IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

Article 4 About Theory Of Distribution Of Prime Numbers

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ABSTRACT

In this article, a theorem about distribution of twin prime numbers is given.

The theorem states that,

Number of pairs of twin prime numbers in the arbitrary (generalized) closed interval [36b+11, 6(b-1)(6b+1)+1] is almost (approximately) equal to

$$(b-2)(6b+1)(3/5)(5/7)(9/11)(11/13)\ldots\ldots\ldots\ldots\ldots(P_r - 2)/P_r$$

Where b is arbitrary natural number, and P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$. And except (b-2) and (6b+1), other factors are in the form (P-2)/P. where P is a prime number such that $P \leq 6(b-1)+1$.

The immediate consequence of above theorem is infiniteness of twin prime numbers, which is proved before three years.

INTRODUCTION

Numbers are wonderful, marvelous creature of human. Numbers are classified into many types. They are Natural numbers, Whole numbers, Integers, Real numbers, Complex numbers, Rational numbers, Irrational numbers.

Natural numbers are classified into two categories

1) Prime numbers,
2) Composite numbers.

Prime numbers are Natural numbers which cannot be expressed in the form of product of two Natural numbers both greater than 1.

Composite numbers are other than Prime numbers. i.e. Which can be expressed in the form of product of two Natural numbers both greater than 1

From the definition of Prime number, 2 and 3 are Prime numbers, but 2x3=6 is a composite number. Multiples 6 are also composite numbers. Numbers in the form 6k±2 are even numbers i.e multiples of 2, hence composite numbers, Numbers in the form 6k±3 are odd multiples of 3 hence composite numbers.

Therefore, Prime numbers except 2 and 3 are in the form of 6k±1. where k is any natural number, but not for all natural numbers. For some Natural number k, 6k+1 is a Prime number but 6k-1 is a Composite number. For some Natural number k, 6k-1 is a Prime number but 6k+1 is a Composite numbers. For some Natural number k, 6k+1 and 6k-1 both are Prime numbers (twin Prime numbers). For some Natural number k, 6k+1 and 6k-1 both are Composite numbers. Hence this k is the key factor that determines Prime numbers and Composite numbers.

Before 2500 years ago Euclid proved that Prime numbers are infinite, Composite numbers generated by their prime factors, but Prime numbers are not generated. They are distributed among the gaps left by Composite numbers. This article is about Theory of distribution of Prime numbers. Distributive rule of Prime numbers is nothing but violation of generating rule of Composite numbers. And since all Prime numbers except 2 and 3 are in the form 6k±1, this k determines the numbers in the form 6k±1.

In my first two article, I have written about this k. Especially In my second article, IJNRD 2304175 I have defined two sets A and B, which are subsets of natural numbers. I want to make remember.

The symbol □ stands for denoting infinity.

$$A = \left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{□} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{□} I_{-n}\right)$$

$$B = \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{□} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{□} J_{-n}\right)$$

Where

$$I_n = \{ x/ x \in N,\ x \in [n]_{6n+1}\ \&\ x > n\}$$

$$I_{-n} = \{ x/ x \in N,\ x \in [-n]_{6n-1}\ \&\ x > n\}$$

$$J_n = \{ x/ x \in N,\ x \in [n]_{6n-1}\ \&\ x > n\}$$

$$J_{-n} = \{ x/ x \in N,\ x \in [-n]_{6n+1}\ \&\ x > n\}$$

It is obvious that two sets A and B are infinite. A and B are union of residue classes of infinite number of prime moduli. so It is very difficult to study the nature of sets A and B. to overcome this drawback, I define this closed interval [1, (b-1)(6b+1)]. the natural numbers of A and B that are contained in this closed interval are union of residue classes of finite number of prime moduli. Therefore, In my third article, IJNRD 2304384 I have proved the following facts.

For any natural number b

i) $$[1, (b-1)(6b+1)] \cap A = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} I_{-n}\right) \right)$$

ii) $[1, (b\text{-}1)(6b+1)] \cap B = [1, (b\text{-}1)(6b+1)] \cap \left(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} J_n \ \cup \ \bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n} \right)$

iii) $[1, (b\text{-}1)(6b+1)] \cap (A \cup B) = [1, (b\text{-}1)(6b+1)] \cap \left(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n}) \cup \bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n) \right)$

This article is an immediate consequence of my third article. Before go to the theorem, l want to show some preliminaries about $I_{\pm n}$ and $J_{\pm n}$ by using theory of probability, which is essential in proving theorem.

We know that,

Probability = Number of favourable outcomes ÷ Total Number of outcomes.

