



“SOME STUDY IN GENERAL THEORY OF RELATIVITY ABOUT CYLINDRICALLY SYMMETRIC MODEL IN PRESENCE OF ELECTROMAGNETIC”

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Abstract: The study of relativistic field equations in the presence of a scalar meson field has drawn the attention of numerous relativists. Brahmchary [3] is credited with coming up with the concept when he looked at the issue of the linked gravitational and zero-rest mass scalar (zerospin) in the situation of a static spherically symmetric field. He has demonstrated that there is no perfect solution to the scalar meson field in purely empty space. He has, however, come up with an approximation that is accurate for a given area. Bergmann Leipnik has also studied the spherically symmetric zero-rest-mass scalar field [1]. Reciprocal static solutions for axial and spherical fields have been created by Buchdahl [4]. Janis, Newman, and Winicokr [7] have further analysed the issue from the perspective of singularities. The analysis demonstrates that the structure of the event horizon corresponding to $g_{44}=0$ and t constant charge changes from a non-singular hyper surface to a singular point with the inclusion of a zero-rest-mass scalar field. Singh [14] has only taken into account plan symmetric fields, but Gautreu [6] has expanded the work to include the case of non-spherical Weyl fields. The scalar meson field (with zero rest mass) interacts with the gravitational field in the study mentioned above.

Key words: relativistic, field, symmetric, zero-rest-mass, scalar, singularities, gravitational, hypersurface.

Introduction : The study of relativistic field equations in the presence of a scalar meson field has drawn the attention of numerous relativists. Brahmchary [3] is credited with coming up with the concept when he looked at the issue of the linked gravitational and zero-rest mass scalar (zerospin) in the situation of a static spherically symmetric field. He has demonstrated that there is no perfect solution to the scalar meson field in purely empty space. He has, however, come up with an approximation that is accurate for a given area. Bergmann Leipnik has also studied the spherically symmetric zero-rest-mass scalar field [1]. Reciprocal static solutions for axial and spherical fields have been created by Buchdahl [4]. Singularity-based analysis of the issue has been done further by Janis, Newman, and Winicokr [7]. The analysis demonstrates that the structure of the event horizon corresponding to $g_{44}=0$ and t constant charge changes from a non-singular hyper surface to a singular point with the inclusion of a zero-rest-mass scalar field. Singh [14] has only taken into account plan symmetric fields, but Gautreu [6] has expanded the work to include the case of non-spherical Weyl fields. The scalar meson field (with zero rest mass) interacts with the gravitational field in the study mentioned above. Stephenson [19] has thought about the issue of the static, spherically symmetric scalar meson field with non-zero rest mass connected with electromagnetic fields. Physically, this condition could be compared to a point source with nuclear g in addition to mass m and an electric charge. He has demonstrated how the coulomb's field is impacted by the existence of nuclear charge on the source and has gotten an approximation of a solution. The gravitational interaction between the electromagnetic fields and the scalar meson causes this effect. . The problem is still open to explore the impacts of the two fields in the case of exact solutions, though, as the approximations might not always paint the right picture of the underlying physical reality.

. Roy and Prakash [10] have built an isotropic magneto-hydrodynamic cosmic model in General Relativity using the cylindrically symmetric metric of Marder [8]. Later, Singh and Yadav [16] built a non-static cylindrically symmetric, spatially homogeneous, non-degenerated prototype I of a cosmological model. They did this by assuming the energy momentum tensor to be that of an ideal fluid with an electromagnetic field. Singh et al. [15] and Roy Tripathi [11] have both studied the cylindrically symmetric solutions of the Einstein-Maxwell equation in great detail. A method for creating solutions to the cylindrically symmetric gravitational field coupled to electromagnetic and massless scalar fields has been given by Singh et al. [17]. The issue of linked gravitational, electromagnetic, and scalar fields was studied by Sharma and Yadav [13]. It is discovered that a cylindrically symmetric gravitational field with two degrees of freedom cannot have its source term in the energy momentum tensor of a massive scalar field. . Roy and Rao [12] have found a similar result for the case of an Einstein-Rosen metric with one degree of freedom that is cylindrically symmetric. A static cylindrically symmetric cosmological model with an electro-magnetic and scalar field has been investigated in general relativity by Yadav and Kumar [20].

