr \pm pr)6 \pm p \pm q \pm r]6 \pm 1$$

$$= 6K \pm 1$$

where $K = 36pqr \pm (pq \pm qr \pm pr)6 \pm p \pm q \pm r$

Hence product of three numbers in the form 6k±1 again gets the form 6k±1. The above argument says multiplication is closed binary operation among set of natural numbers in the form 6k±1.It is obvious 2 and 3 cannot be a factor of the numbers in the form 6k± 1.

Hence number in the 6k±1 is Composite if and only if 6k ± 1 can be factored as

$$6k \pm 1 = (6p \pm 1)(6q \pm 1)$$

Where $6p \pm 1$ and $6q \pm 1$ are two natural numbers in the form $6k \pm 1$, p and q not necessarily distinct.

Hence the number in the form $6k \pm 1$ is Prime number if and only if it cannot be factored as above.

The following two theorems are realized by myself before five years. But now I have heard that two theorems are already proved. For better continuity and better understanding, I reproduce the two theorem with proof once.

THEOREM (1)

Let n=6k+1, where k is any Natural number. n=6k+1 is Composite number if and only if k can be expressed in the form k=6ab±(a+b). where a and b are Natural numbers not necessarily distinct.

PROOF

Let n=6k+1 is a Composite number. From the above argument n can be factored as

$$n=6k+1= (6a\pm1)(6b\pm1)$$

where a and b Natural numbers not necessarily distinct.

On comparing the constant term, two cases arise.

(i) n= 6k+1= (6a+1)(6b+1) (or)
(ii) n= 6k+1= (6a-1)(6b-1)

Case(i)
n= 6k+1= (6a+1)(6b+1)
6k+1= 36ab+(a+b)6+1
6k+1= [6ab+a+b]6+1
Implies k=6ab+a+b

Case(ii)
n=6k+1=(6a-1)(6b-1)
6k+1=36ab-(a+b)6+1
6k+1= [6ab-(a+b)]6+1
Implies k=6ab-(a+b)

From Case(i) and Case(ii)

k=6ab±(a+b)

Conversely let k=6ab±(a+b)
Two cases arise
Case(i) k=6ab+a+b (or)
Case (ii) k=6ab-(a+b)

Case(i)
n=6k+1=6[6ab+a+b]+1
n=6k+1=36ab+(a+b)6+1
n= 6k+1=(6a+1)(6b+1)
implies n is Composite number

Case(ii)
n=6k+1=6[6ab-(a+b)]+1
n=6k+1=36ab-(a+b)6+1
n=6k+1=(6a-1)(6b-1)
implies n is Composite number

From Case(i) and Case(ii)
n is a Composite number

Hence theorem (1) is proved

Theorem (1) impose a rule on k, for which 6k+1 a Composite number. Violation of this rule is distributive rule for Prime numbers in the form 6k+1.

DISTRIBUTIVE RULE (1)

Natural number n=6k+1, where k is any Natural number. Then n=6k+1 is a Prime number if and only if k cannot be expressed in the form k=6ab±(a+b) i.e n=6k+1 is a Prime number if and only if k≠6ab±(a+b) where a and b are Natural numbers not necessarily distinct.

THEOREM (2)

Let n=6k-1, where k is any Natural number. n=6k-1 is a Composite number if and only if k can be expressed in the form k=6ab±(a-b). where a and b are Natural numbers not necessarily distinct.

PROOF

Let n=6k-1 is Composite number. As in theorem (1) 6k-1 can be factored as

n=6k-1=(6a±1) (6b±1)

where a and b are Natural number not necessarily distinct.

On comparing constant term two cases arise

Case(i) n=6k-1=(6a+1)(6b-1) (or)

Case(ii) n=6k-1=(6a-1)(6b+1)

Case(i)
n=6k-1=(6a+1)(6b-1)
6k-1=36ab+(b-a)6-1
6k-1=[6ab-a+b]6-1
Implies k=6ab-(a-b)

Case(ii)
n=6k-1=(6a-1)(6b+1)
6k-1=36ab+(a-b)6-1
6k-1=[6ab+(a-b)]6-1
Implies k=6ab+(a-b)

From Case(i) and Case(ii)

k=6ab±(a-b)

