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ARTICLE 6 ABOUT THEORY OF DISTRIBUTION OF PRIME NUMBERS

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ABSTRACT

Let

G = { x | x∈N and 6x-1, 6x+1, 6x+5 & 6x+7 are all prime numbers}

In other word,

$G = \{x \mid x \in (A \cup B)^c \text{ and } x+1 \in (A \cup B)^c\}$

Where A and B are two subsets of set of natural numbers as defined in my second article IJNRD 2304175. Set of natural numbers as universal set.

i.e

G = {17, 32, 137, 247, 312, 347, 542, 577, 942, 1572, ………………………}

In this article, Theorem states that,

n([6b+1, (b-1)(6b+1)-1]∩G)

$\approx (b-2)(6b+1)(1/5)(3/7)(7/11)(9/13)\ldots\ldots\ldots\ldots((P_r-4)/P_r)$.

Where P_r is greatest prime number such that $P_r \le 6(b-1)+1$.

Infiniteness of pairs of two consecutive pairs of twin prime numbers in the form 6k-1 & 6k+1 and 6k+5 & 6k+7 is the immediate consequence of the above theorem.

INTRODUCTION

Numbers are wonderful, marvelous creature of human. Numbers are classified into many types. They are Natural numbers, Whole numbers, Integers, Real numbers, Complex numbers, Rational numbers, Irrational numbers.

Natural numbers are classified into two categories

1) Prime numbers,
2) Composite numbers.

Prime numbers are Natural numbers which cannot be expressed in the form of product of two Natural numbers both greater than 1.

Composite numbers are other than Prime numbers. i.e. Which can be expressed in the form of product of two Natural numbers both greater than 1

From the definition of Prime number, 2 and 3 are Prime numbers, but 2x3=6 is a composite number. Multiples 6 are also composite numbers. Numbers in the form 6k±2 are even numbers i.e multiples of 2, hence composite numbers, Numbers in the form 6k±3 are odd multiples of 3 hence composite numbers.

Therefore, Prime numbers except 2 and 3 are in the form of 6k±1. where k is any natural number, but not for all natural numbers. For some Natural number k, 6k+1 is a Prime number but 6k-1 is a Composite number. For some Natural number k, 6k-1 is a Prime number but 6k+1 is a Composite numbers. For some Natural number k, 6k+1 and 6k-1 both are Prime numbers (twin Prime numbers). For some Natural number k, 6k+1 and 6k-1 both are Composite numbers. Hence this k is the key factor that determines Prime numbers and Composite numbers.

Before 2500 years ago Euclid proved that Prime numbers are infinite, Composite numbers generated by their prime factors, but <u>Prime numbers are not generated. They are distributed among the gaps left by Composite numbers</u>. This article is about Theory of distribution of Prime numbers. <u>Distributive rule of Prime numbers is nothing but violation of generating rule of Composite numbers</u>. And since all Prime numbers except 2 and 3 are in the form 6k±1, this k determines the numbers in the form 6k±1.

In my first two article, I have written about this k. Especially In my second article, IJNRD 2304175 I have defined two sets A and B, which are subsets of natural numbers. I want to make remember.

$$A = \bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\infty} I_n \ \cup \bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\infty} I_{-n}$$

$$B = \left(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{\infty} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{\infty} J_{-n}\right)$$

Where

$$I_n = \{ x / x \in N,\ x \in [n]_{6n+1} \ \&\ x > n \}$$

$$I_{-n} = \{ x / x \in N,\ x \in [-n]_{6n-1} \ \&\ x > n \}$$

$$J_n = \{ x / x \in N,\ x \in [n]_{6n-1} \ \&\ x > n \}$$

$$J_{-n} = \{ x / x \in N,\ x \in [-n]_{6n+1} \ \&\ x > n \}$$

It is obvious that two sets A and B are infinite. A and B are union of residue classes of infinite number of prime moduli. so It is very difficult to study the nature of sets A and B. to overcome this drawback, I define this closed interval [1, (b-1)(6b+1)]. the natural numbers of A and B that are contained in this closed interval are union of residue classes of finite number of prime moduli. Therefore, In my third article, IJNRD 2304384 I have proved the following facts.

For any natural number b

i) $[1, (b-1)(6b+1)] \cap A =$

$$[1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} I_n\right) \cup \left(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} I_{-n}\right) \right)$$

ii) $[1, (b-1)(6b+1)] \cap B$

$$= [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} J_n\right) \cup \left(\bigcup_{\substack{n=1\ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} J_{-n}\right) \right)$$

iii) $[1, (b-1)(6b+1)] \cap (A \cup B)$

$$= [1, (b-1)(6b+1)] \cap \left(\left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)\right) \right)$$

Let us continue from here to develop new concept about theory of distribution of prime numbers. For that we need some probability ideas, which is essential in proving theorem.

