IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

ABOUT THEORY OF DISTRIBUTION OF PRIME NUMBERS

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ABSTRACT

In my previous article I have defined two sets A and B. which are two subsets of natural number. using this subsets, I have proved the following result. for any natural number k, if k∈A implies 6k+1 a composite number, and k ∉ A implies 6k+1 a prime number. if k∈B implies 6k-1 a composite number, and k ∉ B implies 6k-1 a prime number. if k∈A∩B implies 6k+1 and 6k-1 both are composite numbers, and k ∉ A∪B implies 6k+1 and 6k-1 both are prime numbers. In this article I have proved two theorems and one corollary about intersection of set natural numbers in the closed interval [1, (b-1)(6b+1)] with sets A, B, and AUB. i.e

i) Theorem (1) is about [1, (b-1)(6b+1)] ∩ A.

ii) Theorem (2) is about [1, (b-1)(6b+1)] ∩ B.

iii) Corollary is about [1, (b-1)(6b+1)] ∩ (A∪B).

INTRODUCTION

Numbers are wonderful, marvelous creature of human. Numbers are classified into many types. They are Natural numbers, Whole numbers, Integers, Real numbers, Complex numbers, Rational numbers, Irrational numbers.

Natural numbers are classified into two categories

1) Prime numbers,
2) Composite numbers.

Prime numbers are Natural numbers which cannot be expressed in the form of product of two Natural numbers both greater than 1.

Composite numbers are other than Prime numbers. i.e. Which can be expressed in the form of product of two Natural numbers both greater than 1

From the definition of Prime number, 2 and 3 are Prime numbers, but 2x3=6 is a composite number. Multiples 6 are also composite numbers. Numbers in the form 6k±2 are even numbers i.e multiples of 2, hence composite numbers, Numbers in the form 6k±3 are odd multiples of 3 hence composite numbers.

Therefore, Prime numbers except 2 and 3 are in the form of 6k±1. where k is any natural number, but not for all natural numbers. For some Natural number k, 6k+1 is a Prime number but 6k-1 is a Composite number. For some Natural number k, 6k-1 is a Prime number but 6k+1 is a Composite numbers. For some Natural number k, 6k+1 and 6k-1 both are Prime numbers (twin Prime numbers). For some Natural number k, 6k+1 and 6k-1 both are Composite numbers. Hence this k is the key factor that determines Prime numbers and Composite numbers.

Before 2500 years ago Euclid proved that Prime numbers are infinite, Composite numbers generated by their prime factors, but Prime numbers are not generated. They are distributed among the gaps left by Composite numbers. This article is about Theory of distribution of Prime numbers. Distributive rule of Prime numbers is nothing but violation of generating rule of Composite numbers. And since all Prime numbers except 2 and 3 are in the form 6k±1, this k determines the numbers in the form 6k±1.

In my first two article, I have written about this k. Especially In my second article, I have defined two sets A and B, which are subsets of natural numbers. I want to make remember.

$$A = \left(\bigcup_{\substack{n=1\ \&\\ 6n+1\text{ a prime number.}}}^{\infty} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1\text{ a prime number.}}}^{\infty} I_{-n}\right)$$

$$B = \left(\bigcup_{\substack{n=1\ \&\\ 6n-1\text{ a prime number.}}}^{\infty} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n+1\text{ a prime number.}}}^{\infty} J_{-n}\right)$$

Where

$I_n = \{ x/ x\in N, x\in[n]_{6n+1}\ \&\ x>n\}$

$I_{-n} = \{ x/ x\in N, x\in[-n]_{6n-1}\ \&\ x>n\}$

$J_n = \{ x/ x\in N, x\in[n]_{6n-1}\ \&\ x>n\}$

$J_{-n} = \{ x/ x\in N, x\in[-n]_{6n+1}\ \&\ x>n\}$

It is obvious that two sets A and B are infinite. A and B are union of residue classes of infinite number of prime moduli. so It is very difficult to study the nature of sets A and B. to overcome this drawback, I define this closed interval [1, (b-1)(6b+1)]. the natural numbers of A and B that are contained in this closed interval are union of residue classes of finite number of prime moduli. For that this two theorems and one corollary are very important. The immediate consequences of this two theorems and one corollary are my next four articles. So for continuity, this article is must. Let us go to Theorem (1)

THEOREM (1)

For any natural number b

$$[1, (b-1)(6b+1)] \cap A = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n+1\text{ a prime number.}}}^{b-1} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1\text{ a prime number.}}}^{b-1} I_{-n}\right) \right)$$

PROOF

Let $k=6ab+a+b$ where a and are natural numbers.

