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ARTICLE 5 ABOUT THEORY OF DISTRIBUTION OF PRIME NUMBERSA.

Gabriel

ABSTRACT

In this article, two theorems and one corollary about theory of distribution of prime numbers are given with proof.

For arbitrary natural number b > 1.

Theorem (1) states that Number of prime numbers in the form 6k+1 contained in the closed interval [36b+11, 6(b-1)(6b+1)+1] is almost (approximately) equal to

$$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19)\ldots\ldots\ldots((P_r-1)/P_r)$$

Theorem (2) states that Number of prime numbers in the form 6k-1 contained in the closed interval [36b+11, 6(b-1)(6b+1)+1] is almost (approximately) equal to

$$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19)\ldots\ldots\ldots((P_r-1)/P_r)$$

Corollary states that Number of prime numbers contained in the closed interval [36b+11, 6(b-1)(6b+1)+1] is almost (approximately) equal to

$$2(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19)\ldots\ldots\ldots((P_r-1)/P_r)$$

Where P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$.

INTRODUCTION

Numbers are wonderful, marvelous creature of human. Numbers are classified into many types. They are Natural numbers, Whole numbers, Integers, Real numbers, Complex numbers, Rational numbers, Irrational numbers.

Natural numbers are classified into two categories

1) Prime numbers,
2) Composite numbers.

Prime numbers are Natural numbers which cannot be expressed in the form of product of two Natural numbers both greater than 1.

Composite numbers are other than Prime numbers. i.e. Which can be expressed in the form of product of two Natural numbers both greater than 1

From the definition of Prime number, 2 and 3 are Prime numbers, but 2x3=6 is a composite number. Multiples 6 are also composite numbers. Numbers in the form 6k±2 are even numbers i.e multiples of 2, hence composite numbers, Numbers in the form 6k±3 are odd multiples of 3 hence composite numbers.

Therefore, Prime numbers except 2 and 3 are in the form of 6k±1. where k is any natural number, but not for all natural numbers. For some Natural number k, 6k+1 is a Prime number but 6k-1 is a Composite number. For some Natural number k, 6k-1 is a Prime number but 6k+1 is a Composite numbers. For some Natural number k, 6k+1 and 6k-1 both are Prime numbers (twin Prime numbers). For some Natural number k, 6k+1 and 6k-1 both are Composite numbers. Hence this k is the key factor that determines Prime numbers and Composite numbers.

Before 2500 years ago Euclid proved that Prime numbers are infinite, Composite numbers generated by their prime factors, but Prime numbers are not generated. They are distributed among the gaps left by Composite numbers. This article is about Theory of distribution of Prime numbers. Distributive rule of Prime numbers is nothing but violation of generating rule of Composite numbers. And since all Prime numbers except 2 and 3 are in the form 6k±1, this k determines the numbers in the form 6k±1.

In my first two article, I have written about this k. Especially In my second article, IJNRD 2304175 I have defined two sets A and B, which are subsets of natural numbers. I want to make remember.

$$A = \left(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\square} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\square} I_{-n}\right)$$

$$B = \left(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\square} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\square} J_{-n}\right)$$

Where

$I_n = \{ x/ x\in N, x\in[n]_{6n+1}\ \&\ x>n\}$

$I_{-n} = \{ x/ x\in N, x\in[-n]_{6n-1}\ \&\ x>n\}$

$J_n = \{ x/ x\in N, x\in[n]_{6n-1}\ \&\ x>n\}$

$J_{-n} = \{ x/ x\in N, x\in[-n]_{6n+1}\ \&\ x>n\}$

It is obvious that two sets A and B are infinite. A and B are union of residue classes of infinite number of prime moduli. so It is very difficult to study the nature of sets A and B. to overcome this drawback, I define this closed interval [1, (b-1)(6b+1)]. the natural numbers of A and B that are contained in this closed interval are union of residue classes of finite number of prime moduli. Therefore, In my third article, IJNRD 2304384 I have proved the following facts.

