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ABOUT THEORY OF DISTRIBUTION OF PRIME NUMBERS

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ABSTRACT

In this article, two sets A and B are defined which are two subsets of natural numbers. Let k be an arbitrary natural number. k belongs to A implies 6k+1 a composite number, and k not belongs A implies 6k+1 a prime number. k belongs to B implies 6k-1 a composite number, and k not belongs to B implies 6k-1 a prime number. k belongs to A∩B implies 6k+1 and 6k-1 both are composite numbers, and k not belongs to A∪B implies 6k+1 and 6k-1 both are prime numbers. By using the above thesis, theorem (3) is proved. Theorem (3) states that any arbitrary natural number n the two closed intervals

[6(p-n-1)+2, 6p-2] and [6p+2, 6(p+n+1)-2] contains no prime number. where

$p=q(6^2-1)(12^2-1)(18^2-1)\ldots\ldots\ldots..(n^26^2-1)$ and q is any natural number.

Theorem (4) and Theorem (5) are given without proof. Since proofs of Theorem (4) and Theorem (5) are similar to proof of Theorem (3).

INTRODUCTION

Numbers are wonderful, marvelous creature of human. Numbers are classified into many types. They are Natural numbers, Whole numbers, Integers, Real numbers, Complex numbers, Rational numbers, Irrational numbers.

Natural numbers are classified into two categories

1) Prime numbers,
2) Composite numbers.

Prime numbers are Natural numbers which cannot be expressed in the form of product of two Natural numbers both greater than 1.

Composite numbers are other than Prime numbers. i.e. Which can be expressed in the form of product of two Natural numbers both greater than 1

From the definition of Prime number, 2 and 3 are Prime numbers, but 2x3=6 is a composite number. Multiples 6 are also composite numbers. Numbers in the form 6k±2 are even numbers i.e multiples of 2, hence composite numbers, Numbers in the form 6k±3 are odd multiples of 3 hence composite numbers.

Therefore, Prime numbers except 2 and 3 are in the form of 6k±1. where k is any natural number, but not for all natural numbers. For some Natural number k, 6k+1 is a Prime number but 6k-1 is a Composite number. For some Natural number k, 6k-1 is a Prime number but 6k+1 is a Composite numbers. For some Natural number k, 6k+1 and 6k-1 both are Prime numbers (twin Prime numbers). For some Natural number k, 6k+1 and 6k-1 both are Composite numbers. Hence this k is the key factor that determines Prime numbers and Composite numbers.

Before 2500 years ago Euclid proved that Prime numbers are infinite, Composite numbers generated by their prime factors, but Prime numbers are not generated. They are distributed among the gaps left by Composite numbers. This article is about Theory of distribution of Prime numbers. Distributive rule of Prime numbers is nothing but violation of generating rule of Composite numbers. And since all Prime numbers except 2 and 3 are in the form 6k±1, this article is an analysis about natural number k, which determines numbers in the form 6k±1.

NUMBERS IN THE FORM 6k±1

Here after Prime number means Prime number in the form 6k±1, Composite number means Composite number in the form 6k±1 unless explicitly stated.

When we see the numbers in the form 6k±1, It is seen like a binomial expression. We can say it as binomial in 6 or binomial in k. In this article, for analysis number in the form 6k±1 is treated as binomial in 6, with constant term ±1. First let us analyze nature of this form.

Let

$P=6p\pm1$ and $Q=6q\pm1$ are two Natural numbers in the form 6k±1.

$$P \times Q = (6p\pm1)(6q\pm1) = 36pq \pm (p \pm q)6 \pm 1$$

$$= [6pq \pm (p \pm q)]\, 6 \pm 1$$

$$= 6K\pm1 \quad \text{Where } K = 6pq \pm (p \pm q)$$

Implies product of two numbers in the form 6k±1 gets the form 6k±1 again.

Let $P = (6p\pm1)$, $Q = (6q\pm1)$ and $R = (6r\pm1)$ are three Natural numbers in the form 6k±1.

$$P \times Q \times R = (6p \pm 1)(6q \pm 1)(6r \pm 1)$$

$$= 216pqr \pm (pq \pm qr \pm pr)36 \pm (p \pm q \pm r)6 \pm 1$$

$$= [36pqr \pm (pq \pm qr \pm pr)6 \pm p \pm q \pm r]6 \pm 1$$

$$= 6K \pm 1$$

where $K = 36pqr \pm (pq \pm qr \pm pr)6 \pm p \pm q \pm r$

Hence product of three numbers in the form 6k±1 again gets the form 6k±1. The above argument says multiplication is closed binary operation among set of natural numbers in the form 6k±1.It is obvious 2 and 3 cannot be a factor of the numbers in the form 6k± 1.