$k=a(6b+1)+b$

Implies $k \in I_b$ i.e $k > b$ and k belongs to $[b]_{6b+1}$

And also $k=6ab+a+b$

$k=b(6a+1)+a$

Implies $k \in I_a$.

From the definition of I_b Natural numbers belongs to I_b are

$6b+1+b, 2(6b+1)+b, 3(6b+1)+b, 4(6b+1)+b, \ldots\ldots\ldots$

From the above argument, let r be an arbitrary whole number. natural numbers belongs to $I_{(b+r)}$ are

$6(b+r)+1+b+r,\quad 2(6(b+r)+1)+b+r,\quad 3(6(b+r)+1)+b+r,\quad 4(6(b+r)+1)+b+r, \ldots\ldots\ldots$

As in above,

$6(b+r)+1+b+r = 7(b+r)+1 = (b+r)(6(1)+1)+1 \in I_1.$

Similarly

$2(6(b+r)+1)+b+r = 13(b+r)+2 \in I_2$

$3(6(b+r)+1)+b+r = 19(b+r)+3 \in I_3$

$\ldots\ldots\ldots$

$\ldots\ldots\ldots$

$(b-1)(6(b+r)+1)+b+r = (b+r)(6(b-1)+1)+b-1 \in I_{(b-1)}$

$b(6(b+r)+1)+b+r = (b+r)(6b+1)+b \in I_b$

therefore natural numbers smaller than $b(6(b+r)+1)+b+r$ and belongs to $I_{(b+r)}$ are contained in

$$\bigcup_{n=1}^{b-1} I_n$$

$b(6(b+r)+1)+b+r = b(6b+6r+1)+b+1 > (b-1)(6b+1)$ implies, natural numbers smaller than $(b-1)(6b+1)$ and belongs to $I_{(b+r)}$ are contained in

$$\bigcup_{n=1}^{b-1} I_n$$

r is an arbitrary whole number. implies natural number $b+r \geq b$. Hence, for any $n \geq b$, natural numbers smaller than $(b-1)(6b+1)$ and belongs to I_n are contained in

$$\bigcup_{n=1}^{b-1} I_n$$

n is arbitrary natural number $\geq b$ implies, natural numbers smaller than $(b-1)(6b+1)$ and belongs to

$\left(\bigcup_{n=b}^{\infty} I_n\right)$ are contained in $\left(\bigcup_{n=1}^{b-1} I_n\right)$

i.e

$$\left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=b}^{\infty} I_n\right) \right) \subset \left(\bigcup_{n=1}^{b-1} I_n\right)$$

implies,

$$\left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=b}^{\infty} I_n\right)\right) \subseteq \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} I_n\right)\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

[since $(X \cap Y) \subset Z$ implies $(X \cap Y) \subseteq (X \cap Z)$]

Therefore,

$$[1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{\infty} I_n\right) = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{n=1}^{b-1} I_n\right) \cup \left(\bigcup_{n=b}^{\infty} I_n\right) \right)$$

$$= \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} I_n\right)\right) \cup \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=b}^{\infty} I_n\right)\right)$$

[since $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$]

$$\subseteq \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} I_n\right)\right) \cup \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} I_n\right)\right)$$

[since from (1)]

$$= [1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} I_n\right)$$

[since $X \cup X = X$]

i.e

$$[1, (b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{\infty} I_n\right) \subseteq [1, (b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{b-1} I_n\right)$$

But

$$\left(\bigcup_{n=1}^{b-1} I_n\right) \subset \left(\bigcup_{n=1}^{\infty} I_n\right)$$

implies,

$$[1, (b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{b-1} I_n\right) \subset [1, (b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{\infty} I_n\right)$$

Therefore,

$$[1, (b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{\infty} I_n\right) = [1, (b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{b-1} I_n\right)$$

…………………………..(2).