. For cylindrically symmetric metrics (Stachel metric [18]), we have found some solutions in this chapter for two different scenarios. Case (1) is solved directly in terms of Fij components, whereas Case (II) is solved in terms of two potentials, 2 and 3. Additionally, we have demonstrated that by appropriately redefining one of the metric coefficients, it is possible to produce an entire class of solutions to the Einstein-Maxwell mass-less scalar field equations starting with any solution to the electro-vacuum field equations. We were able to solve the Einstein-Maxwell scalar field equations by applying our method to an existing solution of the Einstein-Maxwell equation. By applying an application of Bonnor's theorem [2] to linearly superposed coupled field, its dual solution is obtained.

THE FIELD EQUATIONS AND THEIR SOLUTIONS

We considered the satchel metric [18] in the form given by

$$ds^2=e^{2n-2\delta}(dt^2 - dr^2) - (X^2e^{2\delta}+r^2e^{-2\delta})d\phi^2 - e^{2\delta}dz^2-2Xe^{2\delta}d\phi dz.....(1)$$

Where n , X and δ are the function of r only and r,φ, z, t correspond respectively x¹,x²,x³, x⁴ coordinates. When X =0 , this goes to the well known Einstein-Rosenmetric [5,9] with one degree of freedom. The distribution consist of an electromagnetic field and a scalar zero rest mass meson field V. Thus the field equation are

$$R_{ij} = - K [\epsilon_{ij+v, i v, j}],.....(2)$$

$$E_{ij} = -g^{ab}F_{ai}F_{bj} + \frac{1}{4}g_{ij}F_{ab}F^{ab},(3)$$

$$F_{[ijk]} = 0,(4)$$

$$F^{ij}{}_{;j} = 0.....(5)$$

$$g^{ij}{}_{;j} = 0.....(6)$$

for the metric (1)the components

R₁₂, R₁₃, R₂₄, and R₃₄ vanish identically . this leads to

$$F_{13}F_{23}\frac{e^{2\delta}}{r^2} = F_{12}F_{23}\frac{Xe^{-2\delta}}{r^2} + F_{14}F_{24}e^{2\delta-2n},(7)$$

$$F_{12}F_{32}\left[e^{+2\delta} + \frac{X^2e^{+2\delta}}{r^2}\right] = F_{13}F_{32}\frac{Xe^{+2\delta}}{r^2} + F_{14}F_{34}e^{2\delta-2n},(8)$$

$$F_{31}F_{41}e^{2\delta-2n} = F_{32}F_{43}\frac{Xe^{+2\delta}}{r^2} - F_{32}F_{42}\left[e^{-2\delta} + \frac{X^2e^{+2\delta}}{r^2}\right].....(9)$$

$$F_{21}F_{41}e^{2\delta-2n} = F_{23}F_{42}\frac{Xe^{+2\delta}}{r^2} - F_{23}F_{43}\frac{1e^{+2\delta}}{r^2}.....(10)$$

In view of (7)-(10) two cases may arise

(i) $F_{12} = F_{13} = F_{24} = F_{34} = 0, F_{23}, F_{14} \neq 0$

(ii) $F_{14} = F_{23} = 0, F_{12}, F_{13}, F_{24}, F_{34} \neq 0$

Case (i) is done directly in term of F_{ij} components, case(ii) is done in term of two potentials θ_2 and θ_3 which give rise to all the non-vanishing components of F_{ij} .

