



Inverse Signed Total Domination of Corona Product of a Path with a Complete Graphs

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Abstract: Graph theory is one of the important branch of mathematics and find the applications in several branches of Science & Technology. In this paper, we study the maximal inverse signed dominating functions, maximal inverse signed total dominating functions of Corona Product graph of a path with a complete graph denoted by $G = P_n \square K_m$, Here P_n denotes the path with n vertices and K_m denotes the complete graph with m vertices. This graph is useful in communication networks and also in internet services.

Keywords: Corona Product Graph, Inverse Signed Dominating Functions, Inverse Signed Total Dominating Functions, Inverse Signed Domination Number, Inverse Signed Total Domination Number.

Subject Classification: 68R10

1. Introduction

Cockayne et al. [3] have studied towards a theory of domination in graphs. Inverse domination and inverse total domination concepts are introduced by Kulli [7,8,9,10]. Allan and Laskar [1] have studied on domination, Independent domination numbers of a graph. Domke et al. [4] have studied the inverse domination number of a graph.

Now we introduce the concept of inverse signed domination as follows:

Let $f : V \rightarrow \{-1, +1\}$ is called an inverse signed dominating function (ISDF) of G , if $f[N(v_i)] = \sum_{u \in N[v_i]} f(u) \leq 0$, for

each $v \in V$. An ISDF f of G is called a Maximal ISDF, if for all $g > f$, g is not an ISDF. The weight of f , denoted $f(G)$, is the sum of the function value of all vertices in G . That is $f(G) = \sum_{x \in V} f(x)$. The inverse signed domination number (ISDN) of G is denoted by $\gamma_s^0(G)$.

We studied inverse signed total domination in [2, 11]. Huang et al. [6] introduce the concept of Inverse signed total domination numbers as follows:

If $f : V \rightarrow \{-1, +1\}$ is called an inverse signed total dominating function (ISTDF) of G , if $f(N(v_i)) = \sum_{u \in N(v_i)} f(u) \leq 0$, for each $v \in V$. An ISTDF f of G is called a Maximal ISTDF, if for all $g > f$, g is not a ISTDF.

The weight of f , denoted $f(G)$, is the sum of the function value of all vertices in G . That is $f(G) = \sum_{x \in V} f(x)$. The inverse

signed total domination number (ISTDN) of G is denoted by $\gamma_{st}^0(G)$. Frucht and Harary [5] introduced a new product on two graphs G_1 and G_2 , called corona product denoted by $G_1 \square G_2$.

2. Corona Product of a Path P_n with a Complete Graph K_m

The corona product of a path P_n with a complete graph K_m is a graph $P_n \square K_m$ obtained by taking one copy of a n -vertex path P_n and n copies of K_m and then joining the i^{th} vertex of P_n to every vertex of i^{th} copy of K_m .

3. Inverse Signed Dominating Functions

Theorem 3.1: A function $f : V \rightarrow \{-1, +1\}$ is defined by $f(v_i) = \begin{cases} +1, & \text{if } 1 \leq i \leq \left(\frac{m}{2}\right) \text{ of each copy of } K_m \text{ in } G \\ -1, & \text{otherwise} \end{cases}$

is a maximal inverse signed dominating function (MISDF) of a graph $G = P_n \square K_m$ and ISDN is $\gamma_s^0(G) = -n$, if m is even.

Proof:

Consider the graph $G = P_n \square K_m$ with $|V|$ number of vertices and $|E|$ number of edges.

Let f be a function defined in the hypothesis.

Case (1): Let $v_i \in P_n$ be such that $d(v_i) = (m+2)$ in G , then $N[v_i]$ contains m vertices of K_m and three vertices of P_n in G .

Thus $\sum_{u \in N[v_i]} f(u) = (-1) + (-1) + (-1) + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}\right)(+1) \right] = -3 \Rightarrow f$ is an ISDF.

Case (2): Let $v_i \in P_n$ be such that $d(v_i) = (m+1)$ in G , then $N[v_i]$ contains m vertices of K_m and two vertices of P_n in G .

Thus $\sum_{u \in N[v_i]} f(u) = (-1) + (-1) + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}\right)(+1) \right] = -2 \Rightarrow f$ is an ISDF.