Hence a number in the 6k±1 is Composite number if and only if 6k ± 1 can be factored as

$$6k \pm 1 = (6p \pm 1)(6q \pm 1)$$

Where $6p \pm 1$ and $6q \pm 1$ are two natural numbers in the form $6k \pm 1$, p and q not necessarily distinct.

Hence the number in the form $6k \pm 1$ is Prime number if and only if it cannot be factored as above.

The following two theorems are realized by myself before five years. But now I have heard that two theorems are already proved. In my previous article "article 1 about theory of distribution of prime numbers." I have given that two theorems with proof. In this article I am giving that two theorems with reformulation.

THEOREM (1)

Let n=6k+1, where k is any Natural number. n=6k+1 is a Composite number if and only if k can be expressed in the form k=6ab±(a+b). where a and b are Natural numbers not necessarily distinct.

REFORMULATION OF THEOREM (1)

Theorem (1) says 6k+1 is a Composite number if and only if k=6ab+a+b (or) k=6ab-(a+b) where a and b are natural numbers not necessarily distinct.

Let k=6ab+a+b

k=a(6b+1)+b

a is any natural number implies k>b and k belongs to residue class [b] modulo 6b+1. i.e k>b and $k \in [b]_{6b+1}$. Hereafter $[x]_y$ denotes residue class [x] modulo y.

let

$I_n = \{ x / x \in N, x \in [n]_{6n+1} \ \& \ x > n \}$

Where n is any natural number. and N is set of Natural number.

Obviously $k \in I_b$

b is any Natural number implies

$$k \in \bigcup_{n=1}^{\square} I_n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Similarly, on the other hand.

Let k=6ab-(a+b)

k=6ab-a-b

k=a(6b-1)-b

a is any Natural number implies

$k \in [-b]_{6b-1}$ & k>b i.e k>b and belongs to residue class[-b] modulo 6b-1

Let

$I_{-n} = \{ x / x \in N, x \in [-n]_{6n-1} \ \& \ x > n \}$

Where n is any natural number

Obviously $k \in I_{-b}$

b is any Natural number implies

$$k \in \bigcup_{n=1}^{\square} I_{-n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Hence from (1) and (2), 6k+1 is a composite number implies,

$$k \in \left(\bigcup_{n=1}^{\square} I_n\right) \cup \left(\bigcup_{n=1}^{\square} I_{-n}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

we know that if 6x±1 is a divisor of 6n±1. then any residue class of modulo 6n±1 is a subset of one of the residue classes of modulo 6x±1. is it true for $I_{\pm n}$ and $I_{\pm x}$?

let k = m(6n+1)+n belongs to I_n. if 6n+1 is a composite number, say 6n+1 = (6x+1)(6y+1) =6(6xy+x+y)+1.

Implies n = 6xy+x+y.

k = m(6x+1)(6y+1)+6xy+x+y.

k = m(6x+1)(6y+1)+x(6y+1)+y.

k = (m(6x+1)+x)(6y+1)+y.

$k \in \mathbf{I_y}$

And also

k = m(6x+1)(6y+1)+6xy+y+x.

k = m(6x+1)(6y+1)+y(6x+1)+x

k = (m(6y+1)+y)(6x+1)+x.

$k \in \mathbf{I_x}$

$I_n \subset I_x$ and also $I_n \subset I_y$

If 6n+1 = (6x-1)(6y-1) = 6(6xy-x-y)+1.

Implies n = 6xy-x-y.

$k = m(6x-1)(6y-1)+6xy-x-y.$

$k = m(6x-1)(6y-1)+x(6y-1)-y.$

$k = (m(6x-1)+x)(6y-1)-y.$

$k \in I_{-y}$

And also

$k = m(6x-1)(6y-1)+6xy-x-y.$

$k = m(6x-1)(6y-1)+y(6x-1)-x.$

$k = (m(6y-1)+y)(6x-1)-x.$

$k \in I_{-x}$

$I_n \subset I_{-x}$ and also $I_n \subset I_{-y}$

Similarly,

let $k = m(6n-1)-n$ belongs to I_{-n}. if $6n-1$ is a composite number, say $6n-1 = (6x+1)(6y-1) = 6(6xy-x+y)-1$.