Conversely let k=6ab±(a-b)
Two cases arise
Case(i) k=6ab+a-b (or) Case (ii) k=6ab-(a-b)

Case(i)
n=6k-1=6[6ab+a-b]-1
n=6k-1=36ab+(a-b)6-1
n=6k-1=(6a-1)(6b+1)
implies n is Composite number

Case(ii)
n=6k-1=6[6ab-(a-b)]-1
n=6k-1=36ab-(a-b)6-1
n=6k-1=(6a+1)(6b-1)
implies n is Composite number

From Case(i) and Case(ii)
n is a Composite number

Hence theorem (2) is proved

Theorem (2) impose a rule on k for which 6k-1 a Composite number. Violation of this rule is distributive rule for Prime numbers in the form 6k-1

DISTRIBUTIVE RULE (2)

Natural number n=6k-1, where k is any Natural number. Then n=6k-1 is a Prime number if and only if k cannot be expressed in the form k=6ab±(a-b) i.e n=6k-1 is a Prime number if and only if k≠6ab±(a-b) where a and b are Natural numbers not necessarily distinct.

NUMBERS IN THE FORM 6ab±(a+b) AND 6ab±(a-b).

Let us discuss the formation of numbers in the form 6ab±(a+b) and 6ab±(a-b). where a and b are natural numbers not necessarily distinct. For that we have to make some assumption. Let us assume that the numbers in the form 6ab±a±b are contributions of multiples 6 say 6ab.

For example

6x1x1-1-1=4, 6x1x1+1-1=6 & 6x1x1+1+1=8 are contributions of 6.

6x1x2-1-2=9, 6x1x2+1-2=11, 6x1x2-1+2=13, & 6x1x2+1+2=15 are contributions 12.

6x1x6-1-6=29, 6x1x6+1-6=31, 6x1x6-1+6=41, 6x1x6+1+6=43

6x2x3-2-3=31, 6x2x3+2-3=35, 6x2x3-2+3=37, 6x2x3+2+3=41 are contributions 36

Some numbers in the form 6ab±a±b, contributed by two or more multiples 6.

Example

6x1x9+1+9=64, 6x6x2-2-6=64, 6x1x13-1-13=64 Hence 64 is contributed by three multiples of 6. i.e 64 contributed 54, 72, &78.

By this assumption let us continue. now we are going to discuss about the greatest and the smallest number contributed by a multiple of 6 say 6p. where p is any natural number. but it is not hard to prove that the greatest number in the form 6ab+a+b contributed by 6p is 6(1)p+p+1=7p+1. If any other number contributed by 6p in the form 6ab+a+b say 6y(p/y)+y+p/y is greater than 7p+1, where y is a factor of p. Then

$6y(p/y)+y+p/y = 6p+y+p/y > 7p+1 = 6p+p+1$

$y+p/y > p+1$

$y^2+p > py+y$

$y^2-y > py-p$

$y(y-1) > p(y-1)$

y>p which is absurd. Since y is divisor of p.

Hence the greatest number in the form 6ab+a+b contributed by 6p is 7p+1. Similarly, the smallest number in the form 6ab-a-b contributed by 6p is 6(1)p-p-1=5p-1. if any other number contributed by 6p in the form 6ab-a-b say 6y(p/y)-p-p/y is smaller than 5p-1. Where y is a factor of p. Then

$6y(p/y)-y-p/y = 6p-y-p/y < 5p-1 = 6p-p-1$

$-y-p/y < -p-1$

$y+p/x > p+1$

$y^2+p > py+y$

$y^2-y > py-p$

$y(y-1) > p(y-1)$

y>p which is absurd

hence the smallest number in the form 6ab-a-b contributed by 6p is 5p-1

Therefore,

The greatest number in the form 6ab+a+b contributed by 6p is 7p+1

The smallest number in the form 6ab-a-b contributed by 6p is 5p-1

And it is obvious that natural numbers x and y are any two divisors of p. then $6p-x-p/x < 6p+y+p/y$. and $5p-1 \leq 6p-x-p/x < 6p+y+p/y \leq 7p+1$

Therefore, n=6k+1 a composite number, theorem (1) implies k=6p+y+p/y or k=6q-x-q/x, where p and q are natural numbers not necessarily distinct, natural number x is any divisor of p, and natural number y is any divisor of q, x and y need not to be necessarily distinct. And from the above arguments.