Let us select a natural number randomly in the set of natural numbers N. we know that N is infinite. Let our favour of selection is that the selected natural number belongs to residue class [n] of modulo 6n+1. i.e selected natural number belongs to $[n]_{6n+1}$. What is the probability of our favourable event? Answer follows.

The set of natural numbers is union of 6n+1 disjoint residue classes of modulo 6n+1. Hence the selected natural number cannot belong to more than one residue class of modulo 6n+1. And each residue class is an infinite set of natural number. Therefore, each residue class has an equal chance in selection. Therefore, the selected natural number should belong to exactly any one residue class of modulo 6n+1. Therefore, Total Number of outcomes are 6n+1. but our favour of selection is that the selected natural number belongs to $[n]_{6n+1}$. So the Number of favourable outcome is 1. Therefore,

probability of selecting a natural number randomly in N such that it belongs to $[n]_{6n+1}$ = 1/6n+1.

Now let us discuss about another event. Let us select a natural number randomly in an arbitrary closed interval [x, y]. where x and y are natural numbers, the natural number $n < x$, and length of [x, y] = number of natural numbers in [x, y] = $y\text{-}x+1 \geq 6n+1$. Here the set of natural number in [x, y] is finite. Let our favour of selection is that the selected natural number belongs to $[n]_{6n+1}$. then what is the probability of our favour of selection?

Let Total Number of natural numbers in [x, y] is p(6n+1)+q. i.e $y\text{-}x+1 = p(6n+1)+q$. where p is natural numbers, q is whole number such that $0 \leq q < 6n+1$. If q = 0, then the Total number of natural numbers in [x, y] is p(6n+1) a multiple of 6n+1. Hence the p(6n+1) consecutive natural numbers in [x, y] are equally distributed among 6n+1 residue classes of modulo 6n+1. Hence each residue class of $[x]_{6n+1}$, $[x+1]_{6n+1}$, $[x+2]_{6n+1}$, …………., $[x+6n]_{6n+1}$ has an equal chance in selection. Therefore, as in above,

probability of selecting a natural number randomly in [x, y] such that it belongs to $[n]_{6n+1}$ = 1/6n+1.

If $q \neq 0$, Then the p(6n+1)+q consecutive natural numbers in [x, y] are not equally distributed among 6n+1 residue classes of modulo 6n+1. i.e each residue class of q residue classes $[x]_{6n+1}$, $[x+1]_{6n+1}$, $[x+2]_{6n+1}$,…….$[x+q-1]_{6n+1}$ contains exactly p+1 natural numbers of [x, y]. And each residue class of other 6n+1-q residue classes $[x+q]_{6n+1}$, $[x+q+1]_{6n+1}$, $[x+q+2]_{6n+1}$, ….. ...$[x+6n]_{6n+1}$ contains exactly p natural numbers of [x, y]. Therefore, each residue class has not an equal chance in selection. i.e Chance in selection of each residue class of $[x]_{6n+1}$, $[x+1]_{6n+1}$, $[x+2]_{6n+1}$,…… $[x+q-1]_{6n+1}$ is greater than the chance in selection of each residue class of $[x+q]_{6n+1}$, $[x+q+1]_{6n+1}$, $[x+q+2]_{6n+1}$, …………. …$[x+6n]_{6n+1}$. But each residue class of $[x]_{6n+1}$, $[x+1]_{6n+1}$,

$[x+2]_{6n+1}$,……… $[x+q-1]_{6n+1}$ has an equal chance in selection. And each residue class of $[x+q]_{6n+1}$, $[x+q+1]_{6n+1}$ $[x+q+2]_{6n+1}$, ….. ...$[x+6n]_{6n+1}$ has an equal chance in selection. hence two cases arise in calculation of probability of our favourable event. Let us discuss one by one.

Case(i)

Let $[n]_{6n+1}$ includes exactly p+1 natural numbers which are in [x, y] i.e $[n]_{6n+1}$ is one of the q residue classes of modulo 6n+1.

$[n]_{6n+1}$ includes exactly p+1 natural numbers which are in [x, y]. implies $n([x, y]\cap[n]_{6n+1})$ is p+1. Here our favour of selection is any one natural number of $([x, y] \cap [n]_{6n+1})$. Therefore, number of favourable outcomes = p+1.

And the randomly selected natural number should be any one natural number of ([x, y] ∩ N). i.e each natural number in [x, y] has an equal chance in selection. Therefore, Total Number of outcomes = n([x, y] ∩ N) = p(6n+1)+q. Hence,

probability of selecting a natural number randomly in [x, y] such that it belongs to $[n]_{6n+1}$ = (p+1)/(p(6n+1)+q).