Similarly

Let $k=6ab-a-b$ where a and b are natural numbers.

$k=a(6b-1)-b$

Implies $k \in I_{-b}$ i.e $k>b$ and k belongs to $[-b]_{6b-1}$

And also $k=6ab-b-a$

$k=b(6a-1)-a$

Implies $k \in I_{-a}$

By the definition of I_{-b} Natural numbers belongs to I_{-b} are

$6b-1-b$, $2(6b-1)-b$, $3(6b-1)-b$, $4(6b-1)-b$, ……….

From the above argument, let r be an arbitrary whole number. natural numbers belongs to $I_{-(b+r)}$ are

$6(b+r)-1-(b+r)$, $2(6(b+r)-1)-(b+r)$, $3(6(b+r)-1)-(b+r)$, $4(6(b+r)-1)-(b+r)$, ………………………………………………

As in above,

$6(b+r)-1-(b+r) = 5(b+r)-1 = (b+r)(6(1)-1)-1 \in I_{-1}$

Similarly,

$2(6(b+r)-1)-(b+r) = 11(b+r)-2 \in I_{-2}$

$3(6(b+r)-1)-(b+r)= 17(b+r)-3 \in I_{-3}$

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……………………………………………………………………………………

$(b-1)(6(b+r)-1)-(b+r) = (b+r)(6(b-1)-1)-(b-1) \in I_{-(b-1)}$

$b(6(b+r)-1)-(b+r) = (b+r)(6b-1)-b \in I_{-b}$

therefore, natural numbers smaller than $b(6(b+r)-1)-(b+r)$ and belongs to $I_{-(b+r)}$ are contained in

$$\bigcup_{n=1}^{b-1} I_{-n}$$

$$\begin{aligned} b(6(b+r)-1)-(b+r) &= b(6b+6r-1)-b-r \\ &= (b-1)(6b+1+6r-2)+6b+6r-1-b-r \\ &> (b-1)(6b+1) \end{aligned}$$

implies, natural numbers smaller than $(b-1)(6b+1)$ and belongs to $I_{-(b+r)}$ are contained in

$$\bigcup_{n=1}^{b-1} I_{-n}$$

r is an arbitrary whole number. implies natural number $b+r \geq b$. Hence, for any $n \geq b$, natural numbers smaller than $(b-1)(6b+1)$ and belongs to I_{-n} are contained in

$$\bigcup_{n=1}^{b-1} I_{-n}$$

n is arbitrary natural number $\geq b$ implies, natural numbers smaller than $(b-1)(6b+1)$ and belongs to

$$\bigcup_{n=b}^{\infty} I_{-n} \quad \text{are contained in} \quad \bigcup_{n=1}^{b-1} I_{-n}$$

i.e

$$\left([1, (b-1)(6b+1)] \cap \bigcup_{n=b}^{\infty} I_{-n} \right) \subset \bigcup_{n=1}^{b-1} I_{-n}$$

implies,

$$\left([1, (b-1)(6b+1)] \cap \bigcup_{n=b}^{\infty} I_{-n}\right) \subseteq \left([1, (b-1)(6b+1)] \cap \bigcup_{n=1}^{b-1} I_{-n}\right)$$

……………………………….(3)

[since $(X \cap Y) \subset Z$ implies $(X \cap Y) \subseteq (X \cap Z)$]

Therefore,

$$[1, (b-1)(6b+1)] \cap \bigcup_{n=1}^{\infty} I_{-n} = [1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} I_{-n} \cup \bigcup_{n=b}^{\infty} I_{-n} \right)$$

$$=\left([1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{b-1} I_{-n}\right)\right)\cup\left([1,(b-1)(6b+1)]\cap\left(\bigcup_{n=b}^{\infty} I_{-n}\right)\right)$$

[since X∩(Y∪Z)=(X∩Y)∪(X∩Z)]

$$\subseteq\left([1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{b-1} I_{-n}\right)\right)\cup\left([1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{b-1} I_{-n}\right)\right)$$

[since from (3)]

$$=[1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{b-1} I_{-n}\right)$$

[since XUX=X]

i.e

$$[1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{\infty} I_{-n}\right)\subseteq[1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{b-1} I_{-n}\right)$$

But

$$\left(\bigcup_{n=1}^{b-1} I_{-n}\right)\subset\left(\bigcup_{n=1}^{\infty} I_{-n}\right)$$

implies,

$$[1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{b-1} I_{-n}\right)\subset[1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{\infty} I_{-n}\right)$$

Therefore,

$$[1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{\infty} I_{-n}\right)=[1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{b-1} I_{-n}\right)$$

…………………………(4).