If k=6p+y+p/y,

$$5p-1 < k \leq 7p+1.$$

If k=6q-x-q/x,

$$5q-1 \leq k < 7q+1.$$

In general if $k=6p\pm(y+p/y)$, where y is divisor of p, then

$$5p-1 \leq k \leq 7p+1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Now we go through theorem (3).

THEOREM (3)

let n=36p+6q+1 is a Natural number, where p and q are natural numbers and $0 \leq q < 6$. n is Composite number if and only if there exist an integer k such that $[(6k-q)^2-4(p+k)]$ a perfect square and $(q-p-1)/7 \leq k \leq (p+q+1)/5$

PROOF

Let n=36p+6q+1=6(6p+q)+1 a Composite number. where p and q are natural numbers and $0 \leq q < 6$. By theorem (1). 6p+q can be expressed in the form $6ab\pm(a+b)$. where a and b are natural numbers not necessarily distinct.

Let ab=m, then b=m/a.

$$6p+q=6m\pm(a+m/a)$$

$$6p+q=6m+a+m/a \quad \text{or} \quad 6p+q=6m-a-m/a.$$

For a and –a we use general integer variable x. since if a=x, m/a=m/x and also –a=x, (-m/a)=(-m/-x)=m/x. Hence for both cases

$$6p+q=6m+x+m/x$$

Let m-p=k, then m=p+k.

Therefore, 6p+q=6(p+k)+x+(p+k)/x

$$6p+q=6p+6k+x+(p+k)/x$$

$$x+6k-q+(p+k)/x=0$$

$$x^2+(6k-q)x+p+k=0 \quad \text{----------(2)}$$

this equation has integer solution, i.e x=a or x=-a satisfies this equation implies,

$(6k-q)^2-4(1)(p+k)$ is a perfect square. Otherwise equation (2) has irrational solution only. Hence

$(6k-q)^2-4(p+k)$ is a perfect square.

Conversely let $(6k-q)^2-4(p+k)$ is a perfect square. then the solution of the above equation is

$$x=(q-6k)/2\pm(\,[\,(6k-q)^2-4(p+k)\,]^{1/2}\,)/2$$

if q-6k is even number obviously x is an integer

if q-6k is odd number then $(6k-q)^2-4(p+k)$ also odd and

$[(6k-q)^2-4(p+k)]^{1/2}$ also odd. Therefore,

[odd number/2 +odd number/2] is an integer. i.e x is an integer.

Hence if $(6k-q)^2-4(p+k)$ is a perfect square, then x is an integer satisfies above equation (2), implies x satisfies

$$6p+q=6(p+k)+x+(p+k)/x$$

But 6p+q and 6m=6(p+k) both are Natural numbers, therefore (p+k)/x is an integer, i.e x divides p+k, hence

$$6p+q=6(x)[(p+k)/x]+x+(p+k)/x$$

m=p+k is a Natural number, if x is positive integer then (p+k)/x also positive integer, implies

6p+q is given in the form 6ab+a+b.

and if x is negative integer then (p+k)/x also negative integer. Implies 6p+q is given in the form 6ab-a-b.

Hence 6p+q can be given in the form 6ab±(a+b). By theorem(1)

6(6p+q)+1 =36p+6q+1 is Composite number.

Next $6p+q=6(p+k)+x+(p+k)/x$

Therefore 6(p+k), a multiple of 6 contributes the number 6p+q in the form 6ab±(a+b)

Hence

$5(p+k)-1 \le 6p+q \le 7(p+k)+1.$ [since from (1)]

First we take right inequality

$$6p+q \le 7(p+k)+1$$
$$6p+q-1 \le 7(p+k)$$
$$6p+q-1 \le 7p+7k$$
$$q-1-p \le 7k$$
$$(q-1-p)/7 \le k \quad \text{------------------(3)}$$

Next we take left inequaity

$$5(p+k)-1 \le 6p+q$$
$$5p+5k \le 6p+q+1$$
$$5k \le p+q+1$$
$$K \le (p+q+1)/5 \quad \text{-----------------(4)}$$

therefore, from (3) and (4)

$$(q-p-1)/7 \le k \le (p+q+1)/5$$

Hence theorem (3) is proved.