Similarly, on the other hand

Case(ii)

Let $[n]_{6n+1}$ includes exactly p natural numbers which are in [x, y]. i.e $[n]_{6n+1}$ is one of the 6n+1-q residue classes of modulo 6n+1.

$[n]_{6n+1}$ includes exactly p natural numbers which are in [x, y]. implies, $n([x, y]\cap[n]_{6n+1})$ is p. Here our favour of selection is any one natural number of $([x, y]\cap[n]_{6n+1})$. Therefore, number of favourable outcomes = p. As in above, Total number of outcomes = p(6n+1)+q. Hence,

probability of selecting a natural number randomly in [x, y] such that it belongs to $[n]_{6n+1}$ = p/(p(6n+1)+q).

both the above two probabilities are different from the probability of selecting a natural number randomly in N such that it belongs to $[n]_{6n+1}$. i.e both the above two probabilities are different from 1/(6n+1).

but it is obvious that both the above two probabilities are almost (approximately) equal to the probability of selecting a natural number randomly in N such that it belongs to $[n]_{6n+1}$ i.e (p+1)/(p(6n+1)+q) and p/(p(6n+1)+q) are almost equal to 1/6n+1.

i.e $1/(6n+1) \approx (p+1)/(p(6n+1)+q)$ and

$1/(6n+1) \approx p/(p(6n+1)+q)$.

The difference (error) of case (i) and 1/(6n+1).

$$(p+1)/(p(6n+1)+q) - 1/(6n+1) = ((p+1)(6n+1) – (p(6n+1)+q)) / (p(6n+1)+q)(6n+1)$$

$$= (6n+1 – q)/ (6n+1)(p(6n+1)+q)$$

$$= (6n+1 – q)/(6n+1) \quad X \quad 1/(p(6n+1)+q)$$

$$> 0$$

$q < 6n+1$. implies, $(6n+1-q)/(6n+1) < 1$, $1/(p(6n+1)+q) < 1$

and it is obvious that when p increases, p(6n+1)+q increases and 1/(p(6n+1)+q) decreases.

p(6n+1)+q is Total Number of natural numbers in [x, y]. i.e p(6n+1)+q is the length of the closed interval [x, y]. therefore, when length of [x, y] increases, difference (error) decreases.

Similarly,

the difference (error) of 1/(6n+1) and case (ii)

$$1/(6n+1) - p/(p(6n+1)+q) = (p(6n+1) + q - p(6n+1))/(p(6n+1) + q)(6n+1)$$

$$= q/(p(6n+1) + q)(6n+1)$$

$$= q/(6n+1) \quad X \quad 1/(p(6n+1) + q)$$

$$> 0$$

$q < 6n+1$. implies, $(q/(6n+1) < 1$, $1/(p(6n+1)+q) < 1$

here is also when p increases, $p(6n+1)+q$ increases and $1/(p(6n+1)+q)$ decreases.

$P(6n+1)+q$ is Total No. of natural numbers in [x, y]. as in above, when length of [x, y] increases, difference (error) decreases.

So far our discussion yields following results, which are essential in proving theorems.

probability of selecting a natural number randomly in the infinite set N such that it belongs to $[n]_{6n+1}$ is almost (approximately) equal to the probability of selecting a natural number randomly in [x, y] such that it belongs to $[n]_{6n+1}$. where [x, y] is arbitrary closed interval, the length of closed interval is $p(6n+1)+q$. p is natural number, q is whole number such that $0 \leq q < y$, and $n < x$.

the difference (error) between probability of selecting a natural number randomly in the infinite set N such that it belongs to $[n]_{6n+1}$ and the probability of selecting a natural number randomly in [x, y] such that it belongs to $[n]_{6n+1}$ is become smaller and smaller when length of the closed interval increases. i.e approximation comes closer, when length of arbitrary interval increases.

Now

From the above result,

The probability of selecting a natural number randomly in [x, y] such that it belongs $[n]_{6n+1}$ is almost (approximately) equal to 1/(6n+1) ………………(1)

and by the definition of I_n,

$n < x$ implies $([x, y] \cap [n]_{6n+1}) \subset I_n$. implies $([x, y] \cap [n]_{6n+1}) \subseteq ([x, y] \cap I_n)$

But $I_n \subset [n]_{6n+1}$ implies $([x, y] \cap I_n) \subset ([x, y] \cap [n]_{6n+1})$

Therefore $\quad [x, y] \cap [n]_{6n+1} = [x, y] \cap I_n$

Hence the event of selecting a natural number in [x, y] such that it belongs to $[n]_{6n+1}$ is nothing but event of selecting a natural number in [x, y] such that it belongs to I_n.