From (2) and (4)

$$\left([1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{\infty} I_{n}\right)\right)\cup\left([1,(b-1)(6b+1)]\cap\left(\bigcup_{n=1}^{\infty} I_{-n}\right)\right)$$

$$=\left([1,(b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} I_n\right)\right) \cup \left([1,(b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} I_{-n}\right)\right)$$

implies,

$$[1,(b-1)(6b+1)] \cap \left(\left(\bigcup_{n=1}^{\infty} I_n\right) \cup \left(\bigcup_{n=1}^{\infty} I_{-n}\right)\right) = [1,(b-1)(6b+1)] \cap \left(\left(\bigcup_{n=1}^{b-1} I_n\right) \cup \left(\bigcup_{n=1}^{b-1} I_{-n}\right)\right)$$

……………………………... (5)

I have already discussed in my previous (second) article, If $6x\pm1$ is a divisor of $6n\pm1$, implies $I_{\pm n}$ is a subset of $I_{\pm x}$. and $6n\pm1$ is a prime number, implies $I_{\pm n}$ is not a proper subset of any $I_{\pm x}$.

therefore, (5) becomes

$$[1,(b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1 \,\&\\ 6n+1 \text{ a prime number.}}}^{\infty} I_n\right) \cup \left(\bigcup_{\substack{n=1 \,\&\\ 6n-1 \text{ a prime number.}}}^{\infty} I_{-n}\right)\right) = [1,(b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1 \,\&\\ 6n+1 \text{ a prime number.}}}^{b-1} I_n\right) \cup \left(\bigcup_{\substack{n=1 \,\&\\ 6n-1 \text{ a prime number.}}}^{b-1} I_{-n}\right)\right)$$

But

$$A = \left(\bigcup_{\substack{n=1 \,\&\\ 6n+1 \text{ a prime number.}}}^{\infty} I_n\right) \cup \left(\bigcup_{\substack{n=1 \,\&\\ 6n-1 \text{ a prime number.}}}^{\infty} I_{-n}\right)$$

Implies,

$$[1,(b-1)(6b+1)] \cap A = [1,(b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1 \,\&\\ 6n+1 \text{ a prime number.}}}^{b-1} I_n\right) \cup \left(\bigcup_{\substack{n=1 \,\&\\ 6n-1 \text{ a prime number.}}}^{b-1} I_{-n}\right)\right)$$

Hence the theorem.

THEOREM (2)

For any arbitrary natural number b

$$[1,(b-1)(6b+1)] \cap B = [1,(b-1)(6b+1)] \cap \Big(\bigcup_{\substack{n=1 \,\&\\ 6n-1 \text{ a prime number.}}}^{b-1} J_n \;\cup \bigcup_{\substack{n=1 \,\&\\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n} \Big)$$

PROOF

Let k=6ab-a+b where a and b are natural numbers.

k=a(6b-1)+b

Implies $k \in J_b$ i.e $k > b$ and k belongs to $[b]_{6b-1}$

And also k=6ab-a+b

k=b(6a+1)-a

Implies $k \in J_{-a}$

By the definition of J_b, Natural numbers belongs to J_b are

6b-1+b, 2(6b-1)+b, 3(6b-1)+b, 4(6b-1)+b, ……….