We know that,

Probability = Number of favourable outcomes ÷ Total Number of outcomes.

Let us select a pair of two consecutive natural numbers randomly in the set of natural numbers N. we know that N is infinite. Let our favour of selection is that the selected pair of two consecutive natural numbers contained in the complement of union of residue class [n] of modulo 6n+1 and residue class [-n] of modulo 6n+1. i.e our favour of selection is that the selected pair of two consecutive natural numbers belongs to $([n]_{6n+1} \cup [-n]_{6n+1})^c$. i.e selected pair of two consecutive natural number does not belong to the following four sets $[n-1]_{6n+1} \cup [n]_{6n+1}$, $[n]_{6n+1} \cup [n+1]_{6n+1}$, $[-n-1]_{6n+1} \cup [-n]_{6n+1}$ & $[-n]_{6n+1} \cup [-n+1]_{6n+1}$. i.e our unfavourable sets are $[n-1]_{6n+1} \cup [n]_{6n+1}$, $[n]_{6n+1} \cup [n+1]_{6n+1}$, $[-n-1]_{6n+1} \cup [-n]_{6n+1}$, & $[-n]_{6n+1} \cup [-n+1]_{6n+1}$. What is the probability of our favourable event? Answer follows.

The set of natural numbers is union of 6n+1 disjoint residue classes of modulo 6n+1. Hence each one of selected two natural numbers cannot belong to more than one residue class of modulo 6n+1. And also, the selected two natural numbers are a pair of two consecutive natural numbers. Hence both cannot belong to a same residue class of modulo 6n+1. It is obvious that the selected two consecutive natural numbers belong to some $[i]_{6n+1} \cup [i+1]_{6n+1}$, where $0 \le i \le 6n$. And each residue class is an infinite set of natural number. Therefore, for each $0 \le i \le 6n$, $[i]_{6n+1} \cup [i+1]_{6n+1}$ is an infinite set. Hence, for each $0 \le i \le 6n$, $[i]_{6n+1} \cup [i+1]_{6n+1}$ has an equal chance in selection. Therefore, the selected two consecutive natural number should belong to exactly any one of following 6n+1 sets, $[0]_{6n+1} \cup [1]_{6n+1}$, $[1]_{6n+1} \cup [2]_{6n+1}$, $[2]_{6n+1} \cup [3]_{6n+1}$, ... $[n-1]_{6n+1} \cup [n]_{6n+1}$, $[n]_{6n+1} \cup [n+1]_{6n+1}$, ... $[-n-1]_{6n+1} \cup [-n]_{6n+1}$, $[-n]_{6n+1} \cup [-n+1]_{6n+1}$, ... $[6n-2]_{6n+1} \cup [6n-1]_{6n+1}$, $[6n-1]_{6n+1} \cup [6n]_{6n+1}$, $[6n]_{6n+1} \cup [0]_{6n+1}$. Therefore, Total Number of outcomes are 6n+1. But our favour of selection is that the selected pair of two consecutive natural numbers should not belong to $[n-1]_{6n+1} \cup [n]_{6n+1}$, $[n]_{6n+1} \cup [n+1]_{6n+1}$, $[-n-1]_{6n+1} \cup [-n]_{6n+1}$, & $[-n]_{6n+1} \cup [-n+1]_{6n+1}$. i.e our favour of selection is other than the above 4 sets. Hence number of favourable outcome is 6n+1-4 = 6n-3.

Probability of our favourable event = (6n-3)/(6n+1).

Next, let us select a pair of two consecutive natural numbers randomly in the closed interval [x, y]. where x and y are natural numbers, natural number n < x, and 6n+1 <

length of [x, y] = y-x+1 = number of natural numbers in [x, y]. it is obvious that the number of natural numbers in [x, y] is finite. Let our favour of selection is that the selected pair of two consecutive natural numbers belongs to $([n]_{6n+1}\cup[-n]_{6n+1})^c$. i.e selected pair of two consecutive natural numbers does not belong to the following four sets $[n-1]_{6n+1}\cup[n]_{6n+1}$, $[n]_{6n+1}\cup[n+1]_{6n+1}$, $[-n-1]_{6n+1}\cup[-n]_{6n+1}$ & $[-n]_{6n+1}\cup[-n+1]_{6n+1}$. i.e the above four sets are unfavourable sets. What is the probability of our favourable event? Answer follows.

