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DATA ENVELOPMENT ANALYSIS & ITS APPLICATIONS:AN OVERVIEW

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Abstract

Data Envelopment Analysis (DEA) is a non-parametric method of measuring the efficiency of a decision–making unit (DMU) such as a firm or a public-sector agency. In contrast to the statistical approach, DEA evaluates DMUs relative to an average DMU. It compares each unit with only the best performing one. In DEA, there are a number of DMUs. These make decisions based on the available set of inputs which produce a set of outputs. Each unit has a varying level of inputs and gives a varying level of outputs. This technique attempts to determine which of the units are most efficient, and to point out specific inefficiencies of the other units thus assisting them to make realistic improvement targets and allocate resources more efficiently and effectively. Farrel [1957] suggested the concept of Frontier Analysis, which forms the basis of DEA. Four basic DEA models can be used to calculate the DEA efficiency score for the decision making units. The optimal objective function values of models, when solved, represent the efficiency score of the DMU. DEA has been used to evaluate efficiency of financial institutions such as banks, manufacturing companies, service organizations, pharmaceutical industries. DEA has also been used to evaluate efficiency of mutual funds, impact of strategies and policies adopted by governments, and for evaluating value of brand names. In the following article we review some of the prominent literatures in the field of DEA. Recent researches in the given field entails the work carried out by Saranga (2009) and Ramanathan (2009). Saranga (2009) carried out the performance analysis of the Indian auto component industry from the perspectives of an original equipment manufacturer and a component supplier. Ramanathan (2009) proposed (DEA) to generate local weights of alternatives from pair-wise comparison judgment matrices used in the analytic hierarchy process (AHP).

Keywords: *Data Envelopment Analysis, Decision Making Units, Constant Return to Scale, Variable Return to Scale, Operational Inefficiencies*

Introduction

DEA is an extreme point method comparing each DMU with only the best DMU.A fundamental assumption behind an extreme point method is that if a given DMU A , is capable of producing Y(A) units of outputs with X(A) inputs, then other DMUs should also be able to do the same if they were to operate efficiently. Similarly, if DMU B is capable of producing Y(B) units of outputs with X(B) inputs, then other DMUs should also be capable of the same production schedule. DMUs A, B and others can then be combined to form a composite DMU with composite inputs and composite outputs. Since this composite DMU does not necessarily exist, it is sometime virtual DMU.

The heart of the analysis lies in finding the best virtual DMU for each real DMU. If the virtual DMU is better than the original DMU by either making more output with same input or making the same output with lesser input then the original DMU is inefficient. Some of the subtleties of DEA are introduced in the various ways that DMUs A and B can be scaled up or down and combined.

Charnes, Cooper and Rhodes[1978] described Data envelopment analysis (DEA) as a linear programming approach for measuring the relative efficiency of peer decision making units (DMUs) that have similar set of units which is based upon Farrell's pioneering work. They generalized the single-output to single-input ratio definition of efficiency to multiple inputs and outputs.

More generally, DEA is a methodology directed to frontiers rather than central tendencies. Instead of trying to fit a regression plane through the ***center*** of the data, one 'floats' a piecewise linear surface to rest on cop of the observations. Because of this unique perspective, DEA proves particularly adapt at uncovering relationships that remain hidden for other methodologies.

In their original DEA model, Charnes, Cooper and Rhodes (CCR model) proposed that the efficiency of a DMU can be obtained as the maximum of a ratio of weighted outputs to weighted inputs, subject to the condition that the same ratio for all DMUs must be less than or equal to one. The DEA model must be run n times, once for each unit, to get the relative efficiency of all DMUs. The CCR model evaluates both technical and scale efficiencies via the optimal value of the ratio form. The envelopment in CCR is constant returns to scale meaning that a proportional increase in inputs results in a proportionate increase in outputs.

DEA has gained too much attention by researchers because of its successful applications and case studies. Assessment of bank branch performance [1], examining bank efficiency [18], analyzing firm's financial statements [10], measuring the efficiency of higher education institutions [9], solving facility layout design (FLD) problem [17] and measuring the efficiency of organizational investments in information technology [15] are examples of using DEA in various areas.