Implies $n = 6xy-x+y$.

$k = m(6x+1)(6y-1)-6xy+x-y.$

$k = m(6x+1)(6y-1)-x(6y-1)-y.$

$k = (m(6x+1)-x)(6y-1)-y.$

$k \in \mathbf{I}_{-y}$

And also

$k = m(6x+1)(6y-1)-6xy-y+x.$

$k = m(6x+1)(6y-1)-y(6x+1)+x$

$k = (m(6y-1)-y)(6x+1)+x.$

$k \in \mathbf{I}_{x}$

$I_{-n} \subset I_x$ and also $I_{-n} \subset I_{-y}$

If $6n+1 = (6x-1)(6y+1).$

Here x and y are just interchanged. Therefore,

$I_{-n} \subset I_{-x}$ and also $I_{-n} \subset I_y$

Therefore, <u>$6x\pm1$ is a divisor of $6n\pm1$, implies $I_{\pm n}$ is a subset of $I_{\pm x}$</u>

And also we know that if $6n\pm1$ is a prime number, then any residue class of modulo $6n\pm1$ is cannot be a proper subset of any residue class of any modulo.

Therefore, <u>$6n+1$ is a prime number implies, I_n is not a proper subset of any $I_{\pm x}$. and $6n-1$ is a prime number implies, I_{-n} is not a proper subset of any $I_{\pm x}$.</u>

Considering all the above argument, (3) becomes,

$$K \in \Big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\square} I_n \Big) \ \cup \ \Big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\square} I_{-n} \Big)$$

Now we define a subset of Natural number N

$$A = \Big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\infty} I_n \Big) \cup \Big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\infty} I_{-n} \Big)$$

Above arguments imply, if n=6k+1 is a composite number, then k belongs to A.

Conversely, let k be any natural number that belongs to

$$A = \Big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\infty} I_n \Big) \cup \Big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\infty} I_{-n} \Big)$$

Implies,

$$K \in \bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\infty} I_n \qquad \textbf{or} \qquad \mathbf{k} \in \bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\infty} I_{-n}$$

Case (1)

$$K \in \bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\infty} I_n$$

Implies k belongs to some I_n. and by the definition of I_n k must be in the form,

$k=m(6n+1)+n = 6mn+m+n.$ where m and n are natural numbers. implies,

6k+1 a composite number by theorem (1).

Similarly,

Case (2)

$$K \in \bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\infty} I_{-n}$$

Implies k belongs to some I_{-n}. and by the definition of I_{-n} k must be in the form,

$k=m(6n-1)-n = 6mn-m-n$ where m and n are natural numbers. implies,

6k+1 a composite number by theorem (1).

Therefore, k belongs to A implies 6k+1 is a composite number.

Now we restate theorem (1) as follows

THEOREM (1)

The number n=6k+1 is Composite number if and only if $k \in A$. where

$$A = \left(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\square} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\square} I_{-n}\right)$$

DISTRIBUTIVE RULE (1)

The number $n=6k+1$ is Prime number if and only if $k \notin A$.

THEOREM (2)

Let $n=6k-1$, where k is any Natural number. $n=6k-1$ is a Composite number if and only if k can be expressed in the form $k=6ab\pm(a-b)$. where a and b are Natural numbers not necessarily distinct.

REFORMULATION OF THEOREM (2)

Theorem (2) says $6k-1$ is Composite number if and only if $k= 6ab+a-b$ (or) $k=6ab-a+b$ where a and b are natural numbers not necessarily distinct.

Let $k=6ab+a-b$

$k=a(6b+1)-b$

a is any natural number implies $k > b$ and k belongs to residue class $[-b]$ modulo $6b+1$. i.e $k\in[-b]_{6b+1}$ and $k > b$.

let

$J_{-n} = \{ x/ x\in N, x\in [-n]_{6n+1} \ \& \ x>n\}$

Where n is any natural number.

Obviously $k \in J_{-b}$

b is any Natural number implies

$$k \in \bigcup_{n=1}^{\square} J_{-n} \qquad \text{----------(4)}$$

Similarly, on the other hand.