Case (3): Let $v_i \in K_m$ be such that $d(v_i) = m$ in G , then $N[v_i]$ contains m vertices of K_m and one vertex of P_n in G and $f(v_i) = -1$ or $+1$.

If $f(v_i) = \pm 1$, then $\sum_{u \in N[v_i]} f(u) = (-1) + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}\right)(+1) \right] = -1 \Rightarrow f$ is an ISDF.

Hence for all the above possibilities, we get $\sum_{u \in N[v_i]} f(u) < 0, \forall v_i \in V$

This implies that the function f is an ISDF.

Now we check for maximality of f , define $g : V \rightarrow \{-1, +1\}$ by

$$g(v_i) = \begin{cases} +1, & \text{if } 1 \leq i \leq \frac{m}{2} \text{ of each copy of } K_m \text{ in } G \\ +1, & \text{if } v_i = v_k \in P_n \text{ in } G \\ -1, & \text{otherwise} \end{cases}$$

Case (1): Let $v_i \in P_n$ be such that $d(v_i) = (m+2)$ in G , then $N[v_i]$ contains m vertices of K_m and three vertices of P_n in G .

If $v_k \in N[v_i]$, then $\sum_{u \in N[v_i]} g(u) = 1 + (-1) + (-1) + \left[\left(\frac{m}{2} \right) (+1) + \left(\frac{m}{2} \right) (-1) \right] = -1$

If $v_k \notin N[v_i]$, then $\sum_{u \in N[v_i]} g(u) = (-1) + (-1) + (-1) + \left[\left(\frac{m}{2} \right) (+1) + \left(\frac{m}{2} \right) (-1) \right] = -3$

In this case g is an ISDF.

Case (2): Let $v_i \in P_n$ be such that $d(v_i) = (m+1)$ in G , then $N[v_i]$ contains m vertices of K_m and two vertices of P_n in G .

If $v_k \in N[v_i]$, then $\sum_{u \in N[v_i]} g(u) = 1 + (-1) + \left[\left(\frac{m}{2} \right) (+1) + \left(\frac{m}{2} \right) (-1) \right] = 0 \Rightarrow g$ is an ISDF.

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Case (3): Let $v_i \in K_m$ be such that $d(v_i) = m$ in G , then $N[v_i]$ contains m vertices of K_m and one vertex of P_n in G and. $g(v_i) = -1$ or $+1$

(i) Let $v_k \in N[v_i]$

If $g(v_i) = \pm 1$, then $\sum_{u \in N[v_i]} g(u) = 1 + \left[\left(\frac{m}{2} \right) (+1) + \left(\frac{m}{2} \right) (-1) \right] = 1 \Rightarrow g$ is not an ISDF.

(ii) Let $v_k \notin N[v_i]$

If $g(v_i) = \pm 1$, then $\sum_{u \in N[v_i]} g(u) = (-1) + \left[\left(\frac{m}{2} \right) (+1) + \left(\frac{m}{2} \right) (-1) \right] = -1 \Rightarrow g$ is an ISDF.

This implies that g is not an ISDF, because $\sum_{u \in N[v_i]} g(u) > 0$, for some $v_i \in V$

Hence f is a maximal inverse signed dominating function on G .

Now $\sum_{u \in V(G)} f(u) = \underbrace{(-1) + \dots + (-1)}_{n\text{-times}} + \underbrace{\left[\left(\frac{m}{2} \right) (+1) + \left(\frac{m}{2} \right) (-1) \right]}_{n\text{-times}} = -n$

Finally, ISDN is $\gamma_s^0(G) = -n$, if m is even.

Theorem 3.2: A function $f : V \rightarrow \{-1, +1\}$ is defined by $f(v_i) = \begin{cases} +1, & \text{if } 1 \leq i \leq \left(\frac{m+1}{2} \right) \text{ of each copy of } K_m \text{ in } G \\ -1, & \text{otherwise} \end{cases}$

is a maximal inverse signed dominating function (MISDF) of a graph $G = P_n \square K_m$ and ISDN is $\gamma_s^0(G) = 0$, if m is odd.

Proof: Let f be a function defined in the hypothesis.