For any natural number b

i) $$[1, (b-1)(6b+1)] \cap A = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} I_{-n}\right) \right)$$

ii) $$[1, (b-1)(6b+1)] \cap B = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n}\right) \right)$$

iii) $$[1, (b-1)(6b+1)] \cap (A \cup B) = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)\right) \right)$$

In my fourth article, IJNRD 2305057. I have shown the following facts.

$[x, y]$ be the closed interval, and X be any subset of N contained in $[x, y]$.

P(X) is the probability of selecting a natural number randomly in $[x, y]$ such that it belongs to X.

For any natural number $n < x$. and $6n+1 \leq y - x + 1$ = length of $[x, y]$.

1) $P(I_n) \approx 1/(6n+1)$
2) $P(I_{-n}) \approx 1/(6n-1)$.
3) $P(J_n) \approx 1/(6n-1)$.
4) $P(J_{-n}) \approx 1/(6n+1)$
5) $P((I_n)^c) \approx 6n/(6n+1)$
6) $P((I_{-n})^c) \approx (6n-2)/(6n-1)$
7) $P((J_n)^c) \approx (6n-2)/(6n-1)$
8) $P((J_{-n})^c) \approx 6n/(6n+1)$
9) $P(I_n \cup J_{-n}) \approx 2/(6n+1)$.
10) $P((I_n \cup J_{-n})^c) \approx (6n-1)/(6n+1)$.
11) $P(I_{-n} \cup J_n) \approx 2/(6n-1)$.
12) $P((I_{-n} \cup J_n)^c) \approx (6n-3)/(6n-1)$.

13) $$P\Big(\bigcap_{i=1}^{n}(I_i \cup J_{-i})\Big) \approx (2/7) \;X\; (2/13) \;X\; (2/19) \;X\; \ldots\ldots\ldots \;X\; 2/(6n+1)$$

14) $$P\Big(\bigcap_{i=1}^{n}(I_{-i} \cup J_{i})\Big) \approx (2/5) \;X\; (2/11) \;X\; (2/17) \;X\; \ldots\ldots\ldots \;X\; 2/(6n-1)$$

15) $$P\Big(\bigcap_{i=1}^{n}(I_i \cup J_{-i})^c\Big) \approx 5/7 \;X\; 11/13 \;X\; 17/10 \;X\; \ldots.. \;X\; (6n-1)/(6n+1)$$

16) $$P\Big(\bigcap_{i=1}^{n}(I_{-i} \cup J_{i})^c\Big) \approx 3/5 \;X\; 9/11 \;X\; 15/17 \;X\; \ldots\ldots\ldots \;X\; (6n-3)/(6n-1).$$

when the length of [x, y] become larger and larger, the difference (error) become smaller and smaller. i.e approximation comes closer, when length of arbitrary interval increases.

Let us continue from here,

Let m1 and m2 are two natural numbers, such that $m1 \le n < x$, $m2 \le n < x$, $6n+1 \le$ length of [x, y].

We know that $I_n = \{ x / x \in N, x \in [n]_{6n+1} \;\&\; x > n\}$

Therefore, I_{m1} and I_{m2} are two residue classes of different moduli. Hence they are not disjoint. i.e $I_{m1} \cap I_{m2} \neq \{\ \}$ i.e selecting a natural number such that it belongs to both I_{m1} and I_{m2} is a possible event. Therefore, event of selecting a natural number such that it belongs to I_{m1} and event of selecting a natural number such that it belongs to I_{m2} are independent events. i.e Occurrence of one event does not affect the occurrence of other event.

Therefore, by the theory of probability,

$$P(I_{m1} \cap I_{m2}) = P(I_{m1}) \;X\; P(I_{m2})$$

[since X and Y are two independent events. Implies, $P(X \cap Y) = P(X) \;X\; P(Y)$]

Similarly, we can show that

$$P(I_{-m1} \cap I_{-m2}) = P(I_{-m1}) \;X\; P(I_{-m2})$$

$$P(J_{m1} \cap J_{m2}) = P(J_{m1}) \;X\; P(J_{m2})$$

$$P(J_{-m1} \cap J_{-m2}) = P(J_{-m1}) \;X\; P(J_{-m2})$$

Let us continuing this argument for any number of natural numbers say m1, m2, m3, ………….mn. such that $m1 \le n$, $m2 \le n$, $m3 \le n$, ………. $mn \le n$.