Hence from (1)

For $n < x$ and $(6n+1) \leq$ length of [x, y]

The probability of selecting a natural number randomly in [x, y] such that it belongs to I_n is almost (approximately) equal to 1/(6n+1)

i.e $\quad P(I_n) \approx 1/(6n+1)$

where $P(I_n)$ is the probability of selecting a natural number in [x, y] such that it belongs I_n,

Similarly, we can show that,

$P(I_{-n}) \approx 1/(6n-1)$.

$P(J_n) \approx 1/(6n-1)$.

$P(J_{-n}) \approx 1/(6n+1)$.

And from the theory of probability, the following statements are obvious.

$P((I_n)^c) \approx 1 – P(I_n) = 1 – 1/(6n+1) = 6n/(6n+1)$

$P((I_{-n})^c) \approx 1 – P(I_{-n}) = 1 – 1/(6n-1) = (6n-2)/(6n-1)$

$P((J_n)^c) \approx 1 – P(J_n) = 1 – 1/(6n-1) = (6n-2)/(6n-1)$

$P((J_{-n})^c) \approx 1 – P(J_{-n}) = 1 – 1/(6n+1) = 6n/(6n+1)$

Next we know that,

$I_n = \{ x/ x\in N, x\in[n]_{6n+1}$ & $x>n\}$

$J_{-n} = \{ x/ x\in N, x\in[-n]_{6n+1}$ & $x>n\}$

Therefore, I_n and J_{-n} are different residue classes of modulo 6n+1. i.e residue classes are different but modulo same, hence I_n and J_{-n} are disjoint.

i.e $I_n \cap J_{-n} = \{ \}$

Therefore $P(I_n \cap J_{-n}) = 0$

i.e event of selecting a natural number such that it belongs to both I_n and J_{-n} is an impossible event. i.e event of selecting a natural number such that it belongs to I_n and the event of selecting a natural number such that it belongs to J_{-n} are dependent events. i.e occurrence one event affects the occurrence of other event.

Therefore, by the algebra of events and by the theory of probability,

$P(I_n \cup J_{-n}) = P(I_n) + P(J_{-n}) – P(I_n \cap J_{-n}) \approx 1/(6n+1) + 1/(6n+1) – 0$

[since $P(X\cup Y) = P(X) + P(Y) – P(X\cap Y)$]

$= 2/(6n+1)$

i.e $P(I_n \cup J_{-n}) \approx 2/(6n+1)$.

Hence $P((I_n \cup J_{-n})^c) \approx 1 – 2/(6n+1) = (6n-1)/(6n+1)$.

Similarly, we can show that,

$P(I_{-n} \cup J_n) \approx 2/(6n-1)$.

$P((I_{-n} \cup J_n)^c) \approx 1 – 2/(6n-1) = (6n-3)/(6n-1)$.

Let m1 and m2 be two different nonzero integers, such that their absolute values are smaller than or equal to n. i.e $|m1| \le n$ and $|m2| \le n$

$(I_{m1} \cup J_{-m1})$ is union of two residue classes of same modulo $(6|m1|\pm1)$, and

$(I_{m2} \cup J_{-m2})$ is union of two residue classes of same modulo $(6|m2|\pm1)$.

Therefore, $(I_{m1} \cup J_{-m1})$ and $(I_{m2} \cup J_{-m2})$ are residue classes of different moduli. Hence, $(I_{m1} \cup J_{-m1}) \cap (I_{m2} \cup J_{-m2}) \neq \{ \}$ i.e $(I_{m1} \cup J_{-m1})$ and $(I_{m2} \cup J_{-m2})$ are not disjoint. i.e event of selecting a natural number such that it belongs to both $(I_{m1} \cup J_{-m1})$ and $(I_{m2} \cup J_{-m2})$ is a possible event. Hence, event of selecting a natural number such that it belongs to $(I_{m1} \cup J_{-m1})$ and event of selecting a natural number such that it belongs to $(I_{m2} \cup J_{-m2})$ are independent events. i.e occurrence of one event does not affect the occurrence of other event.

Therefore, from the theory of probability,

$P((I_{m1} \cup J_{-m1}) \cap (I_{m2} \cup J_{-m2})) = P(I_{m1} \cup J_{-m1})$ X $P(I_{m2} \cup J_{-m2})$.

[since X and Y are independent events, implies $P(X\cap Y)= P(X)$ X $P(Y)$]

Similarly, we can show that,

$P((I_{m1} \cup J_{-m1})^c \cap (I_{m2} \cup J_{-m2})^c) = P((I_{m1} \cup J_{-m1})^c)$ X $P((I_{m2} \cup J_{-m2})^c)$.