Case (1): Let $v_i \in P_n$ be such that $d(v_i) = (m+2)$ in G , then $N[v_i]$ contains m vertices of K_m and three vertices of P_n in G .

Thus $\sum_{u \in N[v_i]} f(u) = (-1) + (-1) + (-1) + \left[\left(\frac{m+1}{2} \right) (+1) + \left(\frac{m-1}{2} \right) (-1) \right] = -2$

Case (2): Let $v_i \in P_n$ be such that $d(v_i) = m+1$ in G , then $N[v_i]$ contains m vertices of K_m and two vertices of P_n in G .

Thus $\sum_{u \in N[v_i]} f(u) = (-1) + (-1) + \left[\left(\frac{m+1}{2} \right) (+1) + \left(\frac{m-1}{2} \right) (-1) \right] = -1$

Case (3): Let $v_i \in K_m$ be such that $d(v_i) = m$ in G , then $N[v_i]$ contains m vertices of K_m and one vertex of P_n in G and $f(v_i) = -1$ or $+1$.

$$\text{If } f(v_i) = \pm 1, \text{ then } \sum_{u \in N[v_i]} f(u) = (-1) + \left[\binom{m+1}{2} (+1) + \binom{m-1}{2} (-1) \right] = 0$$

Hence for all the above possibilities, we get $\sum_{u \in N[v_i]} f(u) \leq 0, \forall v_i \in V$

This implies that the function f is an ISDF.

Now we check for maximality of f , define $g: V \rightarrow \{-1, +1\}$ by

$$g(v_i) = \begin{cases} +1, & \text{if } 1 \leq i \leq \frac{m+1}{2} \text{ of each copy of } K_m \text{ in } G \\ +1, & \text{if } v_i = v_k \in P_n \text{ in } G \\ -1, & \text{otherwise} \end{cases}$$

Case (1): Let $v_i \in P_n$ be such that $d(v_i) = (m+2)$ in G , then $N[v_i]$ contains m vertices of K_m and three vertices of P_n in G .

$$\text{If } v_k \in N[v_i], \text{ then } \sum_{u \in N[v_i]} g(u) = 1 + (-1) + (-1) + \left[\binom{m-1}{2} (-1) + \binom{m+1}{2} (+1) \right] = 0$$

$$\text{If } v_k \notin N[v_i], \text{ then } \sum_{u \in N[v_i]} g(u) = (-1) + (-1) + (-1) + \left[\binom{m-1}{2} (-1) + \binom{m+1}{2} (+1) \right] = -2$$

Case (2): Let $v_i \in P_n$ be such that $d(v_i) = m+1$ in G , then $N[v_i]$ contains m vertices of K_m and two vertices of P_n in G .

$$\text{If } v_k \in N[v_i], \text{ then } \sum_{u \in N[v_i]} g(u) = 1 + (-1) + \left[\binom{m-1}{2} (-1) + \binom{m+1}{2} (+1) \right] = 1$$

$$\text{If } v_k \notin N[v_i], \text{ then } \sum_{u \in N[v_i]} g(u) = (-1) + (-1) + \left[\binom{m-1}{2} (+1) + \binom{m+1}{2} (-1) \right] = -3$$

Case (3): Let $v_i \in K_m$ be such that $d(v_i) = m$ in G , then $N[v_i]$ contains m vertices of K_m and one vertex of P_n in G and $g(v_i) = -1$ or $+1$.

(i) Let $v_k \in N[v_i]$

$$\text{If } g(v_i) = \pm 1, \text{ then } \sum_{u \in N[v_i]} g(u) = 1 + \left[\binom{m-1}{2} (-1) + \binom{m+1}{2} (+1) \right] = 2$$

(ii) Let $v_k \notin N[v_i]$

$$\text{If } g(v_i) = \pm 1, \text{ then } \sum_{u \in N[v_i]} g(u) = (-1) + \left[\binom{m-1}{2} (-1) + \binom{m+1}{2} (+1) \right] = 0$$

This implies that g is not an ISDF, because $\sum_{u \in N[v_i]} g(u) > 0$, for some $v_i \in V$. Hence f is a maximal inverse signed dominating function on G .

$$\text{Now } \sum_{u \in V(G)} f(u) = \underbrace{(-1) + \dots + (-1)}_{n\text{-times}} + \left[\underbrace{\binom{m+1}{2} (+1) + \binom{m-1}{2} (-1)}_{n\text{-times}} \right] = 0$$

Finally, ISDN is $\gamma_s^0(G) = 0$, if m is odd.