Let $k=6ab-(a-b)$

$k=6ab-a+b$

$k=a(6b-1)+b$

a is any Natural number implies

$k \in [b]_{6b-1} \ \& \ k>b$

i.e $k>b$ and k belongs to residue class$[b]$ modulo $6b-1$

Let $J_n = \{ x/ x\in N, x\in [n]_{6n-1} \ \& \ x>n\}$

Where n is any natural number.

Obviously $k \in J_b$

b is any Natural number implies

$$k \in \bigcup_{n=1}^{\square} J_n \qquad \text{----------(5)}$$

Hence from (4) and (5), 6k-1 is a composite number implies,

$$K \in \left(\bigcup_{n=1}^{\square} J_n\right) \cup \left(\bigcup_{n=1}^{\square} J_{-n}\right) \ldots\ldots\ldots\ldots(6)$$

we know that if $6x\pm1$ is a divisor of $6n\pm1$. then any residue class of modulo $6n\pm1$ is a subset of one of the residue classes of modulo $6x\pm1$. is it true for $J_{\pm x}$ and $J_{\pm n}$.

let $k = m(6n+1)-n$ belongs to J_{-n}. if $6n+1$ is a composite number, say $6n+1 = (6x+1)(6y+1) =6(6xy+x+y)+1$.

Implies $n = 6xy+x+y$.

$k = m(6x+1)(6y+1)-6xy-x-y.$

$k = m(6x+1)(6y+1)-x(6y+1)-y.$

$k = (m(6x+1)-x)(6y+1)-y.$

$k \in J_{-y}$

And also

$k = m(6x+1)(6y+1)-6xy-y-x.$

$k = m(6x+1)(6y+1)-y(6x+1)-x$

$k = (m(6y+1)-y)(6x+1)-x.$

$k \in J_{-x}$

$J_{-n} \subset J_{-x}$ and also $J_{-n} \subset J_{-y}$

If $6n+1 = (6x-1)(6y-1) = 6(6xy-x-y)+1.$

Implies $n = 6xy-x-y.$

$k = m(6x-1)(6y-1)-6xy+x+y.$

$k = m(6x-1)(6y-1)-x(6y-1)+y.$

$k = (m(6x-1)-x)(6y-1)+y.$

$k \in J_y$

And also

$k = m(6x-1)(6y-1)-6xy+x+y.$

$k = m(6x-1)(6y-1)-y(6x-1)+x.$

$k = (m(6y-1)-y)(6x-1)+x.$

$k \in J_x$

$J_{-n} \subset J_x$ and also $J_{-n} \subset J_y$

let $k = m(6n-1)+n$ belongs to J_n. if $6n-1$ is a composite number, say $6n-1 = (6x+1)(6y-1) =6(6xy-x+y)-1.$

Implies $n = 6xy-x+y.$

$k = m(6x+1)(6y-1)+6xy-x+y.$

$k = m(6x+1)(6y-1)+x(6y-1)+y.$

$k = (m(6x+1)+x)(6y-1)+y.$

$k \in J_y$

And also

$k = m(6x+1)(6y-1)+6xy+y-x.$

$k = m(6x+1)(6y-1)+y(6x+1)-x$

$k = (m(6y-1)+y)(6x+1)-x.$

$k \in J_{-x}$

$J_n \subset J_{-x}$ and also $J_n \subset J_y$

If $6n+1 = (6x-1)(6y+1).$

Here x and y are just interchanged. therefore,

$J_n \subset J_x$ and also $J_n \subset J_{-y}$

Therefore, <u>6x±1 is a divisor of 6n±1, implies $J_{\pm n}$ is a subset of $J_{\pm x}$</u>

And also we know that if 6n±1 is a prime number, then any residue class of modulo 6n±1 is cannot be a proper subset of any residue class of any modulo.

Therefore, <u>6n+1 is a prime number implies, J_{-n} is not a proper subset of any $J_{\pm x}$. and 6n-1 is a prime number implies, J_n is not a proper subset of any $J_{\pm x}$.</u>

Considering all the above arguments (6) becomes.

□ □
K ∈ (UJ_n) U (UJ_{-n})
n=1 & n=1 &
6n-1 a prime number. 6n+1 a prime number.

Now we define a subset of Natural number N

□ □
B = (UJ_n) U (UJ_{-n})
n=1 & n=1 &
6n-1 a prime number. 6n+1 a prime number.

Above arguments imply, if n=6k-1 is a composite number, then k belongs to B.