And also, it can be shown for any number of different nonzero integers, say m1, m2, m3, ……, mn. Such that their absolute values smaller than or equal to n. i.e $|m1| \leq n$, $|m2| \leq n$, $|m3| \leq n$, …………… $|mn| \leq n$.

As in above, $(I_{m1} \cup J_{-m1})$, $(I_{m2} \cup J_{-m2})$, $(I_{m3} \cup J_{-m3})$, ……………., $(I_{mn} \cup J_{-mn})$ are residue classes of different moduli. Hence,

$$(I_{m1} \cup J_{-m1}) \cap (I_{m2} \cup J_{-m2}) \cap (I_{m3} \cup J_{-m3}) \cap \ldots\ldots \cap (I_{mn} \cup J_{-mn}) \neq \{\ \}$$

i.e

$$\bigcap_{i=1.}^{n}(I_{mi} \cup J_{-mi}) \neq \{\ \}$$

Therefore, similar to above arguments, the following two results are obvious.

First one

$$P\left(\bigcap_{i=1.}^{n}(I_{mi} \cup J_{-mi})\right) = P(I_{m1} \cup J_{-m1})\ X\ P(I_{m2} \cup J_{-m2})\ X\ P(I_{m3} \cup J_{-m3})\ X \ldots X\ P(I_{mn} \cup J_{-mn})$$

Hence, for $n < x$ and $6n \pm 1 \leq y-x+1$ = length of [x, y].

$$P\left(\bigcap_{i=1.}^{n}(I_i \cup J_{-i})\right) = P(I_1 \cup J_{-1})\ X\ P(I_2 \cup J_{-2})\ X\ P(I_3 \cup J_{-3})\ X \ldots\ X\ P(I_n \cup J_{-n})$$

$$\approx (2/7)\ X\ (2/13)\ X\ (2/19)\ X\ \ldots\ldots\ X\ 2/(6n+1)$$

And

$$P\left(\bigcap_{i=1.}^{n}(I_{-i} \cup J_i)\right) = P(I_{-1} \cup J_1)\ X\ P(I_{-2} \cup J_2)\ X\ P(I_{-3} \cup J_3)\ X \ldots\ X\ P(I_{-n} \cup J_n)$$

$$\approx (2/5)\ X\ (2/11)\ X\ (2/17)\ X\ \ldots\ldots\ X\ 2/(6n-1)$$

Next one

$$P\left(\bigcap_{i=1.}^{n}(I_{mi} \cup J_{-mi})^c\right) = P(\ (I_{m1} \cup J_{-m1})^c\ )\ X\ P(\ (I_{m2} \cup J_{-m2})^c\ )\ X\ P(\ (I_{m3} \cup J_{-m3})^c\ )\ X \ldots\ X\ P(\ (I_{mn} \cup J_{-mn})^c\ ).$$

Hence, for $n < x$ and $6n \pm 1 \leq y-x+1$ = length of [x, y].

$$P\left(\bigcap_{i=1.}^{n}(I_i \cup J_{-i})^c\right) = P(\ (I_1 \cup J_{-1})^c\ )\ X\ P(\ (I_2 \cup J_{-2})^c\ )\ X\ P(\ (I_3 \cup J_{-3})^c\ )\ X \ldots\ X\ P(\ (I_n \cup J_{-n})^c\ ).$$

$$\approx 5/7\ X\ 11/13\ X\ 17/19\ X\ \ldots\ X\ (6n-1)/(6n+1)$$

$$P\left(\bigcap_{i=1.}^{n}(I_{-i} \cup J_i)^c\right) = P(\ (I_{-1} \cup J_1)^c\ )\ X\ P(\ (I_{-2} \cup J_2)^c\ )\ X\ P(\ (I_{-3} \cup J_3)^c\ )\ X \ldots\ X\ P(\ (I_{-n} \cup J_n)^c\ ).$$

$\approx 3/5 \times 9/11 \times 15/17 \times \ldots\ldots\ldots\ldots \times (6n-3)/(6n-1)$.

Now let us summarise all the above results.

[x, y] be the closed interval, and X be any subset of N contained in [x, y].

P(X) is the probability selecting a natural number randomly in [x, y] such that it belongs to X.

For any natural number $n < x$. and $6n+1 \leq y - x + 1$ = length of [x, y].

1) $P(I_n) \approx 1/(6n+1)$

2) $P(I_{-n}) \approx 1/(6n-1)$.

3) $P(J_n) \approx 1/(6n-1)$.

4) $P(J_{-n}) \approx 1/(6n+1)$

5) $P((I_n)^c) \approx 6n/(6n+1)$

6) $P((I_{-n})^c) \approx (6n-2)/(6n-1)$

7) $P((J_n)^c) \approx (6n-2)/(6n-1)$

8) $P((J_{-n})^c) \approx 6n/(6n+1)$

9) $P(I_n \cup J_{-n}) \approx 2/(6n+1)$.