From the above argument, let r be an arbitrary whole number. natural numbers belongs to $J_{(b+r)}$ are

6(b+r)-1+b+r, 2(6(b+r)-1)+b+r, 3(6(b+r)-1)+b+r, 4(6(b+r)-1)+b+r, ……………………………………………

As in above,

$6(b+r)-1+b+r = 7(b+r)-1 = (b+r)(6(1)+1)-1 \in J_{-1}$.

similarly

$2(6(b+r)-1)+b+r = 13(b+r)-2 \in J_{-2}$

$3(6(b+r)-1)+b+r = 19(b+r)-3 \in J_{-3}$

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$(b-1)(6(b+r)-1)+b+r = (b+r)(6(b-1)+1)-(b-1) \in J_{-(b-1)}$

$b(6(b+r)-1)+b+r = (b+r)(6b+1)-b \in J_{-b}$

therefore natural numbers smaller than b(6(b+r)-1)+b+r and belongs to $J_{(b+r)}$ are contained in

$$\bigcup_{n=1}^{b-1} J_{-n}$$

$$b(6(b+r)-1)+b+r = b(6b+6r+1-2)+b+r$$
$$= b(6b+1)+b(6r-2)+b+r$$
$$> (b-1)(6b+1)$$

implies, natural numbers smaller than (b-1)(6b+1) and belongs to $J_{(b+r)}$ are contained in

$$\bigcup_{n=1}^{b-1} J_{-n}$$

r is an arbitrary whole number. implies natural number b+r ≥ b. Hence, for any n ≥ b, natural numbers smaller than (b-1)(6b+1) and belongs to J_n are contained in

$$\left(\bigcup_{n=1}^{b-1} J_{-n}\right)$$

n is arbitrary natural number ≥ b implies, natural numbers smaller than (b-1)(6b+1) and belongs to

$$\left(\bigcup_{n=b}^{\infty} J_{n}\right) \text{ are contained in } \left(\bigcup_{n=1}^{b-1} J_{-n}\right)$$

i.e

$$\left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=b}^{\infty} J_{n}\right)\right) \subset \left(\bigcup_{n=1}^{b-1} J_{-n}\right)$$

implies,

$$\left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=b}^{\infty} J_{n}\right)\right) \subseteq \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} J_{-n}\right)\right)$$

……………………………..(1)

[since $(X \cap Y) \subset Z$ implies $(X \cap Y) \subseteq (X \cap Z)$]

Therefore,

$$[1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{\infty} J_{n}\right) = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{n=1}^{b-1} J_{n}\right) \cup \left(\bigcup_{n=b}^{\infty} J_{n}\right) \right)$$

$$= \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} J_{n}\right)\right) \cup \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=b}^{\infty} J_{n}\right)\right)$$

$$\subseteq \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} J_{n}\right)\right) \cup \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} J_{-n}\right)\right)$$

[since from (1)]

$$= [1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} J_n \cup \bigcup_{n=1}^{b-1} J_{-n} \right)$$

[since (X∩Y)U(X∩Z)=X∩(YUZ)]

i.e

$$\left([1, (b-1)(6b+1)] \cap \bigcup_{n=1}^{\infty} J_n\right) \subseteq [1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} J_n \cup \bigcup_{n=1}^{b-1} J_{-n} \right)$$

……………………(2)

Similarly,

Let $k=6ab+a-b$ where a and b are natural numbers
$k=a(6b+1)-b$
Implies $k \in J_{-b}$ i.e $k>b$ and k belongs to $[-b]_{6b+1}$
And also $k=6ab-b+a$
$k=b(6a-1)+a$
Implies $k \in J_a$
By the definition of J_{-b}, Natural numbers belongs to J_{-b} are
$6b+1-b, 2(6b+1)-b, 3(6b+1)-b, 4(6b+1)-b,$ ……….

From the above argument, let r be an arbitrary whole number. natural numbers belongs to $J_{-(b+r)}$ are

$6(b+r)+1-(b+r)$, $2(6(b+r)+1)-(b+r)$, $3(6(b+r)+1)-(b+r)$, $4(6(b+r)+1)-(b+r)$, ………………………………………………

As in above,

$6(b+r)+1-(b+r) = 5(b+r)+1 = (b+r)(6(1)-1)+1 \in J_1$.

similarly

$2(6(b+r)+1)-(b+r) = 11(b+r)+2 \in J_2$

$3(6(b+r)+1)-(b+r)= 17(b+r)+3 \in J_3$

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$(b-1)(6(b+r)+1)-(b+r) = (b+r)(6(b-1)-1)+(b-1) \in J_{(b-1)}$