Violation of theorem (3) is the distributive rule for Prime numbers in the form 36p+6q+1

DISTRIBUTIVE RULE (3)

Let n=36p+6q+1 is Natural number, where p and q are Natural numbers $0 \le q < 6$.Then n is Prime number if and only if There exist no k, such that $(6k-q)^2-4(p+k)$ a perfect square and $(q-p-1)/7 \le k \le (p+q+1)/5$

Similarly,

The greatest number in the form 6ab+a-b contributed by 6p is 6(1)p+p-1=7p-1. if any other number contributed by 6p in the form 6ab+a-b say 6y(p/y)+y-p/y is greater than 7p-1. Where y is a factor of p. Then

$$6y(p/y)+y-p/y = 6p+y-p/y > 7p-1 = 6p+p-1$$

$$y-p/y > p-1$$

$$y^2-p > py-y$$

$$y^2+y > py+p$$

$$y(y+1) > p(y+1)$$

$y > p$ but y is a divisor of p. which is absurd.

Hence the greatest number in the form 6ab+a-b contributed by 6p is 7p-1. Similarly, the smallest number in the form 6ab+a-b contributed by 6p is 6(1)p-p+1=5p+1. If any other number contributed by 6p in the form 6ab+a-b say 6y(p/y)+y-p/y is smaller than 5p+1. Where y is a factor of p. Then

$$6y(p/y)+y-p/y = 6p+y-p/y < 5p+1 = 6p-p+1$$

$$y-p/y < 1-p$$

y is a divisor of p, implies p/y is a natural number say z.

(p/y) = z, implies y = p/z. then the inequality becomes.

$$p/z - z < 1-p$$

$$p-z^2 < z-pz$$

$$p+pz < z+z^2$$

$$p(1+z) < z(1+z)$$

$p < z$ which is absurd. [since y is a divisor of p implies p/y=z also a divisor of p.]

Therefore,

The greatest number in the form 6ab+a-b contributed by 6p is 7p-1

The smallest number in the form 6ab+a-b contributed by 6p is 5p+1.

Hence if y is any divisor of p. $5p+1 \le 6p+y-p/y \le 7p-1$

Therefore, n=6k-1 a composite number implies k=6p+y-p/y, where p and y are natural numbers, and y is divisor of p, by theorem (2). And from the above arguments.

$$5p+1 \le k \le 7p-1. \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Now we go through theorem (4)

THEOREM (4)

Let n=36p+6q-1 is a natural number, where p and q are natural number, $0 \leq q < 6$.then n is a Composite number if and only if there exist an integer k such that $(6k-q)^2+4(p+k)$ a prefect square, and $(q-p+1)/7 \leq k \leq (p+q-1)/5$.

PROOF

Let n=36p+6q-1=6(6p+q)-1 a Composite number.

By theorem (2). 6p+q can be expressed in the form 6ab±(a-b).

Let ab=m, then b=m/a.

Then 6p+q=6m±(a-m/a)

6p+q=6m+a-m/a or 6p+q=6m-a+m/a.

For a and –a we use general integer variable x. since if a=x, m/a=m/x and also –a=x, m/a = m/(-x) = -m/x. Hence for both cases

6p+q=6m+x-m/x

Let m-p=k, then m=p+k.

Therefore 6p+q=6(p+k)+x-(p+k)/x

6p+q=6p+6k+x-(p+k)/x

x+6k-q-(p+k)/x=0

$x^2+(6k-q)x-(p+k)=0$ ----------(6)

this equation has integer solution, i.e x=a or x=-a satisfies this equation implies,

$(6k-q)^2+4(1)(p+k)$ is a perfect square. Otherwise the equation (6) has irrational solution only. Hence,

$(6k-q)^2+4(p+k)$ is a perfect square.

Conversely let $(6k-q)^2+4(p+k)$ is a perfect square. then the solution of the above equation is

$x = (q-6k)/2 \pm ([(6k-q)^2+4(p+k)]^{½})/2$

if q-6k is even number obviously x is an integer

if q-6k is odd number then $(6k-q)^2+4(p+k)$ also odd and

$[(6k-q)^2+4(p+k)]^{½}$ also odd. therefore [odd number/2 + odd number/2] is an integer. i.e x is an integer.