In the present paper we discuss the mathematical programming approach to efficient frontier estimation known as DEA. Section 2 of this paper gives a historical perspective on the origins of DEA. Section 3 provides a description of the original CCR ratio model of Charnes, Cooper, and Rhodes (1978) and contrasts the CCR ratio model with more recent DEA models proposed. Section 4 carried out the performance analysis of the Indian auto component industry from the perspectives of an original equipment manufacturer and a component supplier by Haritha Saranga [2009]. Section 5 explains (DEA) to generate local weights of alternatives from pair-wise comparison judgment matrices used in the analytic hierarchy process (AHP) proposed by Ramanathan [2009].

2.. **Background**

As Farrell (1957) originally argued, information concerning the frontier and the relative efficiency of DMUs has many policy applications. He also argued that while attempts to solve the problem usually produced careful measurements, they failed to combine the measurements of the multiple inputs into any satisfactory overall measure of efficiency. Responding to these inadequacies of separate indices of labor productivity, capital productivity, etc., Farrell proposed an activity analysis approach that could more adequately deal with the problem. His measures were intended to be applicable to any productive organization; in his words, ' . . . from a workshop to a whole economy'.

Our focus in this paper is on nonparametric linear programming models for measuring the efficiency of a DMU *relative* to similar DMUs and thus estimating a 'best practice' frontier. The initial DEA model was originally presented in Charnes, Cooper, and Rhodes (CCR) (1978), and built on the earlier work of FarreIl (1957). They generalized the single-output to single-input ratio definition of efficiency to multiple inputs and outputs. Researchers in a number of fields have quickly now recognized that DEA is an excellent methodology for modeling operational processes. After that, Banker et al. [1984] developed the BCC model to estimate the pure technical efficiency of decision making units with reference to the efficient frontier. Thus we can say that BCC model is a specific form of CCR.

At present, DEA actually encompasses a variety of alternate (but related) approaches to evaluating performance. Extensions to the original CCR work have resulted in a deeper analysis of both the multiplier side and the envelopment side of the mathematical duality structure.

3. DEA Models

The present paper focuses attention on four DEA models, the CCR ratio model [Charnes, Cooper, and Rhodes (1978)], the BCC model [Banker, Charnes, and Cooper (1984)], Additve model[1985] & Slack based Model (SBM)[1997, 2001].

3.1 Charnes, Cooper & Rhodes [1978] (CCR Model)

Consider a set of n DMUs, with each DMU_j, (j = 1,. . .,n) using m inputs $x_{ij}(i = 1, 2, \dots . m)$ and generating s outputs $y_{rj}(r = 1, 2, \dots . s)$. If the multipliers $\bar{u}_r, \bar{v}_i$ associated with outputs r and inputs i, respectively, are known, then one could express the efficiency of DMU_j as the ratio of weighted outputs to weighted inputs, i.e.

$$\sum_r \bar{u}_r y_{rj} / \sum_i \bar{v}_i x_{ij}$$

This ratio is, of course, the basis for the standard engineering ratio of productivity. In the absence of known multipliers, Charnes et al. (1978) proposed deriving appropriate multipliers for a given DMU by solving a particular non-linear programming problem. Specifically, if DMU_o is under consideration, the Charnes et al model. for measuring the technical efficiency of that DMU is given by the solution to the fractional programming problem (FPP):

$$\max_{v,u} \frac{u_1 y_{1o} + u_2 y_{2o} + \cdots + u_s y_{so}}{v_1 x_{1o} + v_2 x_{2o} + \cdots + v_m x_{mo}}$$

Subject to (2.1)

$$\frac{u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj}} \leq 1 \ (j = 1, 2, \dots \dots, n)$$

$$v_1, v_2, \dots . . v_m \geq 0$$

$$u_1, u_2, \dots . . u_s \geq 0$$

We point out that this model involving the ratio of outputs to inputs is referred to as the input-oriented model. One could, as well, invert this ratio and solve the corresponding output-oriented minimization problem. We will generally deal with the input oriented model herein. Problem (2.1) is referred to as the CCR (Charnes, Cooper and Rhodes) model, and provides for constant returns to scale (CRS).