As in above, $I_{m1}, I_{m2}, I_{m3}, \ldots\ldots\ldots\ldots I_{mn}$ are n different residue classes of n different modulo. Therefore, they are not disjoint. i.e

$$I_{m1} \cap I_{m2} \cap I_{m3} \cap \ldots\ldots\ldots\ldots \cap I_{mn} \neq \{\ \}$$

i.e

$$\bigcap_{i=1}^{n} I_{mi} \neq \{\ \}$$

i.e selecting a natural number such that it belongs to $I_{m1} \cap I_{m2} \cap I_{m3} \cap \ldots\ldots. \cap I_{mn}$ is a possible event. Therefore, events of selecting a natural number such that it belongs to each I_{mi} are independent events. i.e Occurrence of one event does not affect the occurrence of other events.

Therefore, by the theory of probability,

$$P\left(\bigcap_{i=1.}^{n} I_{mi}\right) = P(I_{m1}) \text{ X } P(I_{m2}) \text{ X } P(I_{m3}) \text{ X } \ldots\ldots\ldots\ldots\ldots \text{ X } P(I_{mn}).$$

Similar argument continues for $(I_{m1})^c$, $(I_{m2})^c$, $(I_{m3})^{c,}$ …………… $(I_{mn})^c$.

i.e $(I_{m1})^c$, $(I_{m2})^c$, $(I_{m3})^{c,}$ …………… $(I_{mn})^c$. are complements of n different residue classes of n different modulo. Therefore, they are not disjoint. i.e

$(I_{m1})^c \cap (I_{m2})^c \cap (I_{m3})^c \cap \ldots\ldots\ldots\ldots \cap (I_{mn})^c \neq \{\ \}$

i.e

$$\bigcap_{i=1}^{n} (I_{mi})^c \neq \{\ \}$$

i.e selecting a natural number such that it belongs to $(I_{m1})^c \cap (I_{m2})^c \cap (I_{m3})^c \cap$ .. …….. $\cap (I_{mn})^c$ is a possible event. Therefore, events of selecting a natural number such that it belongs to each $(I_{mi})^c$ are independent events. i.e Occurrence of one event does not affect the occurrence of other events.

Therefore, by the theory of probability,

$$P\left(\bigcap_{i=1.}^{n} (I_{mi})^c\right) = P((I_{m1})^c) \text{ X } P((I_{m2})^c) \text{ X } P((I_{m3})^c) \text{ X } \ldots\ldots\ldots\ldots\ldots \text{ X } P((I_{mn})^c).$$

Therefore, for $n < x$, and $6n+1 \leq$ length of $[x, y]$

$$P\left(\bigcap_{i=1.}^{n} (I_i)^c\right) = P((I_1)^c) \text{ X } P((I_2)^c) \text{ X } P((I_3)^c) \text{ X } \ldots\ldots\ldots\ldots\ldots \text{ X } P((I_n)^c).$$

$$\approx (6/7) \text{ X } (12/13) \text{ X } (18/19) \text{ X } \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ X } (6n/(6n+1))$$

Similarly, we can show the following facts.