Let the Total Number of natural numbers in [x, y] is p(6n+1)+q+1. i.e y-x+1 = p(6n+1)+q+1. where p is a natural number, q is a whole number such that $0 \leq q < 6n+1$. [x, y] is a closed interval implies that p(6n+1)+q+1 natural numbers contained in [x, y] are consecutive. Hence total number of pairs of two consecutive natural numbers in [x, y] is p(6n+1)+q. If q = 0, then the Total number of pairs of two consecutive natural numbers in [x, y] is p(6n+1) a multiple of 6n+1. Hence the p(6n+1) pairs of two consecutive natural number in [x, y] are equally distributed among $[x]_{6n+1}\cup[x+1]_{6n+1}$, $[x+1]_{6n+1}\cup[x+2]_{6n+1}$, $[x+2]_{6n+1}\cup[x+3]_{6n+1}$, ………………………, $[x+6n-1]_{6n+1}\cup[x+6n]_{6n+1}$, $[x+6n]_{6n+1}\cup[x]_{6n+1}$. Hence, for each $0 \leq i \leq 6n$, $[x+i]_{6n+1}\cup[x+i+1]_{6n+1}$ has an equal chance in selection. Therefore, as in above, Total number of outcomes = 6n+1. And Number of favourable outcomes = 6n-3.

probability of our favourable event = 6n-3/6n+1.

If $q \neq 0$, Then the p(6n+1)+q pairs of two consecutive natural numbers in [x, y] are not equally distributed among $[x]_{6n+1}\cup[x+1]_{6n+1}$, $[x+1]_{6n+1}\cup[x+2]_{6n+1}$, $[x+2]_{6n+1}\cup[x+3]_{6n+1}$, ………, $[x+6n-1]_{6n+1}\cup[x+6n]_{6n+1}$, $[x+6n]_{6n+1}\cup[x]_{6n+1}$. since each of the following q sets $[x]_{6n+1}\cup[x+1]_{6n+1}$, $[x+1]_{6n+1}\cup[x+2]_{6n+1}$, ………… , $[x+q-1]_{6n+1}\cup[x+q]_{6n+1}$ contains exactly p+1 pairs of two consecutive natural numbers which are in [x, y]. And each of the other 6n+1-q sets $[x+q]_{6n+1}\cup[x+q+1]_{6n+1}$, $[x+q+1]_{6n+1}\cup[x+q+2]_{6n+1}$, $[x+q+2]_{6n+1}\cup[x+q+3]_{6n+1}$, ……………………….. $[x+6n-1]_{6n+1}\cup[x+6n]_{6n+1}$, $[x+6n]_{6n+1}\cup[x]_{6n+1}$ contains exactly p pairs of two consecutive natural numbers, which are in [x, y]. Therefore, each set of 6n+1 sets has not an equal chance in selection. i.e the Chance in selection of each set of $[x]_{6n+1}\cup[x+1]_{6n+1}$, $[x+1]_{6n+1}\cup[x+2]_{6n+1}$, $[x+2]_{6n+1}\cup[x+3]_{6n+1}$,………………… $[x+q-1]_{6n+1}\cup[x+q]_{6n+1}$ is greater than the chance in selection of each set of $[x+q]_{6n+1}\cup[x+q+1]_{6n+1}$, $[x+q+1]_{6n+1}\cup[x+q+2]_{6n+1}$, $[x+q+2]_{6n+1}\cup[x+q+3]_{6n+1}$, ………………. $[x+6n]_{6n+1}\cup[x]_{6n+1}$. But each set of $[x]_{6n+1}\cup[x+1]_{6n+1}$, $[x+1]_{6n+1}\cup[x+2]_{6n+1}$, $[x+2]_{6n+1}\cup[x+3]_{6n+1}$,…………… $[x+q-1]_{6n+1}\cup[x+q]_{6n+1}$ has an equal chance in selection. And each set of $[x+q]_{6n+1}\cup[x+q+1]_{6n+1}$, $[x+q+1]_{6n+1}\cup[x+q+2]_{6n+1}$, $[x+q+2]_{6n+1}\cup[x+q+3]_{6n+1}$, ……………… $[x+6n]_{6n+1}\cup[x]_{6n+1}$ has an equal chance in selection. hence five cases arise in the calculation of probability of our favourable event. Let us discuss one by one.