4. Inverse Signed Total Dominating Functions

Theorem 4.1: A function $f : V \rightarrow \{-1, +1\}$ is defined by $f(v_i) = \begin{cases} +1, & \text{if } 1 \leq i \leq \left(\frac{m}{2}\right) \text{ of each copy of } K_m \text{ in } G \\ -1, & \text{otherwise} \end{cases}$

is a maximal inverse signed total dominating function (MISTDF) of a graph $G = P_n \square K_m$ and $\gamma_{st}^0(G) = -n$, if m is even.

Proof:

Consider the graph $G = P_n \square K_m$ with $|V|$ number of vertices and $|E|$ number of edges.

Let f be a function defined in the hypothesis.

Case (1): Let $v_i \in P_n$ be such that $d(v_i) = (m+2)$ in G , then $N(v_i)$ contains m vertices of K_m and two vertices of P_n in G .

Thus $\sum_{u \in N(v_i)} f(u) = (-1) + (-1) + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}\right)(+1) \right] = -2 \Rightarrow f$ is an ISTDF.

Case (2): Let $v_i \in P_n$ be such that $d(v_i) = (m+1)$ in G , then $N(v_i)$ contains m vertices of K_m and one vertex of P_n in G .

Thus $\sum_{u \in N(v_i)} f(u) = (-1) + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}\right)(+1) \right] = -1 \Rightarrow f$ is an ISTDF.

Case (3): Let $v_i \in K_m$ be such that $d(v_i) = m$ in G , then $N(v_i)$ contains $(m-1)$ vertices of K_m and one vertex of P_n in G and $f(v_i) = -1$ or $+1$.

If $f(v_i) = -1$, then $\sum_{u \in N(v_i)} f(u) = (-1) + \left[\left(\frac{m}{2}-1\right)(-1) + \left(\frac{m}{2}\right)(+1) \right] = 0 \Rightarrow f$ is an ISTDF.

If $f(v_i) = +1$, then $\sum_{u \in N(v_i)} f(u) = (-1) + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}-1\right)(+1) \right] = -2 \Rightarrow f$ is an ISTDF.

Hence for all the above possibilities, we get $\sum_{u \in N(v_i)} f(u) \leq 0, \forall v_i \in V$

This implies that the function f is an ISTDF. Now we check for maximality of f , define $g : V \rightarrow \{-1, +1\}$ by

$$g(v_i) = \begin{cases} +1, & \text{if } 1 \leq i \leq \frac{m}{2} \text{ of each copy of } K_m \text{ in } G \\ +1, & \text{if } v_i = v_k \in P_n \text{ in } G \\ -1, & \text{otherwise} \end{cases}$$

Case (1): Let $v_i \in P_n$ be such that $d(v_i) = (m+2)$ in G , then $N(v_i)$ contains m vertices of K_m and two vertices of P_n in G .

If $v_k \in N(v_i)$, then $\sum_{u \in N(v_i)} g(u) = 1 + (-1) + \left[\left(\frac{m}{2}\right)(+1) + \left(\frac{m}{2}\right)(-1) \right] = 0$

If $v_k \notin N(v_i)$, then $\sum_{u \in N(v_i)} g(u) = (-1) + (-1) + \left[\left(\frac{m}{2}\right)(+1) + \left(\frac{m}{2}\right)(-1) \right] = -2$

In this case g is an ISTDF.

Case (2): Let $v_i \in P_n$ be such that $d(v_i) = (m+1)$ in G , then $N(v_i)$ contains m vertices of K_m and one vertex of P_n in G .

If $v_k \in N(v_i)$, then $\sum_{u \in N(v_i)} g(u) = 1 + \left[\binom{m}{2}(+1) + \binom{m}{2}(-1) \right] = +1 \Rightarrow g$ is not an ISTDF.