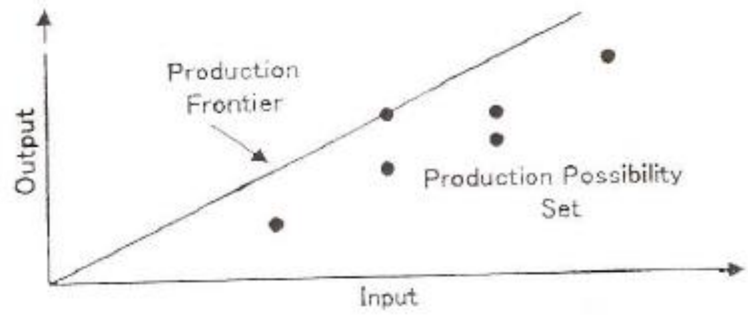

Figure 1 CCR model

Applying the Charnes and Cooper (1962) theory of fractional programming, we get

(LP) $max_{\mu,v}\ uy_o$

Subject to (2.2)

$vx_o = 1$

$-vX + uY \leq 0$

$v \geq 0, u \geq 0$

The dual of above fractional programming problem is expressed with a real variable θ & a non- negative vector $(\lambda_1, \lambda_2, \ldots \lambda_n)^t$ of variables as follows (Envelopment Form):

(DLP_o) $min_{\theta,\lambda}\ \theta$

Subject to (2.3)

$\theta x_o - X\lambda \geq 0$

$Y\lambda \geq y_o$

$\lambda \geq 0$

Problem (2.3) is referred to as the envelopment or dual problem, and (2.2) the multiplier or primal problem. (DLP_o) has a feasible solution $\theta = 1, \lambda_o = 1, \lambda_j = 0\ (j \neq 0)$. Hence the optimal θ, denoted by θ^*, is not greater than 1.

The constraint space of (2.3) defines the production possibility set P. That is

P = $\{(x, y) | x \geq X\lambda, y \leq Y\lambda, \lambda \geq 0\}$

Where λ is a semi-positive vector in R^n

A DMU_o is CCR – Efficient if $\boldsymbol{\theta}^*$= 1 and there exists at least one optimal $(\boldsymbol{v}^*, \boldsymbol{u}^*)$, with $\boldsymbol{v}^* > 0$ and $\boldsymbol{u}^* > \mathbf{0}$. Otherwise, $\boldsymbol{DMU_o}$ is CCR- Inefficient.

We define the *input excesses* $s^- \epsilon R^m$ and the *output shortfalls* $s^+ \epsilon R^s$ and identify them as "slack" vectors by:

$$s^- = \theta x_o - X\lambda, \quad s^+ = Y\lambda - y_o$$

with $s^- \geq 0, s^+ \geq 0$ for any feasible solution (θ, λ) of (DLP_o)

To discover the possible input excesses and output shortfalls, we solve the following two-phase LP problem.

Phase I

We solve (DLP_o). Let the optimal objective value be θ^*. By the duality theorem of Linear Programming, θ^* is equal to the optimal objective value of (LP_o) and is the CCR-efficiency value, also called "Farrell Efficiency", after M.J.Farrell (1957). See below. This value of θ^* is incorporated in the following Phase II extension of (DLP_o).

Phase II

Using our knowledge of θ^*, we solve the following LP using $(\boldsymbol{\lambda}, s^-, s^+)$ as variables:

$$\max_{\lambda, s^-, s^+} \quad w = es^- + es^+$$

Subject

to

$$s^- = \theta^* x_o - X\lambda$$

$$s^+ = Y\lambda - y_o \qquad \lambda \geq 0, s^- \geq 0, s^+ \geq 0$$

where $e = (1, 1, \dots \dots \dots 1)$ (a vector of ones) so that $es^- = \sum_{i=1}^{m} s_i^-$ *and* $es^+ = \sum_{r=1}^{s} s_r^+$.

The objective of Phase II is to find a solution that maximizes the sum of input excesses and output shortfalls while keeping $\theta = \theta^*$.

3.2 Banker Charnes Cooper (BCC Model) [1984]

Banker et al. (1984) (BCC), extended the earlier work of Charnes et al. (1978) by providing for variable returns to scale (VRS). This is pictured in the redrawn version of Fig. 1 in the form of Fig. 2.