Case (1)

Let, $[n-1]_{6n+1}\cup[n]_{6n+1}$ includes exactly p+1 pairs of two consecutive natural numbers, which are in [x, y], and each of $[n]_{6n+1}\cup[n+1]_{6n+1}$, $[-n-1]_{6n+1}\cup[-n]_{6n+1}$, & $[-n]_{6n+1}\cup[-n+1]_{6n+1}$ includes exactly p pairs of two consecutive natural numbers, which are in [x, y].

Therefore, Number of unfavourable outcomes = p+1+3p = 4p+1

Since the sets $[n-1]_{6n+1}\cup[n]_{6n+1}$, $[n]_{6n+1}\cup[n+1]_{6n+1}$, $[-n-1]_{6n+1}\cup[-n]_{6n+1}$, & $[-n]_{6n+1}\cup[-n+1]_{6n+1}$ are unfavourable sets. i.e the selected pair of two consecutive natural numbers should not belong to the above 4 sets.

And the randomly selected pair of two consecutive natural numbers should be any one pair of p(6n+1)+q pairs of two consecutive natural numbers of [x, y]. Hence each pair of p(6n+1)+q pairs has an equal chance in selection. Therefore,

Total number of outcomes = p(6n+1)+q

Therefore, Number of favourable outcomes

= Total number of outcomes – number of unfavourable outcomes.

= p(6n+1)+q - 4p-1.

Therefore,

Probability of our favourable event = (p(6n+1)+q- 4p-1)/(p(6n+1)+q).

Case (2)

Let each of $[n-1]_{6n+1} \cup [n]_{6n+1}$ & $[n]_{6n+1} \cup [n+1]_{6n+1}$ includes exactly p+1 pairs of two consecutive natural numbers, which are in [x, y], And each of $[-n-1]_{6n+1} \cup [-n]_{6n+1}$, & $[-n]_{6n+1} \cup [-n+1]_{6n+1}$ includes exactly p pairs of two consecutive natural numbers, which are in [x, y].

Therefore, Number of unfavourable outcomes = 2(p+1)+2p = 4p+2.

Similar to the above argument,

Probability of our favourable event = (p(6n+1)+q- 4p-2)/(p(6n+1)+q).

Case (3)

Let each of $[n-1]_{6n+1} \cup [n]_{6n+1}$, $[n]_{6n+1} \cup [n+1]_{6n+1}$ & $[-n-1]_{6n+1} \cup [-n]_{6n+1}$, includes exactly p+1 pairs of two consecutive natural numbers, which are in [x, y], And $[-n]_{6n+1} \cup [-n+1]_{6n+1}$ includes exactly p pairs of two consecutive natural numbers, which are in [x, y],

Therefore, Number of unfavourable outcomes = 3(p+1)+p = 4p+3.

Similar to the above argument,

Probability of our favourable event =(p(6n+1)+q- 4p-3)/(p(6n+1)+q).

Case (4)

Let each of $[n-1]_{6n+1} \cup [n]_{6n+1}$, $[n]_{6n+1} \cup [n+1]_{6n+1}$, $[-n-1]_{6n+1} \cup [-n]_{6n+1}$, & $[-n]_{6n+1} \cup [-n+1]_{6n+1}$ includes exactly p+1 pairs of two consecutive natural numbers, which are in [x, y].

Therefore, Number of unfavourable outcomes = 4(p+1) = 4p+4

Similar to the above argument,

Probability of our favourable event = (p(6n+1)+q- 4p-4)/(p(6n+1)+q).

Case (5)

Let each of $[n-1]_{6n+1}\cup[n]_{6n+1}$, $[n]_{6n+1}\cup[n+1]_{6n+1}$, $[-n-1]_{6n+1}\cup[-n]_{6n+1}$, & $[-n]_{6n+1}\cup[-n+1]_{6n+1}$ includes exactly p pairs of two consecutive natural numbers, which are in [x, y].

Therefore, Number of unfavourable outcomes = 4p

Similar to the above argument,

Probability of our favourable event = (p(6n+1)+q- 4p)/(p(6n+1)+q).

the probabilities of above five cases are different from the probability of selecting a pair of two consecutive natural numbers randomly in N such that it belongs to $([n]_{6n+1}\cup[-n]_{6n+1})^c$ i.e All the above probabilities are different from 6n-3/(6n+1).

but it is obvious that all the above five probabilities are almost (approximately) equal to the probability of selecting a pair of two consecutive natural numbers randomly in N such that it belongs to $([n]_{6n+1}\cup[-n]_{6n+1})^c$. i.e the above five probabilities are almost equal to 6n-3/6n+1.