For $n < x$ and $6n+1 \leq$ length of $[x, y]$

$$P\left(\bigcap_{i=1.}^{n} (I_{-i})^c\right) = P((I_{-1})^c) \text{ X } P((I_{-2})^c) \text{ X } P((I_{-3})^c) \text{ X } \ldots\ldots\ldots\ldots\ldots \text{ X } P((I_{-n})^c).$$

$$\approx (4/5) \text{ X } (10/11) \text{ X } (16/17) \text{ X } \ldots\ldots\ldots\ldots\ldots\ldots \text{ X } ((6n-2)/(6n-1))$$

$$P\left(\bigcap_{i=1}^{n}(J_i)^c\right) = P((J_1)^c) \times P((J_2)^c) \times P((J_3)^c) \times \ldots \times P((J_n)^c).$$

$$\approx (4/5) \times (10/11) \times (16/17) \times \ldots \times ((6n-2)/(6n-1))$$

$$P\left(\bigcap_{i=1}^{n}(J_{-i})^c\right) = P((J_{-1})^c) \times P((J_{-2})^c) \times P((J_{-3})^c) \times \ldots \times P((J_{-n})^c).$$

$$\approx (6/7) \times (12/13) \times (18/19) \times \ldots \times (6n/(6n+1))$$

……………………………………………………(1).

Now let us go to the theorem (1).

THEOREM (1)

Number of natural numbers contained in $[6b+2, (b-1)(6b+1)] \cap A^c$ is almost (approximately) equal to

$$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots ((P_r-1)/P_r).$$

In other words, Number of prime numbers in the form 6k+1 contained in [36b+11, 6(b-1)(6b+1)+1] is almost (approximately) equal to

$$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots ((P_r-1)/P_r).$$

Where b is the arbitrary natural number greater than 1.

Where P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$

PROOF

In my third article IJNRD 2304384, I have proved the theorem,

For any natural number b

$$[1, (b-1)(6b+1)] \cap A = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} I_{-n}\right) \right)$$

For b > 1, Let

$$C = \left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} I_{-n}\right)$$

[since, if b=1, implies b-1=0, but I_{+n} and I_{-n} are defined only for natural numbers. i.e $I_{+(b-1)}$ and $I_{-(b-1)}$ are cannot be defined. Therefore, C cannot be defined for b=1]

Therefore, for $b > 1$, Above theorem becomes

$[1, (b-1)(6b+1)] \cap A = [1, (b-1)(6b+1)] \cap C$

Implies,

$[1, (b-1)(6b+1)] \cap A^c = [1, (b-1)(6b+1)] \cap C^c$

[since $X\cap Y=X\cap Z$ implies, $X\cap Y^c=X\cap Z^c$]

Implies,

$[6b+2, (b-1)(6b+1)] \cap A^c = [6b+2, (b-1)(6b+1)] \cap C^c$

[since $Y\subset X$, $X\cap P=X\cap Q$ implies, $Y\cap P=Y\cap Q$.]

$n([6b+2, (b-1)(6b+1)] \cap A^c) = n([6b+2, (b-1)(6b+1)] \cap C^c)$

Let

$P(C^c)$ is the Probability of selecting a natural number randomly in $[6b+2, (b-1)(6b+1)]$ Such that it belongs to C^c.

By the basic definition of probability,

$P(C^c) = n([6b+2, (b-1)(6b+1)] \cap C^c) \div n([6b+2, (b-1)(6b+1)])$

It is obvious that,

$n([6b+2, (b-1)(6b+1)]) = (b-2)(6b+1)$.

Therefore,

$P(C^c) = n([6b+2, (b-1)(6b+1)] \cap C^c) \div (b-2)(6b+1)$.

Transposing yields

$n([6b+2, (b-1)(6b+1)] \cap C^c) = (b-2)(6b+1) \times P(C^c)$

……………………………………….(2)

Next,

$$P(C^c) = P\Big(\big(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} I_n \big) \cup \big(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} I_{-n} \big)^c \Big)$$

$$= P\Big(\big(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} I_n \big)^c \cap \big(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} I_{-n} \big)^c \Big)$$

[since $P((X\cup Y)^c)=P(X^c\cap Y^c)$]

b-1 b-1

$= P((UI_n)^c) \text{ X } P((UI_{-n})^c)$

n=1 & n=1 &

6n+1 a prime number. 6n-1 a prime number.

[since

b-1 b-1

$(UI_n)^c$ & $(UI_{-n})^c$

n=1 & n=1 &

6n+1 a prime number. 6n-1 a prime number.