SECTION-I

Here we will consider case (i). The field equation (2)-(6) for the metric

(1) are,

$$R_{11} = -K \left[-\frac{1}{2} \left\{ e^{2\delta-2n} F_{14}^2 + \frac{e^{2n-2\delta}}{r^2} F_{23}^2 \right\} + V_1^2 \right] \dots\dots\dots(11)$$

$$R_{22} = -K \left[-\frac{1}{2} \left\{ e^{4\delta-4n} (X^2 e^{2\delta} + r^2 e^{-2\delta}) F_{14}^2 \right\} + (X^2 e^{2\delta} + r^2 e^{-2\delta}) \frac{F_{23}^2}{2r^2} \right] \dots\dots\dots(12)$$

$$R_{33} = -K \left[e^{6\delta-4n} F_{14}^2 + \frac{e^{+2\delta}}{2r^2} F_{23}^2 \right] \dots\dots\dots(13)$$

$$R_{44} = -K \left[\frac{1}{2} \left\{ \frac{e^{2\delta-2n}}{2} F_{14}^2 + \frac{e^{2n-2\delta}}{r^2} F_{23}^2 \right\} \right] \dots\dots\dots(14)$$

$$R_{23} = -K \left[-\frac{X}{2} \left\{ e^{6\delta-4n} F_{14}^2 + \frac{e^{+2\delta}}{r^2} F_{23}^2 \right\} \right] \dots\dots\dots(15)$$

$$F_{23} = H \dots\dots\dots(16)$$

$$F_{14} = K_1 \dots\dots\dots(17)$$

$$V_{11} + \frac{V_1}{r} = 0 \dots\dots\dots(18)$$

Equation (h) on integration yields

$$V = K_2 \rho \dots\dots\dots(19)$$

Where we have defined the new variables ρ by means of the equation,

$$\frac{dr}{d\rho} = r \text{ or } \log_e^r = \rho$$

The equation (11)-(15) are five equation in F_{14} and F_{23} from these equation , we get the following four differential equation

$$R_{11} + R_{44} + K V_{,1}^2 = 0 \dots\dots\dots(20)$$

$$R_{11} + e^{2n-4\delta} R_{44} + K V_{,1}^2 = 0 \dots\dots\dots(21)$$

$$R_{11} + \frac{e^{2n}}{r} R_{33} - \frac{e^{+2n}}{r} R_{23} + K V_{,1}^2 = 0 \dots\dots\dots(22)$$

$$R_{23} - X R_{22} = 0 \dots\dots\dots(23)$$

Equation (j)-(m) give rise to the following differential equation.

$$2\delta_1^2 - \frac{2n_1}{r} + X_1^2 \frac{e^{4\delta}}{2r^2} + K V_{,1}^2 = 0 \dots\dots\dots(24)$$

$$2\delta_1^2 + n_{11} \frac{n_1}{r} + KV_{,1}^2 = 0 \dots\dots\dots(25)$$

$$2\delta_1^2 - \frac{2\delta_1}{r} - \frac{n_1}{r} + n_{11} - 2\delta_{11} + \frac{e^{4\delta}}{2r^2} \left[X_1^2 + \frac{1}{2}XX_{11} + 2XX_1\delta - \frac{XX_1}{2r} \right] + KV_{,1}^2 = 0 \dots\dots(26)$$

$$2X_1\delta_1 + \frac{1}{2}X_{11} - \frac{1}{2} \frac{X_1}{r} = 0 \dots\dots\dots(27)$$

From equation (25), (26) and (27), we get

$$X_1^2 \frac{e^{4\delta}}{r} - \frac{2\delta_1}{r} - 2\delta_{11} = 0 \dots\dots\dots(28)$$

Equation (27) on integration gives

$$\frac{X_1}{r} e^{4\delta} = a \dots\dots\dots(29)$$

Where a is integration constant from equation (24) and (29) we obtained

$$\eta_1 = r\delta_1^2 + \frac{a^2 e^{-4\delta r}}{4} + \frac{KV_{,1}^2 r}{2} = 0 \dots\dots\dots(30)$$