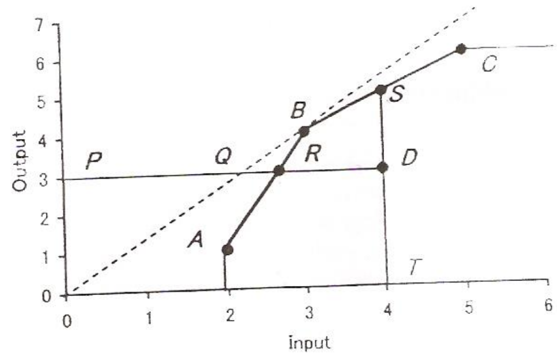

Figure 2

Shown are the original CRS frontier, and the VRS frontier, here represented by the line segments. The BCC ratio model differs from (2.3), by way of an additional variable, i.e.

$$\text{(BCCo)} \quad \min_{\theta_B,\lambda} \quad \theta_B$$

$$\text{Subject to} \quad \theta_B x_o - X\lambda \geq 0 \qquad (2.4)$$

$$Y\lambda \geq y_o$$

$$e\lambda = 1 \qquad \lambda \geq 0$$

where θ_B is a scalar.

The dual multiplier form of this linear program (BCC_o) is expressed as:

$$\max_{v,u,u_o} z = uy_o - u_o$$

$$\text{Subject to} \quad vx_o = 1 \qquad (2.5)$$

$$-vX + uY - u_o e \leq 0 \qquad v \geq 0, u \geq 0, u_o \; free \; in \; sign,$$

where v *and* u are vectors and z *and* u_o are scalars and the latter, being "free in sign", may be positive or negative (or zero). The equivalent BCC fractional program is obtained from the dual program as:

$$\max \quad \frac{uy_o - u_o}{vx_o} \quad \text{Subject to} \quad \frac{uy_j - u_o}{vx_j} \leq 1 \; (j = 1, 2, \ldots \ldots, n) \qquad (2.6)$$

$$v \geq 0, u \geq 0, u_o \; free.$$

It is noted that (2.6) differs from (2.3) in that it has the additional convexity constraint on the λ_j, namely $\sum_j \lambda_j = 1$.

As with the CRS model, If an optimal solution $(\theta_B^*, \lambda^*, s^{-*}, s^{+*})$ obtained for (BCC_o) satisfies $\theta_B^* = 1$ and has no slack $(s^{-*} = 0, s^{+*} = 0)$, then the $DMU_o$is called BCC-Efficient, otherwise it is BCC-Inefficient.

The primal problem (BCC_o) is solved using a two-phase procedure similar to the CCR case. In the first phase, we minimize θ_B and, in the second phase, we maximize the sum of the input excesses and output shortfalls, keeping $\theta_B = \theta_B^*$ (the optimal objective value obtained in Phase one). The evaluations secured from the CCR and BCC models are also related to each other as follows. An optimal solution for (BCC_o) is represented by $(\theta_B^*, \lambda^*, s^{-*}, s^{+*})$, where $s^{-*} and \; s^{+*}$ represent the maximal input excesses and output shortfalls, respectively. Notice that θ_B^* is not less than the optimal objective value θ^* of the CCR model, since (BCC_o) imposes one additional constraint, $e\lambda = 1$, so its feasible region is a subset of feasible region for the CCR model.

3.3 Additive Model (Charnes et al. (1985)

Charnes et al.(1985) introduced the additive or Pareto–Koopmans (PK) model which, to an extent, combines both orientations. The previous two efficiency models are radial projection constructs. Specifically, in the input-oriented case, inputs are proportionally reduced while outputs remain fixed. (For the output-oriented case, outputs are proportionally increased while inputs are held constant).

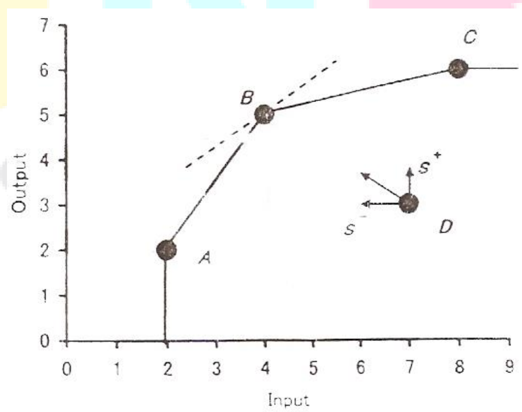

Figure 3

To explain this model we use Figure 3, where four DMUs A, B, C and D, each with one input and one output, are depicted. Since,the model (ADD_o) has the same production possibility set as the BCC model, the efficient frontier, which is continuous, consists of the line segments $\overline{AB}$ and $\overline{BC}$. Now consider how DMU D might be evaluated. A feasible replacement of D with s^- *and* s^+ is denoted by the arrows s^- *and* s^+ in the figure. As shown by the dotted line in the figure, the maximal value of $s^- + s^+$ is attained at B. It is clear that this model considers the input excess and the output shortfall simultaneously in arriving at a point on the efficient frontier which is most distant from D.