Are two complements of unions of residue classes of different prime moduli. Hence they are not disjoint. i. e selecting a natural number such that it belongs to both complements is a possible event. i.e event of selecting a natural number such that it belongs to

b-1

$(UI_n)^c$

n=1 &

6n+1 a prime number.

And the event of selecting a natural number such that it belongs to

b-1

$(UI_{-n})^c$

n=1 &

6n-1 a prime number.

Are independent events. i.e Occurrence one event does not affect the occurrence of other event.]

And Here C is union of I_n and I_{-n} such that $1 \leq n \leq b-1 < 6b+2$.

And $6n\pm1 \leq 6(b-1)+1 < (b-2)(6b+1)$ = length of [6b+2, (b-1)(6b+1)]. Therefore, the results of (1) what we have found from above probability analysis obeys in the closed interval [6b+2, (b-1)(6b+1)] for any $n \leq b-1$.

Therefore,

b-1 b-1

$P(C^c) = P((UI_n)^c) \text{ X } P((UI_{-n})^c)$

n=1 & n=1 &

6n+1 a prime number. 6n-1 a prime number.

$$P(C^c) = P\Big(\bigcap_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n)^c\Big) \ X\ P\Big(\bigcap_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n})^c\Big)$$

[since complement of union of sets is equal to intersection of complements of sets]

$$\approx (6/7)\ X\ (12/13)\ X\ (18/19)\ X\ \ldots\ldots\ldots\ldots\ldots\ X\ (P_s\text{-}1/P_s)$$

$$X\ (4/5)\ X\ (10/11)\ X\ (16/17)\ X\ \ldots\ldots\ X\ (P_r\text{-}1)/P_r).$$

[since from (1)

Where P_s is the greatest prime number in the form 6k+1 such that $P_s \leq 6(b-1)+1$ and P_r is the greatest prime number in the form 6k-1 such that $P_r \leq 6(b-1)-1$. Note: each factor in $P(C^c)$ is in the form (P-1)/P where P is prime number smaller than or equal to 6(b-1)+1. After rearrangement of all factors in ascending order,

$$P(C^c) \approx (4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots ((P_r-1)/P_r)$$

P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$.

Now (2) becomes,

$$n(\ [6b+2, (b-1)(6b+1)] \cap C^c\ ) = (b-2)(6b+1)\ \ X\ \ P(C^c)$$

$$\approx (b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots ((P_r-1)/P_r).$$

But

$$n(\ [6b+2, (b-1)(6b+1)] \cap C^c\ ) = n(\ [6b+2, (b-1)(6b+1)] \cap A^c\ ).$$

Implies,

$$n(\ [6b+2, (b-1)(6b+1)] \cap A^c\ ) \approx (b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots ((P_r-1)/P_r).$$

P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$

Hence the theorem is proved.

But in my second article IJNRD 2304175 "About theory of distribution of prime numbers.", I have shown that the natural number k determines the nature of natural numbers in the form 6k±1. i.e nature of natural numbers in the form 6k±1 depends on natural number k. Therefore, the natural numbers in [6b+2, (b-1)(6b+1)] determines the nature of natural numbers in the form 6k±1 contained in the closed interval

$$[6(6b+2)-1, 6(b-1)(6b+1)+1] = [36b+11, 6(b-1)(6b+1)+1].$$

Especially arbitrary natural number k belongs to A^c implies 6k+1 a prime number. Therefore every natural number belongs to $[6b+2, (b-1)(6b+1)] \cap A^c$ determines exactly one prime numbers in the form 6k+1 contained in the closed interval [36b+11, 6(b-1)(6b+1)+1].

Therefore, $n(\ [6b+2, (b-1)(6b+1)] \cap A^c\ )$ is equal to Number of prime numbers in the form 6k+1 contained in the closed interval [36b+11, 6(b-1)(6b+1)+1]. Hence the theorem can be restated as,

Number of prime numbers in the form 6k+1 contained in [36b+11, 6(b-1)(6b+1)+1] is almost (approximately) equal to

$$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots\ldots\ldots\ldots ((P_r-1)/P_r).$$

Where P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$.