10) $P((I_n \cup J_{-n})^c) \approx (6n-1)/(6n+1)$.

11) $P(I_{-n} \cup J_n) \approx 2/(6n-1)$.

12) $P((I_{-n} \cup J_n)^c) \approx (6n-3)/(6n-1)$.

13) $P(\bigcap_{i=1.}^{n}(I_i \cup J_{-i})) \approx (2/7) \times (2/13) \times (2/19) \times \ldots\ldots\ldots \times 2/(6n+1)$

14) $P(\bigcap_{i=1.}^{n}(I_{-i} \cup J_i)) \approx (2/5) \times (2/11) \times (2/17) \times \ldots\ldots\ldots \times 2/(6n-1)$

15) $P(\bigcap_{i=1.}^{n}(I_i \cup J_{-i})^c) \approx 5/7 \times 11/13 \times 17/19 \times \ldots\ldots \times (6n-1)/(6n+1)$

16) $P(\bigcap_{i=1.}^{n}(I_{-i} \cup J_i)^c) \approx 3/5 \times 9/11 \times 15/17 \times \ldots\ldots\ldots \times (6n-3)/(6n-1)$.

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$

Note, when the length of [x, y] become larger and larger, the difference (error) become smaller and smaller. i.e approximation comes closer, when length of arbitrary interval increases.

Now let us go to the theorem.

THEOREM

For any natural number $b > 1$,

Number of natural numbers belongs to $[6b+2, (b-1)(6b+1)]\cap(A\cup B)^c$ is almost (approximately) equal to

$$(b-2)(6b+1)(3/5)(5/7)(9/11)(11/13) \ldots\ldots\ldots\ldots ((P_r-2)/P_r).$$

Where P_r is the greatest prime number, such that $P_r \leq 6(b-1)+1$

In other words, Number of pairs of twin prime numbers that belongs to the closed interval $[36b+11, 6(b-1)(6b+1)+1]$ is almost equal to

$$(b-2)(6b+1)(3/5)(5/7)(9/11)(11/13) \ldots\ldots\ldots\ldots ((P_r-2)/P_r).$$

REMARK

The fact, twin prime numbers are infinite is the immediate consequence of the above theorem.

PROOF

In my third article "article 3 about theory of distribution prime numbers.",

I have proved the corollary,

$$[1, (b-1)(6b+1)] \cap (A\cup B) = [1, (b-1)(6b+1)] \cap \Big(\big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})\big) \cup \big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)\big) \Big)$$

For $b > 1$, Let

$$C = \big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})\big) \cup \big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)\big)$$

[since if b=1, implies b-1=0, but $I_{\pm n}$ and $J_{\pm n}$ are defined only for natural number. i.e $I_{\pm(b-1)}$ and $J_{\pm(b-1)}$ are cannot be defined if b = 1. Therefore C cannot be defined for b=1]

Therefore, The above corollary becomes,

$$[1, (b-1)(6b+1)] \cap (A\cup B) = [1, (b-1)(6b+1)] \cap C.$$

Implies,

$$[1, (b-1)(6b+1)] \cap (A\cup B)^c = [1, (b-1)(6b+1)] \cap C^c.$$

[since $X\cap Y = X\cap Z$ implies $X\cap Y^c = X\cap Z^c$]

Implies,

$$[6b+2, (b-1)(6b+1)] \cap (A\cup B)^c = [6b+2, (b-1)(6b+1)] \cap C^c.$$

[since $Y\subset X$, $X\cap P = X\cap Q$ implies, $Y\cap P = Y\cap Q$].

Therefore,

$n([6b+2, (b-1)(6b+1)] \cap (AUB)^c) = n([6b+2, (b-1)(6b+1)] \cap C^c)$

By the basic definition of probability,

Probability of selecting a natural number randomly in [6b+2, (b-1)(6b+1)] such that it belongs to C^c

$$= n([6b+2, (b-1)(6b+1)] \cap C^c) \div n([6b+2, (b-1)(6b+1)])$$

i.e

$$P(C^c) = n([6b+2, (b-1)(6b+1)] \cap C^c) \div n([6b+2, (b-1)(6b+1)])$$

Where $P(C^c)$ is the probability of selecting a natural number randomly in [6b+2, (b-1)(6b+1)] such that it belongs to C^c

And it is obvious that

$$n([6b+2, (b-1)(6b+1)]) = (b-2)(6b+1)$$

Hence,

$$P(C^c) = n([6b+2, (b-1)(6b+1)] \cap C^c) \div (b-2)(6b+1)$$

Transposing yields

$n([6b+2, (b-1)(6b+1)] \cap C^c) = (b-2)(6b+1) X P(C^c)$

…………………………………………(3)