$b(6(b+r)+1)-(b+r) = (b+r)(6b-1)+b \in J_b$

therefore natural numbers smaller than $b(6(b+r)+1)-(b+r)$ and belongs to $J_{-(b+r)}$ are contained in

$$\bigcup_{n=1}^{b-1} J_n$$

$$b(6(b+r)+1)-(b+r) = b(6b+6r+1)-b-r$$

$$= (b-1)(6b+1+6r)+6b+6r+1-b-r$$

$$> (b-1)(6b+1)$$

implies, natural numbers smaller than $(b-1)(6b+1)$ and belongs to $J_{-(b+r)}$ are contained in

$$\left(\bigcup_{n=1}^{b-1} J_n\right)$$

r is an arbitrary whole number. implies natural number $b+r \geq b$. Hence, for any $n \geq b$, natural numbers smaller than $(b-1)(6b+1)$ and belongs to J_{-n} are contained in

$$\left(\bigcup_{n=1}^{b-1} J_n\right)$$

n is arbitrary natural number $\geq b$ implies, natural numbers smaller than $(b-1)(6b+1)$ and belongs to

$$\left(\bigcup_{n=b}^{\infty} J_{-n}\right) \text{ are contained in } \left(\bigcup_{n=1}^{b-1} J_n\right)$$

i.e

$$\left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=b}^{\infty} J_{-n}\right)\right) \subset \left(\bigcup_{n=1}^{b-1} J_n\right)$$

implies,

$$\left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=b}^{\infty} J_{-n}\right)\right) \subseteq \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} J_n\right)\right)$$

……………………………..(3)

[since $(X \cap Y) \subset Z$ implies $(X \cap Y) \subseteq (X \cap Z)$]

Therefore,

$$[1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{\infty} J_{-n}\right) = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{n=1}^{b-1} J_{-n}\right) \cup \left(\bigcup_{n=b}^{\infty} J_{-n}\right)\right)$$

$$= \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} J_{-n}\right)\right) \cup \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=b}^{\infty} J_{-n}\right)\right)$$

[since $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$]

$$\subseteq \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} J_{-n}\right)\right) \cup \left([1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{b-1} J_{n}\right)\right)$$

[since from (1)]

$$= [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{n=1}^{b-1} J_{-n}\right) \cup \left(\bigcup_{n=1}^{b-1} J_{n}\right) \right)$$

[since $(X\cap Y)\cup(X\cap Z)=X\cap(Y\cup Z)$]

i.e

$$[1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{\infty} J_{-n}\right) \subseteq [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{n=1}^{b-1} J_{n}\right) \cup \left(\bigcup_{n=1}^{b-1} J_{-n}\right) \right) \quad \text{.......................(4)}$$

(2) and (4) implies,

$$[1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{\infty} J_{n}\right) \ \& \ [1, (b-1)(6b+1)] \cap \left(\bigcup_{n=1}^{\infty} J_{-n}\right)$$

both are subsets of $[1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{n=1}^{b-1} J_{n}\right) \cup \left(\bigcup_{n=1}^{b-1} J_{-n}\right) \right)$

hence their union also subset of

$$[1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{n=1}^{b-1} J_{n}\right) \cup \left(\bigcup_{n=1}^{b-1} J_{-n}\right) \right)$$

i.e

$$([1,\ (b-1)(6b+1)]\cap(\bigcup_{n=1}^{\infty}J_n)) \cup ([1,\ (b-1)(6b+1)]\cap(\bigcup_{n=1}^{\infty}J_{-n})) = [1,\ (b-1)(6b+1)] \cap \left((\bigcup_{n=1}^{\infty}J_n) \cup (\bigcup_{n=1}^{\infty}J_{-n}) \right)$$

$$\subseteq [1,\ (b-1)(6b+1)] \cap \left((\bigcup_{n=1}^{b-1}J_n) \cup (\bigcup_{n=1}^{b-1}J_{-n}) \right)$$