Next theorem (2) and its proof are similar to the theorem (1) and its proof. For better understanding we redo it. let us go to theorem (2).

THEOREM (2)

Number of natural numbers contained in $[6b+2, (b-1)(6b+1)] \cap B^c$ is almost (approximately) equal to

$$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots\ldots\ldots\ldots. ((P_r-1)/P_r).$$

In other words, Number of prime numbers in the form 6k-1 contained in [36b+11, 6(b-1)(6b+1)+1] is almost (approximately) equal to

$$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots\ldots\ldots\ldots. ((P_r-1)/P_r).$$

Where b is arbitrary natural number such that b is greater than 1

Where P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$

PROOF

In my third article IJNRD 2304384, I have proved the theorem,

For any natural number b

$$[1, (b-1)(6b+1)] \cap B = [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n}\right) \right)$$

For b > 1, Let

$$C = \left(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n}\right)$$

[since if b=1, implies b-1=0, but J_{+n} and J_{-n} are defined only for natural number. i.e $J_{+(b-1)}$ and $J_{-(b-1)}$ are cannot be defined. Therefore C cannot be defined for b=1]

Therefore, for b > 1, Above theorem becomes

$[1, (b-1)(6b+1)] \cap B = [1, (b-1)(6b+1)] \cap C$

Implies,

$[1, (b-1)(6b+1)] \cap B^c = [1, (b-1)(6b+1)] \cap C^c$

[since $X \cap Y = X \cap Z$ implies, $X \cap Y^c = X \cap Z^c$]

Implies,

$[6b+2, (b-1)(6b+1)] \cap B^c = [6b+2, (b-1)(6b+1)] \cap C^c$

[since $Y \subset X$, $X \cap P = X \cap Q$ implies, $Y \cap P = Y \cap Q$.]

$n([6b+2, (b-1)(6b+1)] \cap B^c) = n([6b+2, (b-1)(6b+1)] \cap C^c)$

Let

$P(C^c)$ is the Probability of selecting a natural number randomly in [6b+2, (b-1)(6b+1)] Such that it belongs to C^c

By the basic definition of probability,

$$P(C^c) = n([6b+2, (b-1)(6b+1)] \cap C^c) \div n([6b+2, (b-1)(6b+1)])$$

And it is obvious that,

$$n([6b+2, (b-1)(6b+1)]) = (b-2)(6b+1).$$

Therefore,

$$P(C^c) = n([6b+2, (b-1)(6b+1)] \cap C^c) \div (b-2)(6b+1).$$

Transposing yields

$$n([6b+2, (b-1)(6b+1)] \cap C^c) = (b-2)(6b+1) \ X \ P(C^c) \qquad \text{...(2)}$$

Next,

$$P(C^c) = P\Big(\Big(\big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} J_n\big) \ \cup \ \big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n}\big) \Big)^c \Big)$$

$$= P\Big(\big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} J_n\big)^c \ \cap \ \big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n}\big)^c \Big)$$

[since $P((X\cup Y)^c)=P(X^c\cap Y^c)$]

$$= P\Big(\big(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} J_n\big)^c \Big) \ X \ P\Big(\big(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n}\big)^c \Big)$$

[since

b-1 b-1

$(\cup J_n)^c$ & $(\cup J_{-n})^c$

n=1 & n=1 &

6n-1 a prime number. 6n+1 a prime number.

Are two complements of unions of residue classes of different prime moduli. Hence they are not disjoint. i.e selecting a natural number such that it belongs to both complements is a possible event. i.e event of selecting a natural number such that it belongs to

b-1

$(\cup J_n)^c$

n=1 &

6n-1 a prime number.

And the event of selecting a natural number such that it belongs to

b-1

$(\cup J_{-n})^c$

n=1 &

6n+1 a prime number.

Are independent events. i.e Occurrence of one event does not affect the occurrence of other event.]