Next,

$$P(C^c) = P(((\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1}(I_nUJ_{-n})) U (\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1}(I_{-n}UJ_n)))^c)$$

$$= P((\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1}(I_nUJ_{-n}))^c \cap (\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1}(I_{-n}UJ_n))^c)$$

[since $P((X\cup Y)^c) = P(X^c \cap Y^c)$]

$$= P\Big(\big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n U J_{-n})\big)^c \Big) \text{ X } P\Big(\big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} U J_n)\big)^c \Big)$$

[since

$$\big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n U J_{-n})\big)^c \quad \& \quad \big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} U J_n)\big)^c$$

Are two complements of union of residue classes of different prime moduli. Hence they are not disjoint. i.e selecting a natural number such that it belongs to both complements is a possible event.

i.e event of selecting a natural number such that it belongs to

$$\big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n U J_{-n})\big)^c$$

And the event of selecting a natural number such that it belongs to

$$\big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} U J_n)\big)^c$$

are independent events].

Here C is union of $I_{\pm n}$ and $J_{\pm n}$ such that $1 \leq n \leq b-1 < 6b+2$.

And $6n\pm1 \leq 6(b-1)+1 < (b-2)(6b+1)$ = length of $[6b+2, (b-1)(6b+1)]$. Therefore, the results of (2) what we have found from above probability analysis obeys in the closed interval $[6b+2, (b-1)(6b+1)]$ for any $n \leq b-1$.

Therefore,

$$P(C^c) = P\Big(\big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n U J_{-n})\big)^c \Big) \text{ X } P\Big(\big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} U J_n)\big)^c \Big)$$

$$= P\Big(\bigcap_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n U J_{-n})^c\Big) \ X\ P\Big(\bigcap_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} U J_n)^c\Big)$$

[since complement of union of sets is equal to intersection of complements of sets]

$$\approx (5/7) \text{ x } (11/13) \text{ x } (17/19) \text{ x} \ldots\ldots\ldots \text{x } ((6s-1)/(6s+1))$$

$$\text{x } (3/5) \text{ x } (9/11) \text{ x } (15/17) \text{ x } \ldots\ldots\ldots \text{ x } ((6r-3)/(6r-1))$$

[since from (2)]

Where 6s+1 is the greatest prime number in the form 6k+1 such that $6s+1 \leq 6(b-1)+1$, similarly 6r-1 is the greatest prime number in the form 6k-1 such that $6r-1 \leq 6(b-1)-1$. Note: each factor in $P(C^c)$ is in the form (P-2)/P. where P is prime number smaller than or equal to 6(b-1)+1. After rearrangement of all factors in ascending order,

$$P(C^c) \approx (3/5) \text{ X } (5/7) \text{ X } (9/11) \text{ X } (11/13) \text{ X } \ldots\ldots\ldots\ldots\ldots\ldots \text{ X } ((P_r - 2)/P_r).$$

Where P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$.

Now (3) becomes,

$$n([6b+2, (b-1)(6b+1)] \cap C^c) \approx (b-2)(6b+1)(3/5)(5/7)(9/11)(11/13) \ldots\ldots\ldots\ldots ((P_r - 2)/P_r).$$

But

$$n([6b+2, (b-1)(6b+1)] \cap C^c) = n([6b+2, (b-1)(6b+1)] \cap (AUB)^c)$$

implies

$$n([6b+2, (b-1)(6b+1)] \cap (AUB)^c) \approx (b-2)(6b+1)(3/5)(5/7)(9/11)(11/13) \ldots\ldots\ldots\ldots ((P_r - 2)/P_r).$$

i.e Number of natural numbers belongs to $[6b+2, (b-1)(6b+1)] \cap (A \cup B)^c$ is almost (approximately) equal to

$$(b-2)(6b+1)(3/5)(5/7)(9/11)(11/13) \ldots\ldots\ldots\ldots ((P_r - 2)/P_r).$$

Hence the theorem is proved.

But from my second article IJNRD 2304175 "About theory of distribution of prime numbers.", I have shown that the natural number k determines the nature of natural numbers in the form 6k±1. i.e nature of natural numbers in the form 6k±1 depends on natural number k. Therefore, the natural numbers in the closed interval [6b+2, (b-1)(6b+1)] determines the nature of natural numbers in the form 6k±1 contained in the closed interval [6(6b+2)-1, 6(b-1)(6b+1)+1] = [36b+11, 6(b-1)(6b+1)+1]

Especially the arbitrary natural number k belongs to $(A \cup B)^c$ implies 6k+1 and 6k-1 both are prime numbers, i.e twin prime numbers. Therefore every natural number belongs to $[6b+2, (b-1)(6b+1)] \cap (A \cup B)^c$ determines exactly one pair of twin prime numbers in the closed interval [36b+11, 6(b-1)(6b+1)+1].