From equation (28) and (29) we obtained

$$\delta_{11} + \delta_1 \frac{1}{r} = \frac{a^2 e^{-4\delta}}{2} \dots\dots\dots(31)$$

From equation (24),(25) and (29), we obtained

$$\eta_{11} + \eta_1 \left(\frac{1}{r}\right) = \frac{a^2 e^{-4\delta}}{2} \dots\dots\dots(32)$$

Equation (31) and (32) lead to

$$(\delta_{11} - \eta_{11}) + (\delta_1 - \eta_1) \left(\frac{1}{r}\right) = 0 \dots\dots\dots(33)$$

Which on integration yields

$$\eta_1 = \delta_1 - \frac{1}{2r} - \frac{b}{r} = \delta_1 - \frac{1}{2} \left(\frac{1}{2} + b\right) \dots\dots\dots(34)$$

Where b is a constant of integration.

From equation (30), and (34), we obtained

$$\left[\delta_1 - \frac{1}{2r} \right]^2 + \frac{3}{4r^2} + \frac{b}{r^2} - \frac{1}{2r^2} + \frac{a^2 e^{-4\delta}}{2} + \frac{KV_{,1}^2}{2} = 0 \dots\dots\dots(35)$$

By the substitution

$$\delta - \frac{1}{2r^2} \log_r = \alpha, \dots\dots\dots(36)$$

Equation (30) reduces to

$$\alpha_{11} + \frac{\alpha_1}{r} - \frac{a^2 e^{-4\alpha}}{2r^2} = 0 \dots\dots\dots(37)$$

Which on integration gives

$$[r \alpha_1]^2 = C_1^2 - \frac{1}{4} \frac{a^2 e^{-4\alpha}}{2r^2}, \dots\dots\dots(38)$$

Let us now use the relation

$$r = \frac{dr}{d\rho} \dots\dots\dots(39)$$

Equation (38) then leads to

$$\frac{d\alpha}{\sqrt{C_1^2 - \frac{1}{4} a^2 e^{-4\alpha}}} = d\rho \dots\dots\dots(40)$$

Which on integration gives

$$e^{2\alpha} = \frac{a}{2c_1} \text{Cosh}[2c_1\rho - \beta] \dots\dots\dots(41)$$

Where β is a constant of integration. Equation (36) and (41) leads to

$$e^{4\delta} = \left[\frac{dr}{d\rho} \right]^2 \frac{a^2}{4C_1} \text{Cosh}^2(2C_1\rho - \beta)$$

$$\text{Or } e^{4\delta} = \frac{r^2 a^2}{44} \text{Cosh}^2(2C_1 \log r - \beta) \dots\dots\dots(42)$$

Equation (34) on integration gives

$$\eta = \alpha - \int \frac{b}{(r)} dr + \frac{1}{2} \log S$$

$$\text{Or } \eta = \alpha - b \log r + \frac{1}{2} \log S \dots\dots\dots(43)$$

Where S is constant of integration and α is given by equation (41)

From equation (29) and (41) we obtained ,

$$\frac{dX}{d\rho} = \frac{4C_1^2}{a} \text{sech}^2[2c_1\rho - \beta] \dots\dots\dots(44)$$

Which on integration gives

$$X = \frac{2C_1}{a} \tanh[2c_1\rho - \beta] + \omega \dots\dots\dots(45)$$

Where ω is a constant of integration. Hence the metric of the space time in given by

$$\frac{s}{\sqrt{C_1}} r^{-(2b+1)} (dt^2 - dr^2) - \left\{ \frac{2C_1}{a} \tanh(2C_1 \log r - \beta) + \omega \right\}^2 \left\{ \frac{ar}{2\sqrt{C_1}} \text{Cosh}(2C_1, \log r - \beta) \right\} + r^2 \frac{2\sqrt{4}}{a} \text{sech}(2 \log r - \beta) \Big] d\phi^2$$

$$\left\{ \frac{ar}{2\sqrt{C_1}} \text{Cosh}(2C_1 \log r - \beta) \right\} dz^2 - 2 \left[\frac{2C_1}{a} \tanh(2C_1 \log r - \beta) + \omega \right]$$

$$\frac{ar}{2\sqrt{C_1}} \text{Cosh}(2C_1 \log r - \beta) d\phi dz \dots\dots\dots(46)$$