If $v_k \notin N(v_i)$, then $\sum_{u \in N(v_i)} g(u) = (-1) + \left[\binom{m}{2}(+1) + \binom{m}{2}(-1) \right] = -1 \Rightarrow g$ is an ISTDF.

Case (3): Let $v_i \in K_m$ be such that $d(v_i) = m$ in G , then $N(v_i)$ contains $(m-1)$ vertices of K_m and one vertex of P_n in G and $g(v_i) = -1$ or $+1$. Let $v_k \in N(v_i)$

If $g(v_i) = -1$, then $\sum_{u \in N(v_i)} g(u) = (+1) + \left[\binom{m}{2}-1(-1) + \binom{m}{2}(+1) \right] = +2 \Rightarrow f$ is not an ISTDF.

If $g(v_i) = +1$, then $\sum_{u \in N(v_i)} g(u) = (+1) + \left[\binom{m}{2}(-1) + \left(\binom{m}{2}-1 \right)(+1) \right] = 0 \Rightarrow f$ is an ISTDF.

Let $v_k \notin N(v_i)$

If $g(v_i) = -1$, then $\sum_{u \in N(v_i)} f(u) = (-1) + \left[\left(\binom{m}{2}-1 \right)(-1) + \binom{m}{2}(+1) \right] = 0 \Rightarrow f$ is an ISTDF.

If $f(v_i) = +1$, then $\sum_{u \in N(v_i)} f(u) = (-1) + \left[\binom{m}{2}(-1) + \left(\binom{m}{2}-1 \right)(+1) \right] = -2 \Rightarrow f$ is an ISTDF.

This implies that g is not an ISTDF, because $\sum_{u \in N(v_i)} g(u) > 0$, for some $v_i \in V$

Hence f is a maximal inverse signed total dominating function on G .

$$\text{Now } \sum_{u \in V(G)} f(u) = \underbrace{(-1) + \dots + (-1)}_{n\text{-times}} + \underbrace{\left[\binom{m}{2}(+1) + \binom{m}{2}(-1) \right]}_{n\text{-times}} = -n$$

Finally, ISTDN is $\gamma_{st}^0(G) = -n$, if m is even.

Theorem 4.2: A function $f : V \rightarrow \{-1, +1\}$ is defined by $f(v_i) = \begin{cases} +1, & \text{if } 1 \leq i \leq \left(\frac{m-1}{2} \right) \text{ of each copy of } K_m \text{ in } G \\ -1, & \text{otherwise} \end{cases}$

is a maximal inverse signed total dominating function (MISTDF) of a graph $G = P_n \square K_m$ and ISTDN is $\gamma_{st}^0(G) = -2n$, if m is odd.

Proof: Let f be a function defined in the hypothesis.

Case (1): Let $v_i \in P_n$ be such that $d(v_i) = (m+2)$ in G , then $N(v_i)$ contains m vertices of K_m and two vertices of P_n in G .

$$\text{Thus } \sum_{u \in N(v_i)} f(u) = (-1) + (-1) + \left[\binom{m-1}{2}(+1) + \binom{m+1}{2}(-1) \right] = -3$$

Case (2): Let $v_i \in P_n$ be such that $d(v_i) = (m+1)$ in G , then $N(v_i)$ contains m vertices of K_m and one vertex of P_n in G .

$$\text{Thus } \sum_{u \in N(v_i)} f(u) = (-1) + \left[\binom{m-1}{2}(+1) + \binom{m+1}{2}(-1) \right] = -2$$

Case (3): Let $v_i \in K_m$ be such that $d(v_i) = m$ in G , then $N(v_i)$ contains $(m-1)$ vertices of K_m and one vertex of P_n in G and $f(v_i) = -1$ or $+1$.

$$\text{If } f(v_i) = -1, \text{ then } \sum_{u \in N(v_i)} f(u) = (-1) + \left[\binom{m}{2} (+1) + \binom{m}{2} (-1) \right] = -1$$

$$\text{If } f(v_i) = +1, \text{ then } \sum_{u \in N(v_i)} f(u) = (-1) + \left[\binom{m-3}{2} (+1) + \binom{m+1}{2} (-1) \right] = -3$$

$$\text{Hence for all the above possibilities, we get } \sum_{u \in N[v_i]} f(u) \leq 0, \forall v_i \in V$$