Therefore,

$$n([6b+2, (b-1)(6b+1)] \cap (A \cup B)^c) = \text{Number of pairs of twin primes numbers that belongs to the closed interval } [36b+11, 6(b-1)(6b+1)+1].$$

Hence the theorem can be restated as

Number of pairs of twin prime numbers that belongs to the closed interval [36b+11, 6(b-1)(6b+1)+1] is almost equal to (b-2)(6b+1)(3/5)(5/7)(9/11)(11/13) ………… $(P_r-2)/P_r$.

Where P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$.

Hence the theorem.

REMARK

1) Immediate consequence of above theorem is, By rearranging the factors,

$$(b-2)(6b+1)(3/5)(5/7)(9/11)(11/13) \ldots\ldots\ldots\ldots (P_r-2)/P_r.$$

$$= 3(b-2)(5/5)(9/7)(11/11)(15/13) \ldots\ldots\ldots\ldots\ldots ((P_r-2)/P_{r-1})((6b+1)/P_r)$$

$$> 3(b-2) \qquad \text{[other factors are greater than or equal to 1]}$$

Therefore for any arbitrary natural number b > 1, [36b+11, 6(b-1)(6b+1)+1] contains more than 3(b-2) pairs of twin prime numbers, which implies infiniteness of twin prime numbers. Hence twin prime numbers are infinite. Which is proved just before three years.

2) When b=1, b-1=0 but $I_{\pm n}$ and $J_{\pm n}$ are defined for natural numbers only. Therefore The above theorem obeys for any arbitrary natural number greater than 1.

3) when b=2 the closed interval is [14, 13], Which is incorrect notation of closed interval. And also though if we consider the interval as [13, 14].

The length of [13, 14] = 2 < 6(2-1)+1 = 6(b-1)+1. i.e (2) does not obey for [13, 14].

However, for [13, 14] the result is obvious. i.e

$n([13, 14] \cap (A\cup B)^c) = (2-2) \text{ X } 13 \text{ X } (3/5)(5/7) = 0$

i.e [13, 14] contains no natural number that belongs to $(A\cup B)^c$.

i.e [77, 85] contains no pair of twin prime numbers.

EXAMPLES

When b=3. The closed interval is [20, 38] here there are six natural numbers 23, 25, 30, 32, 33 and 38 belongs to $(AUB)^c$. but (3-2) x 19 x (3/5) x (5/7) x (9/11) x (11/13) = 5.64. which is nearly.

When b=4. The closed interval is [26, 75] here there are 11 natural numbers 30, 32, 33, 38, 40, 45, 47, 52, 58, 70, 72 belongs to $(AUB)^c$. but (4-2) x 25 x (3/5) x (5/7) x (9/11) x (11/13) x (15/17) x (17/19) x =11.71, which is nearly.

When b=5. The closed interval is [32, 124] here there are 17 natural numbers 32, 33, 38, 40, 45, 47, 52, 58, 70, 72, 77, 87, 95, 100, 103, 107, 110 belongs to $(AUB)^c$. but (5-2) x 31 x (3/5) x (5/7) x (9/11) x (11/13) x (15/17) x (17/19) x (21/23) = 19.88, which is nearly.

When b=16. The closed interval is [98, 1455] here there are 159 natural numbers belongs to $(AUB)^c$. but

$$(16-2)(97)(3/5)(5/7)(9/11)(11/13) \ldots\ldots\ldots (87/89) = 159.30$$

When b 17. The closed interval is [104, 1648] here there are 176 natural numbers belongs to $(AUB)^c$. but

$$(16-2)(97)(3/5)(5/7)(9/11)(11/13) \ldots\ldots\ldots (95/97)=177.50.$$

CONCLUTION

My name is **A. GABRIEL** a distance educated post graduate in mathematics. The thesis what we discussed above is myself realized one. Here I have submitted my completed concepts only. I am continuing my research about **THEORY OF DISTRIBUTION OF PRIME NUMBERS** by analyzing numbers which can be expressed in form 6ab±a±b, and which cannot be expressed in the form 6ab±a±b. i.e by analyzing the sets A, B, A^c, B^c, $A \cup B$, $(A \cup B)^c$, $A \cap B$, and $(A \cap B)^c$, where A and B are as defined above and the set of Natural numbers as universal set. then I conclude.

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