There are several versions of the additive model, the most basic being given by the linear optimization problem shown as (2.7).

(ADDo) $\max_{\lambda,s^-,s^+} z = es^- + es^+$

Subject to $X\lambda + s^- = x_o$ (2.7)

$Y\lambda - s^+ = y_o$

$e\lambda = 1$

$\lambda \geq 0, s^- \geq 0, s^+ \geq 0$

The convexity condition on the λ_j variables implies that we are using the VRS technology. The frontier generated by model (2.7) is identical to that arising from the corresponding VRS structure (2.6), hence a DMU is additive-efficient or PK efficient (all slacks equal to zero at the optimum in (2.7)) if and only if it is VRS-efficient.

Since the various inputs and outputs may be measured in non-commensurate units, (Russell, 1988), it may not be practical in certain contexts to use the simple sum of

slacks as the objective in (2.7). Moreover, model (2.7) does not provide for an actual measure of inefficiency as in the case for the BCC and CCR models. To overcome this latter problem, Charnes et al. (1985b) proposed the dual of the above problem as

$\min_{v,u,u_o} w = vx_o - uy_o + u_o$ Subject to

$vX - uY + u_o e \geq 0$ (2.8)

$v \geq e$

$u \geq e$

u_o *free.*

3.4 Slack Based Model (Introduced by Tone [1997, 2001]

We now augment the Additive models by introducing a measure that makes its efficiency evaluation, as effected in the objective, invariant to the units of measure used for the different inputs and outputs. That is, we would like this summary measure to assume the form of a scalar that yields the same efficiency value when distances are measured in either kilometers or miles. More generally, we want this measure to be the same when x_{io} and x_{ij} are replaced by $k_i x_{io} = \hat{x}_{io}, k_i x_{ij} = \hat{x}_{ij}$ and y_{ro} and y_{rj} are replaced by $c_r y_{ro} = \hat{y}_{ro}, c_r y_{rj} = \hat{y}_{rj}$ where the k_i and c_r are arbitrary positive constraints, $i = 1, \ldots.., m$; $r = 1, \ldots \ldots., s$.

This property is known by names such as "dimension free" and "units invariant". Now, we will introduce such a measure for Additive models in the form of a single scalar called "SBM", (Slacks-Based Measure) which was introduced by Tone (1997, 2001) and has the following important properties:

1. (P1) The measure is invariant with respect to the unit of measurement of each input and output item. (Units Invariant)

2. (P2) The measure is monotone decreasing in each input and output slack. (Monotone)

In order to estimate the efficiency of a DMU (x_o, y_o), we formulate the following fractional program in λ, s^- and s^+

(SBM) $$\min_{\lambda, s^-, s^+} \quad \rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{io}}{1 + \frac{1}{s}\sum_{r=1}^{s} s_r^+ / y_{ro}}$$ Subject to $x_o = X\lambda + s^-$ (2.9)

$$y_o = Y\lambda - s^+$$

$$\lambda \geq 0, s^- \geq 0, s^+ \geq 0$$

In this model, we assume that $X \geq 0$. If $x_{io} = 0$, then we delete the term s_i^- / x_{io} in the objective function. If $y_{io} \leq 0$, then we replace it by a very small positive number so that the term s_r^+ / y_{ro} plays a role of penalty.

Furthermore, we have

$$0 \leq \rho \leq 1$$

Solving SBM

(SBM) as formulated in (2.9) can be transformed into the program below by introducing a positive scalar variable t.