Conversely, let k belongs to

□ □
B = (UJ_n) U (UJ_{-n})
n=1 & n=1 &
6n-1 a prime number. 6n+1 a prime number.

Implies,

□ □
K ∈ UJ_n **or** **k** ∈ UJ_{-n}
n=1 & n=1 &
6n-1 a prime number. 6n+1 a prime number.

Case (1) □
K ∈ UJ_n
n=1 &
6n-1 a prime number.

Implies k belongs to some J_n. and by the definition of J_n k must be in the form,

k=m(6n-1)+n = 6mn-m+n where m and n are natural numbers. implies,

6k-1 a composite number by theorem (2).

Similarly,

Case (2)

$K \in \bigcup J_{-n}$

n=1 &

6n+1 a prime number.

Implies k belongs to some J_{-n}. and by the definition of J_{-n} k must be in the form,

k=m(6n+1)-n = 6mn+m-n where m and n are natural numbers. implies,

6k-1 a composite number by theorem (2).

Therefore, k belongs to B implies 6k-1 is a composite number.

Now we restate theorem (2) as follows

THEOREM (2)

The number n=6k-1 is Composite number if and only if $k \in B$.

where

$$B = \left(\bigcup J_n\right) \cup \left(\bigcup J_{-n}\right)$$

n=1 & 6n-1 a prime number. n=1 & 6n+1 a prime number.

DISTRIBUTIVE RULE (2)

The number n=6k-1 is Prime number if and only if $k \notin B$.

theorem (1), theorem (2), distribution rule (1) and distribution rule (2) imply the following facts about theory of distribution prime numbers

THEORY OF DISTRIBUTION OF PRIME NUMBERS

A and B are subsets of natural numbers as defined above and k is any natural number.

i) if $k \in A$ implies 6k+1 is a Composite number.
ii) if $k \in B$ implies 6k-1 is a Composite number.
iii) if $k \in A \cap B$ implies 6k+1 and 6k-1 both are Composite numbers.
iv) if $k \notin A$ implies 6k+1 is a Prime number.
v) if $k \notin B$ implies 6k-1 is a Prime number.

vi) if $k \notin [A \cup B]$ implies 6k+1 and 6k-1 both are Prime number i.e twin Prime number. In other word if $k \neq 6ab \pm a \pm b$ where a and b are natural number not necessarily distinct, then 6k+1 and 6k-1 both are Prime numbers. Intuitively it is obvious that not all the numbers at infinite can be expressed in the form $6ab \pm a \pm b$. i.e set of natural numbers which cannot be expressed in the form $k=6ab \pm a \pm b$ is infinite. But this infinite subset of natural numbers determines twin Prime numbers. Hence twin Prime numbers are infinite

vii) From the above thesis it is clear 1, 2, & 3 are not belongs to $A \cup B$. Hence 5 & 7, 11 & 13, & 17 & 19 are three consecutive pairs of twin Prime numbers. But except 1, 2, & 3 for any other three consecutive Natural numbers greater than 1, at least One must be contained in $[1]_5$ or $[-1]_5 = [4]_5$. i.e at least one must be contained in I_{-1} of A or J_1 of B. Therefore, no other three consecutive Natural numbers exist such that not belongs to A∪B. Hence except 5 & 7, 11 & 13, & 17 & 19, no other three consecutive pairs of twin Prime numbers exist.

viii) Similarly, for every five consecutive Natural numbers say k_1, k_2, k_3, k_4 & k_5 all greater than 1, at least one must be contained in I_{-1} of A, and at least one must be contained in J_1 of B, hence among $6k_1+1$, $6k_2+1$, $6k_3+1$, $6k_4+1$, & $6k_5+1$ at least one must be a Composite number, and among $6k_1-1$, $6k_2-1$, $6k_3-1$, $6k_4-1$, & $6k_5-1$ at least one must be a Composite number.

ix) 4 is the smallest Natural number contained in A i.e $4 \in I_{-1}$, hence 6(4)+1=25 is the smallest Composite number in the form 6k+1, similarly, 6 is the smallest Natural number contained in B i.e $6 \in J_1$,hence 6(6)-1=35 is the smallest Composite number in the form 6k-1.

x) 20 smallest Natural number contained in A∩B i.e $20 \in I_{-2}$ of A & $20 \in J_{-1}$ of B . Hence 6(20)-1=119 & 6(20)+1=121 is the smallest pair Composite number in the form 6k-1 & 6k+1

xi) The above thesis shows clearly that the set of Natural numbers belongs to A are irregularly distributed. Therefore, Composite numbers in the form 6k+1 are irregularly generated, hence Prime numbers in the form 6k+1 are irregularly distributed.

xii) Similarly, set of Natural numbers belongs to B are irregularly distributed. Therefore, Composite numbers in the form 6k-1 are irregularly generated, hence Prime numbers in the form 6k-1 are irregularly distributed. Since Prime numbers just fill the gaps left by Composite numbers.