SECTION-2

Here we will consider case (II) where $F_{14}=F_{23}=0$, F_{12} , F_{13} , F_{24} , $F_{34} \neq 0$

Case (II) is done in terms of two potentials θ_2 and θ_3 which give rise to all the non-vanishing components of F_{ij} . Here Q_i is electromagnetic potential. Hence

$$F_{ij} = \theta_{i,j} - \theta_{j,i} \dots \dots \dots (47)$$

Thus we have

$$F_{12} = -\phi_1, F_{13} = \Psi_1, F_{24} = -\phi_4, F_{34} = \Psi_4$$

Where

$$\theta_2 = \phi, \theta_3 = \Psi \text{ and}$$

$$\theta_1 = \phi, i\Psi_i = \Psi, i.$$

Here also we are taking the same live elements (1) but here η , X , δ are

Function of r and t both. Also equation (6) leads to

$$\nabla^2 V \equiv V_{,11} - V_{,44} + \left(\frac{V_{,1}}{r}\right) = 0$$

The field equation (2), (5) and (6) leads to

$$\eta_1 = r(\delta_1^2 + \delta_4^2) + \frac{e^{4\delta}}{4r} (X_1^2 + X_4^2) + \frac{1}{2} \left[r(V_1^2 + V_4^2) + \frac{C^{2\delta}}{r} (\phi_1^2 + \phi_4^2) + \left(\frac{X^2 e^{2\delta}}{r^2} + r e^{-2\delta}\right) (\Psi_1^2 + \Psi_4^2) - 2e^{2\delta} (\phi_1 \Psi_1 + \phi_4 \Psi_4) \right] \dots \dots \dots (48)$$

$$\eta_4 = \frac{X_1 X_4 e^{4\delta}}{2r} + 2r\delta_1 \delta_4 + K \left[r(V_{,1} V_{,4}) + \frac{e^{2\delta}}{r} (\phi_1 \phi_4) - \frac{X e^{2\delta}}{r} (\Psi_1 \phi_4) \right] \dots \dots \dots (49)$$

$$\left[(\delta_1^2 - \delta_4^2) - \frac{e^{4\delta}}{4r^2} (X_1^2 - X_4^2) + (\eta_{11} - \eta_{44}) \right] \\ = \frac{1K}{2} \left[(V_{,4}^2 - V_{,1}^2) + \frac{e^{2\delta}}{r^2} (\phi_1^2 - \phi_4^2) - \left(-\frac{X^2 e^{2\delta}}{r^2} + e^{-2\delta}\right) (\Psi_1^2 - \Psi_4^2) + \frac{2X}{r^2} e^{2\delta} (\phi_1 \Psi_4 + \phi_4 \Psi_1) \right] \dots \dots \dots (50)$$

$$\left[(X_{11} - X_{44}) + 4(\delta_1 X_1 - \delta_4 X_4) - \frac{X_1}{r} \right] \\ = 2K e^{-2\delta} \left[(\phi_4 \Psi_4 - \phi_1 \Psi_1) + X(\Psi_1^2 - \Psi_4^2) \right] \dots \dots \dots (51)$$

$$\left[(\delta_{11} - \delta_{44}) + \frac{e^{4\delta}}{2r^2} (X_4^2 - X_1^2) + \frac{X_1}{r} \right] \\ = \frac{K}{2} \left[\frac{e^{2\delta}}{r^2} (\phi_1^2 - \phi_4^2) + \left(\frac{X^2 e^{2\delta}}{r^2} - e^{-2\delta}\right) (\Psi_1^2 - \Psi_4^2) - \frac{2X e^{2\delta}}{r^2} (\phi_1 \Psi_1 - \phi_4 \Psi_4) \right] \dots \dots \dots (52)$$