And Here C is union of J_n and J_{-n} such that $1 \leq n \leq b\text{-}1 < 6b+2$.

And $6n\pm1 \leq 6(b\text{-}1)+1 < (b\text{-}2)(6b+1)$ = length of [6b+2, (b-1)(6b+1)]. Therefore, the results of (1) what we have found from the above probability analysis obeys in the closed interval [6b+2, (b-1)(6b+1)] for any $n \leq b\text{-}1$.

Therefore,

b-1 b-1

$P(C^c) = P((\cup J_n)^c) \text{ X } P((\cup J_{-n})^c)$

n=1 & n=1 &

6n-1 a prime number. 6n+1 a prime number.

b-1 b-1

$= P(\cap (J_n)^c) \text{ X } P(\cap (J_{-n})^c)$

n=1 & n=1 &

6n-1 a prime number. 6n+1 a prime number.

[since complement of union of sets is equal to intersection of complements of sets]

$\approx (4/5) \text{ X } (10/11) \text{ X } (16/17) \text{ X } \ldots\ldots\ldots \text{ X } (P_r\text{-}1)/P_r).$

$\text{X } (6/7) \text{ X } (12/13) \text{ X } (18/19) \text{ X } \ldots\ldots\ldots\ldots \text{ X } (P_s\text{-}1/P_s)$

[since from (1)

Where P_s is the greatest prime number in the form 6k+1 such that $P_s \leq 6(b-1)+1$ and P_r is the greatest prime number in the form 6k-1 such that $P_r \leq 6(b-1)-1$. Note: each factor in $P(C^c)$ is in the form (P-1)/P where P is prime number smaller than or equal to 6(b-1)+1. After rearrangement of all factors in ascending order,

$$P(C^c) \approx (4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots ((P_r-1)/P_r)$$

P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$

Now (2) becomes,

$$n([6b+2, (b-1)(6b+1)] \cap C^c) = (b-2)(6b+1) \text{ X } P(C^c)$$

$$\approx (b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots ((P_r-1)/P_r).$$

But

$$n([6b+2, (b-1)(6b+1)] \cap C^c) = n([6b+2, (b-1)(6b+1)] \cap B^c).$$

Implies,

$$n([6b+2, (b-1)(6b+1)] \cap B^c) \approx (b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots ((P_r-1)/P_r).$$

P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$

Hence the theorem is proved.

But from my second article IJNRD 2304175 "About theory of distribution of prime numbers.", I have shown that the natural number k determines the nature of natural numbers in the form 6k±1. i.e nature of natural numbers in the form 6k±1 depends on natural number k. Therefore, the natural numbers in [6b+2, (b-1)(6b+1)] determines the nature of natural numbers in the form 6k±1 contained in the closed interval

$$[6(6b+2)-1, 6(b-1)(6b+1)+1] = [36b+11, 6(b-1)(6b+1)+1].$$

Especially arbitrary natural number k belongs to B^c implies 6k-1 a prime number. Therefore every natural number belongs to $[6b+2, (b-1)(6b+1)] \cap B^c$ determines exactly one prime numbers in the form 6k-1 contained in the closed interval [36b+11, 6(b-1)(6b+1)+1].

Therefore, $n([6b+2, (b-1)(6b+1)] \cap B^c)$ is equal to Number of prime numbers in the form 6k-1 contained in the closed interval [36b+11, 6(b-1)(6b+1)+1]. Hence the theorem can be restated as,

Number of prime numbers in the form 6k-1 contained in [36b+11, 6(b-1)(6b+1)+1] is almost (approximately) equal to

$$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots\ldots\ldots\ldots\ldots ((P_r-1)/P_r).$$

Where P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$

COROLLARY

Number of prime numbers contained in [36b+11, 6(b-1)(6b+1)+1] is almost (approximately) equal to

$$2(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13)(16/17)(18/19) \ldots\ldots\ldots\ldots\ldots\ldots ((P_r-1)/P_r).$$

Where P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$

PROOF

Corollary is nothing but combined form of above two theorems.