(SBM t) $$\min_{t, \lambda, s^-, s^+} \quad \tau = t - \frac{1}{m}\sum_{i=1}^{m} t s_i^- / x_{io}$$

$$\text{Subject to} \quad 1 = t + \frac{1}{s}\sum_{r=1}^{s} ts_r^+/y_{ro} \qquad (2.10)$$

$$x_o = X\lambda + s^-$$

$$y_o = Y\lambda - s^+$$

$$\lambda \geq 0, s^- \geq 0, s^+ \geq 0, t > 0$$

Now let us define

$$S^- = ts^-, S^+ = ts^+, and\ \Lambda = t\lambda$$

Then (SBM t) becomes the following linear program in $t, S^-, S^+, and\ \Lambda$:

$$\text{(LP)} \qquad \min \quad \tau = t - \frac{1}{m}\sum_{i=1}^{m} S_i^-/x_{io}$$

$$\text{Subject to} \quad 1 = t + \frac{1}{s}\sum_{r=1}^{s} S_r^+/y_{ro} \qquad (2.11)$$

$$tx_o = X\Lambda + S^-$$

$$ty_o = Y\Lambda - S^+$$

$$□ \geq 0, □^- \geq 0, □^+ \geq 0, □ \geq 0$$

Note that the choice $□ > 0$ means that the transformation is reversible. Thus, let an optimal solution of (LP) be $(□^*, □^*, □^*, □^{-*}, □^{+*})$.

We then have an optimal solution of (SBM) defined by,

$$□^* = □^*, □^* = □^*/□^*, □^{-*} = □^{-*}/□^*, □^{+*} = □^{+*}/□^*$$

4. Application of DEA in Evaluation of work Safety Supervision [2010].

Qu et. Al.[32] introduced the DEA method to address two major shortcomings of conventional approaches of evaluation of the work safety supervision which are : the difficulty in assigning weights criteria and ignoring the different risky levels of districts.

The proposed DEA models with weight constraints is applied to evaluate the work safety supervision in 18 districts. First, AHP method is applied to combine 23 sub-indices to 6 indices. Several weight constraints are used to ensure the efficient DMUs have balanced outputs. The lower bound of each weight represents the importance grade of each output given by decision makers. The redundancy rates and the output insufficiency rates are calculated to help decision makers to find the fields which need improvement. This new approach is used to evaluate the 18 districts in

12 months of 2009.Compared with the conventional AHP based approach , this AHP+DEA approach can make more objective evaluation, which is confirmed by experts.

5. Application of DEA to Indian Cement Industry[2009]

This paper [31] makes an attempt to estimate energy use efficiency of the Indian cement industry at the state level for the period 2000-01 to 2005-05 , using Data Envelopment Analysis (DEA). Since, cement industry is a major producer of environmentally detrimental carbon dioxide gas as an undesirable by-product, a special emphasis is given to that undesirable output while evaluating energy use efficiency. The major focus of the study has been to answer two empirical questions. First, whether exclusion of undesirable output from the analysis results in biased estimates of energy use efficiency. Secondly, whether environmental regulation has any reinforcing impact on energy use efficiency or not.To answer the first question, we have estimated energy use efficiency considering both desira ble and undesirable output in the first case and only desirable output in the other to examine whether omitting undesirable output results in biased estimates of energy use efficiency. To answer the second question we have assumed both weak and strong disposability with respect to disposal of undesirable output to examine whether environmental regulation aimed at reducing energy related emissions is able to bring about further improvement in energy use efficiency also. Empirical results demonstrate that energy efficiency measures are biased if only desirable output is considered, implying that undesirable output indeed matters while evaluating energy use efficiency Moreover, average energy use efficiency is higher in presence environmental regulation than that obtained in absence of it. Therefore, we conclude by claiming that environmental regulation has the potential in terms of positively impacting energy use efficiency in addition to reducing higher pollution levels, implying that if we formulate our model correctly with introduction of environmental regulation it will result in higher efficiency scores. Higher energy use efficiency in presence environmental regulation suggests that the government can introduce environmental regulation in the form of inst itutional instruments such as pollution taxes which would induce the firms to internalize the external costs (including environmental) of energy consumption.

Conclusion

This paper has attempted to provide a brief sketch of some of the important areas of research in DEA that have emerged over the past three decades. Data envelopment analysis (DEA) has been proposed in this paper for deriving weights from the judgment matrices of the analytic hierarchy process (AHP). If the decision maker is not able, categorically, to decide whether one alternative is better than another, he will not be in a position to think that one is more important than the other, and this is the logic employed by DEA for calculating the weights. It has been proved that DEA calculates true weights for consistent judgment matrices. DEA is further used to aggregate local weights of alternatives in terms of different criteria in AHP to final weights. DEA also explains its applications in various fields.

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