$$\left[\eta_{11} - \eta_{44} + \delta_1^2 - \delta_4^2 - \frac{e^{4\delta}}{4r^2} (X_1^2 - X_4^2) \right]$$

$$= \frac{k}{2} \left[(V_4^2 - V_1^2) + \frac{e^{2\delta}}{r^2} (\phi_1^2 - \phi_4^2) + \left(-\frac{X^2 e^{2\delta}}{r^2} - e^{-2\delta} \right) (\Psi_1^2 - \Psi_4^2) + \frac{2X e^{2\delta}}{r^2} (\phi_4 \Psi_4 - \phi_1 \Psi_1) \right] \dots \dots \dots (53)$$

$$(X_4 \Psi_4 - X_1 \Psi_1) + X(\Psi_{44} - \Psi_{11}) - (\phi_{44} - \phi_{11}) + 2X(\Psi_4 \delta_4 - \Psi_1 \delta_1) \frac{1}{r} (X \Psi_1 - \phi_1) = 0 \dots \dots \dots (54)$$

$$[X^2(\Psi_{11} - \Psi_{44}) + 2C(X_1 \Psi_1 - X_4 \Psi_4) - X(\phi_{11} - \phi_{44}) - (\phi_1 X_1 - \phi_4 X_4) + 2\{(X^2 \Psi_1 - X \phi_1) \delta_1 + (-X^2 \Psi_4 + X \phi_4) \delta_4\} - \frac{1}{r} (X^2 \Psi_1 - X \phi_1) + r 2 e^{-4\delta} (\Psi_{11} - \Psi_{44}) - 2r^2 e^{-4\delta} (\Psi_1 \delta_1 - \Psi_4 \delta_4) + r e^{-4\delta} \Psi_1 = 0 \dots \dots \dots (55)$$

$$V_{11} - V_{44} + \frac{V_1}{r} = 0 \dots \dots \dots (56)$$

Equation (48), (49) and (53) enable us to define

$$\eta = \eta^v + \eta^r$$

where η^v , corresponding to EM field equation, can be evaluated by integration. once a solution for δ , X , ϕ and Ψ

are known and η^v depends only on the scalar fields V as

$$(\eta^v)_1 = \frac{1}{2} k r (V_1^2 + V_4^2) \dots \dots \dots (57)$$

$$(\eta^v)_4 = k r V_1 V_4 \dots \dots \dots (58)$$

Which again can be evaluated by integration once a solution of equation (5.2.52) is specified. Thus we may state the following theorem (Singh et.al [17] whose application gives a solution of the EMS field equation for every solution of the EM equation.

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Conclusion

The title “SOME STUDY IN GENERAL THEORY OF RELATIVITY ABOUT CYLINDRICALLY SYMMETRIC MODEL IN PRESENCE OF ELECTROMAGNETIC”, explains the analytical solution of Einstein’s field equation for static anisotropic fluid sphere by assuming that space time is conformally flat with a choice of energy density.

This model is physically reasonable and free from Singularity hence various physical parameters can be calculated. The solution of Einstein-Maxwell field equation for Static conformally flat charged perfect fluid sphere by using a suitable form of mass density.

The result gives uniform charge density and uniform mass density distribution also. Various physical parameters can be calculated by using different boundary conditions. In spherical symmetric metric, we have solved Einstein- Maxwell field equation by taking a suitable form of matter density and charge density hence various parameters can also be calculated by putting various conditions. Solutions of Electromagnetic equation and scalar field for cylindrically symmetric metric (Sachder metric) in two different cases (i) directly solved in terms of F_{ij} components (ii) in terms of two potentials θ_2 and θ_3 .