THEOREM (3)

For any natural number n, the following two closed intervals [6(p-n-1)+2, 6p-2] and [6p+2, 6(p+n+1)-2] contains no prime number. where $p=q(6^2-1)(2^26^2-1)(3^26^2-1)……….(n^26^2-1)$, and q is any natural number.

PROOF

Let $p=q(6^2-1)(2^26^2-1)(3^26^2-1)…….(n^26^2-1)$

i.e $p = q \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times …….\times (6n-1) \times (6n+1)$.

let r is any natural number, such that $r \le n$.

obviously r^26^2-1 is a factor of p.

i.e $p/(r^26^2-1)$ a natural number.

$$p = (r^26^2-1) \times p/(r^26^2-1)$$
$$p - r = (r^26^2-1) \times p/(r^26^2-1) - r$$
$$p - r = (6r-1) \times (6r+1) \times p/(r^26^2-1) - r$$

p - r belongs to $[-r]_{6r-1}$ and obviously (p-r) > r. p - r belongs to I_{-r}.
as we have seen in reformulation of theorem (1), for any natural number r, $I_{\pm r} \subset A$.
Hence p-r belongs to A. implies

6(p-r)+1 is a composite number.

And also p-r can be rewritten as,

$$p - r = (6r+1) \times (6r-1) \times p/(r^26^2-1) - r$$

p - r belongs to $[-r]_{6r+1}$ and obviously (p-r) > r. p - r belongs to J_{-r}. as we have seen in reformulation of theorem (2), for any natural number r, $J_{\pm r} \subset B$.
Hence p-r belongs to B. implies

6(p-r)-1 is a composite number.

Therefore for any $r \le n$, i.e any natural number $r \in [1, n]$, 6(p-r)-1 and 6(p-r)+1 both are composite number.

hence for $k \in [p-n, p-1]$, 6k+1 and 6k-1 both are composite number. and since all the prime numbers greater than 3 are in the form 6k±1 implies, the closed interval, [6(p-n)-1, 6(p-1)+1] contains no prime number. and 6(p-n)-4, 6(p-n)-3, 6(p-n)-2, & 6(p-1)+2, 6(p-1)+3, & 6(p-1)+4 are also composite numbers. Implies the closed interval [6(p-n)-4, 6(p-1)+4] contains no prime number. but 6(p-n)-4=6(p-n-1)+2 and 6(p-1)+4=6p-2 implies the closed interval

[6(p-n-1)+2, 6p-2] contains no prime number.

Similarly,

$$p + r = (r^2 6^2 - 1) \times p/(r^2 6^2 - 1) + r$$
$$p + r = (6r-1) \times (6r+1) \times p/(r^2 6^2 - 1) + r$$

p + r belongs to $[r]_{6r-1}$ and obviously (p+r) > r. p + r belongs to J_r. as we have seen in reformulation of theorem (2), for any natural number r, $J_{\pm r} \subset B$.

Hence p+r belongs to B. implies

6(p+r)-1 a composite number.

And also p + r can be rewritten as,

$$p + r = (6r+1) \times (6r-1) \times p/(r^2 6^2 - 1) + r$$

p + r belongs to $[r]_{6r+1}$ and obviously (p+r) > r. p + r belongs to I_r. as we have seen in reformulation of theorem (1), for any natural number r, $I_{\pm r} \subset A$.

Hence p+r belongs to A. implies

6(p+r)+1 a composite number.

therefore for any $r \leq n$, i.e any natural number $r \in [1, n]$, 6(p+r)-1 and 6(p+r)+1 both are composite number.