i.e

Number of prime numbers contained in [36b+11, 6(b-1)(6b+1)+1]

$$=$$

Number of prime numbers in the form 6k+1 contained in [36b+11, 6(b-1)(6b+1)+1]

$$+$$

Number of prime numbers in the form 6k-1 contained in [36b+11, 6(b-1)(6b+1)+1]

$$\approx$$

$$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13) \ldots\ldots ((P_r-1)/P_r).$$

$$+$$

$$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13) \ldots\ldots ((P_r-1)/P_r).$$

$$=$$

$$2(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13) \ldots\ldots ((P_r-1)/P_r).$$

[since from theorem (1) and theorem (2)]

Where P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$.

REMARKS

1) Immediate consequences of theorem (1) is, By rearranging the factors,

$(b-2)(6b+1)(4/5)(6/7)(10/11)(12/13) \ldots\ldots (P_r-1)/P_r.$

$= 4(b-2)(6/5)(10/7)(12/11)(16/13)\ldots\ldots((P_r-1)/P_{r-1})((6b+1)/P_r)$

$> 4(b-2)$ [since other factors are greater than 1]

Therefore, by theorem (1), for any arbitrary natural number b, [36b+11, 6(b-1)(6b+1)+1] contains more than 4(b-2) prime numbers in the form 6k+1, which implies infiniteness of prime numbers in the form 6k+1. Similarly, theorem (2) implies infiniteness of prime numbers in the form 6k-1.

2) When b=1, b-1=0 but $I_{\pm n}$ and $J_{\pm n}$ are defined for natural numbers only. Therefore The above theorems and corollary obey for any arbitrary natural number greater than 1.

3) when b=2, the closed interval is [14, 13], Which is incorrect notation of closed interval, if we take the interval as [13, 14] then also length of [13, 14] is 2 < 6(2-1)+1 = 6(b-1)+1. Therefore, results of (1) does not obey for closed interval [13, 14]. However, the result is obvious for b = 2.

Only one number 13 belongs to A^c, and Only one number 14 belongs to B^c,

But (2-2) X 13 X (4/5) X (6/7) = 0, which is nearly.

EXAMPLES

When b=3. Then The closed interval in the form [36b+11, 6(b-1)(6b+1)+1] is [119, 229]. In this closed interval, there are 20 prime numbers.

but 2(3-2) x 19 x (4/5) x (6/7) x (10/11) x (12/13) = 21.86 which is nearly.

When b=4. Here the closed interval is [155, 451] here there are 51 prime numbers.

but 2(4-2) x 25 x (4/5) x (6/7) x (10/11) x (12/13) x (16/17) x (18/19) x =51.30, which is nearly.

When b=5. Here the closed interval is [191, 745] here there are 90 prime numbers.

but 2(5-2) x 31 x (4/5) x (6/7) x (10/11) x (12/13) x (16/17) x (18/19) x (22/23) = 91.28, which is nearly.

When b=16. Here the closed interval is [587, 8731]. here there are 982 prime numbers.

but 2(16-2)(97)(4/5)(6/7)(10/11)(12/13)……..(88/89) = 990.50, which is nearly.

When b = 17. Here the closed interval is [623, 9889]. here there are 1106 prime numbers.

but 2(17-2)(103)(4/5)(6/7)(10/11)(12/13)……..(96/97)=.1115.34 which is nearly.

See, when the interval length increases, the error percentage with respect interval length decreases.

CONCLUTION

My name is **A. GABRIEL** a distance educated post graduate in mathematics. The thesis what we discussed above is myself realized one. Here I have submitted my completed concepts only. I am continuing my research about **THEORY OF DISTRIBUTION OF PRIME NUMBERS** by analyzing numbers which can be expressed in form 6ab±a±b, and which cannot be expressed in the form 6ab±a±b. i.e by analyzing the sets A, B, A^c, B^c, A∪B, $(A∪B)^c$, A∩B, and $(A∩B)^c$, where A and B are as defined above and the set of Natural numbers as universal set. then I conclude.

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