Hence if $(6k-q)^2+4(p+k)$ is a perfect square, then x is an integer satisfies above equation (6), implies x satisfies

6p+q=6(p+k)+x-(p+k)/x

But 6p+q and 6m=6(p+k) both are Natural numbers, therefore (p+k)/x is an integer, i.e x divides p+k. hence,

6p+q=6(x)[(p+k)/x]+x-(p+k)/x

m=p+k is a Natural number, if x is positive integer then -(p+k)/x is a negative integer, implies

6p+q is given in the form 6ab+a-b.

and if x is negative integer then -(p+k)/x is a positive integer. Implies 6p+q is given in the form 6ab-a+b.

Hence 6p+q can be given in the form 6ab±(a-b). By theorem (2)

6(6p+q)-1 =36p+6q-1 is Composite number.

Next $6p+q=6(p+k)+x-(p+k)/x$

Therefore $6(p+k)$, a multiple of 6 contributes a number $6p+q$ in the form $6ab+a-b$.

Hence

$5(p+k)+1 \leq 6p+q \leq 7(p+k)-1$ [since from (5)]

First we take right inequality

$6p+q \leq 7(p+k)-1$

$6p+q+1 \leq 7(p+k)$

$(6p+q+1) \leq 7p+7k$

$q+1-p \leq 7k$

$(q+1-p)/7 \leq k$ ------------------(7)

Next we take left inequaity

$5(p+k)+1 \leq 6p+q$

$5p+5k \leq 6p+q-1$

$5k \leq p+q-1$

$K \leq (p+q-1)/5$ -----------------(8)

therefore, from (7) and (8)

$(q-p+1)/7 \leq k \leq (p+q-1)/5$

Hence theorem (4) is proved.

Violation of theorem (4) is the distributive rule for Prime numbers in the form 36p+6q-1

DISTRIBUTIVE RULE (4)

Let $n=36p+6q-1$ is Natural number, where p and q are Natural numbers $0 \leq q < 6$. Then n is a Prime number if and only if There exist no k, such that $(6k-q)^2+4(p+k)$ a perfect square and $(q-p+1)/7 \leq k \leq (p+q-1)/5$

Examples

1) let p=10 and q=0,

$(q-p-1)/7 = (0-10-1)/7 = -11/7 = -1.57$ and

$(p+q+1)/5 = (0+10+1)/5 = 11/5 = 2.2$

here -1 lies in between -1.57 and 2.2. i.e $1.57 \leq -1 \leq 2.2$ such that

$(6(-1)-0)^2-4(10+(-1)) = 0$ a perfect square. Hence

$36(10)+6(0)+1 = 361$ a composite number.

But

$(q-p+1)/7 = (0-10+1)/7 = -9/7 = -1.28$ and

$(p+q-1)/5 = (0+10-1)/5 = 9/5 = 1.8$ here no integer k exist such that

$-1.28 \leq k \leq 1.8$ and $(6k-q)^2+4(p+q)$ a perfect square. Hence

$36(10)+6(0)-1 = 359$ a prime number.

2) let p=276 and q=5,

(q-p-1)/7 = (5-276-1)/7 = -272/7 = -38.85

(p+q+1)/5 = (5+276+1)/7 = 282/5 = 56.4

here no integer k exist such that $-38.85 \leq k \leq 56.4$ and

(6k-q)2-4(p+k) a perfect square. Hence

36(276)+6(5)+1 = 9967 a prime number.

But

(q-p+1)/7 = (5-276+1)/7 = -270/7 = 38.57

(p+q-1)/5 = (5+276-1)/7 = 280/5 = 56

here $-38.57 \leq 56 \leq 56$ and

$(6(56)-5)^2+4(276+56) = 331^2+4(332) = 110889$

$=333^2$ a perfect square. hence

36(276)+6(5)-1 = 9965 a composite number.

CONCLUSION

My name is **A. GABRIEL** a distance educated post graduate in mathematics. I am not guided by any professor in mathematics and any other university professional. I am continuing my research about **THEORY OF DISTRIBUTION OF PRIME NUMBERS**. then I conclude.

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