This implies that the function f is an ISTDF. Now we check for maximality of f , define $g: V \rightarrow \{-1, +1\}$ by

$$g(v_i) = \begin{cases} +1, & \text{if } 1 \leq i \leq \frac{m-1}{2} \text{ of each copy of } K_m \text{ in } G \\ +1, & \text{if } v_i = v_k \in P_n \text{ in } G \\ -1, & \text{otherwise} \end{cases}$$

Case (1): Let $v_i \in P_n$ be such that $d(v_i) = (m+2)$ in G , then $N(v_i)$ contains m vertices of K_m and two vertices of P_n in G .

$$\text{If } v_k \in N(v_i), \text{ then } \sum_{u \in N(v_i)} g(u) = 1 + (-1) + \left[\binom{m+1}{2} (-1) + \binom{m-1}{2} (+1) \right] = -1$$

$$\text{If } v_k \notin N(v_i), \text{ then } \sum_{u \in N(v_i)} g(u) = (-1) + (-1) + \left[\binom{m+1}{2} (-1) + \binom{m-1}{2} (+1) \right] = -3$$

Case (2): Let $v_i \in P_n$ be such that $d(v_i) = (m+1)$ in G , then $N(v_i)$ contains m vertices of K_m and one vertex of P_n in G .

$$\text{If } v_k \in N(v_i), \text{ then } \sum_{u \in N(v_i)} g(u) = 1 + \left[\binom{m+1}{2} (-1) + \binom{m-1}{2} (+1) \right] = 0$$

$$\text{If } v_k \notin N(v_i), \text{ then } \sum_{u \in N(v_i)} g(u) = (-1) + \left[\binom{m-1}{2} (+1) + \binom{m+1}{2} (-1) \right] = -2$$

Case (3): Let $v_i \in K_m$ be such that $d(v_i) = m$ in G , then $N(v_i)$ contains $(m-1)$ vertices of K_m and one vertex of P_n in G and $g(v_i) = -1$ or $+1$.

$$\text{Let } v_k \in N(v_i)$$

$$\text{If } g(v_i) = -1, \text{ then } \sum_{u \in N(v_i)} g(u) = (+1) + \left[\binom{m}{2} (+1) + \binom{m}{2} (-1) \right] = 1$$

$$\text{If } g(v_i) = +1, \text{ then } \sum_{u \in N(v_i)} g(u) = (+1) + \left[\binom{m-3}{2} (+1) + \binom{m+1}{2} (-1) \right] = -1$$

$$\text{Let } v_k \notin N(v_i)$$

$$\text{If } g(v_i) = -1, \text{ then } \sum_{u \in N(v_i)} g(u) = (-1) + \left[\binom{m}{2} (+1) + \binom{m}{2} (-1) \right] = -1$$

$$\text{If } g(v_i) = +1, \text{ then } \sum_{u \in N(v_i)} g(u) = (+1) + \left[\binom{m-3}{2} (+1) + \binom{m+1}{2} (-1) \right] = -1$$

This implies that g is not an ISTDF, because $\sum_{u \in N[v_i]} g(u) > 0$, for some $v_i \in V$

Hence f is a maximal inverse signed total dominating function on G .

$$\text{Now } \sum_{u \in V(G)} f(u) = \underbrace{(-1) + \dots + (-1)}_{n\text{-times}} + \left[\underbrace{\left(\frac{m-1}{2}\right)(+1) + \left(\frac{m+1}{2}\right)(-1)}_{n\text{-times}} \right] = -2n$$

Finally, ISTDN is $\gamma_{st}^0(G) = -2n$, if m is odd.

Conclusion

In this paper, we studied about the inverse signed domination number and inverse signed total domination number of a corona product of path with a complete graph. Here ISDN and ISTDN are equal, if m is even, i.e. $\gamma_s^0(G) = \gamma_{st}^0(G) = -n$. ISDN and ISTDN are different, if m is odd, i.e. $\gamma_s^0(G) = 0$ and $\gamma_{st}^0(G) = -2n$.

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