Hence for $k \in [p+1, p+n]$, 6k+1 and 6k-1 both are composite numbers. and since all the prime numbers greater than 3 are in the form 6k±1 implies, the closed interval, [6(p+1)-1, 6(p+n)+1] contains no prime number. and 6(p+1)-4, 6(p+1)-3, 6(p+1)-2, & 6(p+n)+2, 6(p+n)+3, 6(p+n)+4 are also composite numbers. Implies [6(p+1)-4, 6(p+n)+4] contains no prime number.

but 6(p+1)-4=6p+2 and 6(p+n)+4=6(p+n+1)-2 implies the closed interval

[6p+2, 6(p+n+1)-2] contains no prime number.

Hence the theorem.

REMARK

Both closed intervals [6(p-n-1)+2, 6p-2] and

[6p+2, 6(p+n+1)-2] are having same length 6n+3.

i.e Each closed interval contains 6n+3 natural numbers.

And

$$p = q(6^2-1)(2^2 6^2-1)(3^2 6^2-1)\ldots\ldots\ldots(n^2 6^2-1),$$

and q is any natural number. hence theorem (3) Implies, for every natural number n, there exist infinite number of closed intervals in the form, [6(p-n-1)+2, 6p-2] & [6p+2, 6(p+n+1)-2] such that contains no prime number, and having length 6n+3.

Similarly, we can prove the following theorems. The following Theorems are given without proof. Since the proofs are similar to the proof of theorem (3).

THEOREM (4)

For any natural number n, the following two closed intervals [6(p-n), 6p-2] and [6p+2, 6(p+n)] contains no prime number. where $p=q(6^2-1)(2^2 6^2-1)(3^2 6^2-1)\ldots\ldots((n-1)^2 6^2-1)(6n-1)$, and q is any natural number.

Here closed intervals [6(p-n), 6p-2] and [6p+2, 6(p+n)] are having length 6n-1. As in above remark, theorem (4) implies, for every natural number n, there exist infinite number of closed intervals in the form, [6(p-n), 6p-2] & [6p+2, 6(p+n)] such that contains no prime number, and having length 6n-1.

THEOREM (5)

For any natural number n, the following two closed intervals [6(p-n-1), 6(p-1)] and [6(p+1), 6(p+n+1)] contains no prime number.

where $p=q(6+1)(2^2 6^2-1)(3^2 6^2-1)\ldots.(n^2 6^2-1)(6(n+1)-1)$, and q is any natural number.

Here closed intervals [6(p-n-1), 6(p-1)] and [6(p+1), 6(p+n+1)] are having length 6n+1. As in above remark, theorem (5) implies, for every natural number n, there exist infinite number of closed intervals in the form, [6(p-n-1), 6(p-1)] & [6(p+1), 6(p+n+1)] such that contains no prime number, and having length 6n+1.
Hence theorem (3), theorem (4) & theorem (5) implies the following facts about.

THEORY OF DISTRIBUTION OF PRIME NUMBERS

For any arbitrary natural number n.

i) there exist infinite number of closed intervals in the form, [6(p-n-1)+2, 6p-2] & [6p+2, 6(p+n+1)-2] such that contains no prime number, and having length 6n+3.
where $p=q(6^2-1)(2^2 6^2-1)(3^2 6^2-1)\ldots\ldots\ldots(n^2 6^2-1)$, and q is any natural number.

ii) there exist infinite number of closed intervals in the form, [6(p-n), 6p-2] & [6p+2, 6(p+n)] such that contains no prime number, and having length 6n-1.
where $p=q(6^2-1)(2^2 6^2-1)(3^2 6^2-1)\ldots\ldots((n-1)^2 6^2-1)(6n-1)$, and q is any natural number.

iii) there exist infinite number of closed intervals in the form, [6(p-n-1), 6(p-1)] & [6(p+1), 6(p+n+1)] such that contains no prime number, and having length 6n+1.

where $p=q(6+1)(2^2 6^2-1)(3^2 6^2-1)\ldots.(n^2 6^2-1)(6(n+1)-1)$, and q is any natural number.

CONCLUTION

My name is **A. GABRIEL** a distance educated post graduate in mathematics. The thesis what we discussed above is myself realized one. Here I have submitted my completed concepts only. I am continuing my research about **THEORY OF DISTRIBUTION OF PRIME NUMBERS** by analyzing numbers which can be expressed in form 6ab±a±b, and which cannot be expressed in the form 6ab±a±b. i.e by analyzing the sets A, B, A^c, B^c, $A \cup B$, $(A \cup B)^c$, $A \cap B$, and $(A \cap B)^c$, where A and B are as defined above and the set of Natural numbers as universal set. then I conclude.

By

A. GABRIEL

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