But

$$\left((\bigcup_{n=1}^{b-1}J_n) \cup (\bigcup_{n=1}^{b-1}J_{-n}) \right) \subset \left((\bigcup_{n=1}^{\infty}J_n) \cup (\bigcup_{n=1}^{\infty}J_{-n}) \right)$$

implies,

$$[1,\ (b-1)(6b+1)] \cap \left((\bigcup_{n=1}^{b-1}J_n) \cup (\bigcup_{n=1}^{b-1}J_{-n}) \right) \subset [1,\ (b-1)(6b+1)] \cap \left((\bigcup_{n=1}^{\infty}J_n) \cup (\bigcup_{n=1}^{\infty}J_{-n}) \right)$$

therefore,

$$[1,\ (b-1)(6b+1)] \cap \left((\bigcup_{n=1}^{\infty}J_n) \cup (\bigcup_{n=1}^{\infty}J_{-n}) \right) = [1,\ (b-1)(6b+1)] \cap \left((\bigcup_{n=1}^{b-1}J_n) \cup (\bigcup_{n=1}^{b-1}J_{-n}) \right)$$

……………………….. (5)

I have already discussed in my previous (second) article, If $6x\pm1$ is a divisor of $6n\pm1$, implies $J_{\pm n}$ is a subset of $J_{\pm x}$. and $6n\pm1$ is a prime number, implies $J_{\pm n}$ is not a proper subset of any $J_{\pm x}$.

therefore, (5) becomes

$$[1,\ (b-1)(6b+1)] \cap \left((\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\infty}J_n) \cup (\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\infty}J_{-n}) \right) = [1,\ (b-1)(6b+1)] \cap \left((\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1}J_n) \cup (\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1}J_{-n}) \right)$$

But

$$B = \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{\infty} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{\infty} J_{-n}\right)$$

Implies,

$$[1, (b-1)(6b+1)] \cap B = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n}\right) \right)$$

Hence the theorem.

COROLLARY

$$[1, (b-1)(6b+1)] \cap (A \cup B) = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)\right) \right)$$

PROOF

Corollary is nothing but combined form above two theorem.

$$[1, (b-1)(6b+1)] \cap (A \cup B) = ([1, (b-1)(6b+1)] \cap A) \cup ([1, (b-1)(6b+1)] \cap B)$$

$$= \left([1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} I_{-n}\right) \right) \right)$$

$$\cup$$

$$\left([1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n}\right) \right) \right)$$

[since from theorem (1) and theorem (2)]

$$= [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})\right) \cup \left(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)\right) \right)$$

Hence the corollary.

REMARK

When b= 1, (b-1)(6b+1)=0 implies the closed interval [1, (b-1)(6b+1)] = [1, 0] but this is error notation of closed interval. Since notation of closed intervals are in the form [x, y]. where x is smaller than y. which means set of numbers n such that $x \leq n \leq y$. so in this case, we take closed interval as [(b-1)(6b+1), 1]. Therefore, when b=1,

Theorem (1) implies $[0, 1] \cap A = \{ \}$

Theorem (1) implies $[0, 1] \cap B = \{ \}$

Corollary implies $[0, 1] \cap (A \cup B) = \{ \}$

where { } means null set.

For $b > 1$, we take closed interval as in theorems and corollary.

I have already noticed that my next four articles are immediate consequences of above theorems and corollary. (b-1)(6b+1) is an Arbitrarily chosen natural number such that it is smaller than $b(6(b+r)\pm1)\pm(b+r)$. so we cannot restrict that [1, (b-1)(6b+1)] is the only closed interval obeys the above theorems and corollary. I have chosen (b-1)(6b+1) for calculations of my next four articles. For example closed intervals [1, (b-1)(6b+1)+1] and [1, (b-1)(b+1)+2] are also obeys above theorems and corollary.

CONCLUTION

My name is **A. GABRIEL** a distance educated post graduate in mathematics. The thesis what we discussed above is myself realized one. Here I have submitted my completed concepts only. I am continuing my research about **THEORY OF DISTRIBUTION OF PRIME NUMBERS** by analyzing numbers which can be expressed in form $6ab\pm a\pm b$, and which cannot be expressed in the form $6ab\pm a\pm b$. i.e by analyzing the sets A, B, A^c, B^c, $A \cup B$, $(A \cup B)^c$, $A \cap B$, and $(A \cap B)^c$, where A and B are as defined above and the set of Natural numbers as universal set. then I conclude.

By

A. GABRIEL

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