The difference (error) between case (5) and 6n-3/(6n+1)

(p(6n+1)+q-4p)/(p(6n+1)+q) – (6n-3)/(6n+1)

= ((6n+1)(p(6n+1)+q-4p)-(6n-3)(p(6n+1)+q)) / (6n+1)(p(6n+1)+q)

= ((6n-3+4)(p(6n+1)+q-4p)-(6n-3)(p(6n+1)+q)) / (6n+1)(p(6n+1)+q)

= (4(p(6n+1)+q-4p)-4p(6n-3)) / (6n+1)(p(6n+1)+q)

= (4p6n+4p+4q-16p-4p6n+12p) / (6n+1)(p(6n+1)+q)

= 4q / (6n+1)(p(6n+1)+q)

= 4q/(6n+1) X 1/(p(6n+1)+q)

Therefore,

0 ≤ The difference (error) between case (5) and 6n-3/(6n+1) < 1

[since, q < 6n+1 implies, 0 ≤ 4q/(6n+1) < 4 & 0 < 1/(p(6n+1)+q) ≤ 1/(p(6(1)+1)+q) ≤ 1/7].

P(6n+1)+q = length of [x, y] -1 implies, when length of [x, y] increases, the difference (error) decreases. i.e approximation comes closer, when length of [x, y] increases.

Similarly,

The difference (error) between case (1) and 6n-3/(6n+1)

(p(6n+1)+q-4p-1)/(p(6n+1)+q) – (6n-3)/(6n+1)

= ((6n+1)(p(6n+1)+q-4p-1)-(6n-3)(p(6n+1)+q)) / (6n+1)(p(6n+1)+q)

= ((6n-3+4)(p(6n+1)+q-4p-1)-(6n-3)(p(6n+1)+q)) / (6n+1)(p(6n+1)+q)

= (4(p(6n+1)+q-4p-1)-(4p+1)(6n-3)) / (6n+1)(p(6n+1)+q)

= (4p6n+4p+4q-16p-4-4p6n+12p-6n+3) / (6n+1)(p(6n+1)+q)

= (4q-(6n+1)) / (6n+1)(p(6n+1)+q)

= (4q-(6n+1))/(6n+1) X 1/(p(6n+1)+q)

Therefore,

-1 ≤ The difference (error) between case (1) and 6n-3/(6n+1) < 1

[since q < 6n+1 implies, -1 ≤ (4q-(6n+1))/(6n+1) < 3 & 0 < 1/(p(6n+1)+q) ≤ 1/7].

P(6n+1)+q = length of [x, y] -1 implies, when length of [x, y] increases, the difference (error) decreases. i.e approximation comes closer, when length of [x, y] increases.

Similarly,

The difference (error) between case (2) and 6n-3/(6n+1)

(p(6n+1)+q-4p-2)/(p(6n+1)+q) – (6n-3)/(6n+1)

= ((6n+1)(p(6n+1)+q-4p-2)-(6n-3)(p(6n+1)+q)) / (6n+1)(p(6n+1)+q)

= ((6n-3+4)(p(6n+1)+q-4p-2)-(6n-3)(p(6n+1)+q)) / (6n+1)(p(6n+1)+q)

= (4(p(6n+1)+q-4p-2)-(4p+2)(6n-3)) / (6n+1)(p(6n+1)+q)

= (4p6n+4p+4q-16p-8-4p6n+12p-12n+6) / (6n+1)(p(6n+1)+q)

= (4q-2(6n+1)) / (6n+1)(p(6n+1)+q)

= (4q-2(6n+1))/(6n+1) X 1/(p(6n+1)+q)

Therefore,

-1 ≤ The difference (error) between case (2) and 6n-3/(6n+1) < 1

[since, q < 6n+1 implies, -2 ≤ (4q-2(6n+1))/(6n+1) < 2 & 0 < 1/(p(6n+1)+q) ≤ 1/7].

Here is also, the approximation comes closer, when the length of [x, y] increases.

Similarly,

The difference (error) between case (3) and 6n-3/(6n+1)

(p(6n+1)+q-4p-3)/(p(6n+1)+q) – (6n-3)/(6n+1)

= (4q-3(6n+1)) / (6n+1)(p(6n+1)+q)

= (4q-3(6n+1))/(6n+1) X 1/(p(6n+1)+q)

Therefore,

-1 ≤ The difference (error) between case (3) and 6n-3/(6n+1) < 1

[since, q < 6n+1 implies, -3 ≤ (4q-3(6n+1))/(6n+1) < 1 & 0 < 1/(p(6n+1)+q) ≤ 1/7].

Here is also approximation comes closer, when length of [x, y] increases.

Similarly,

The difference (error) between case (4) and 6n-3/(6n+1)

(p(6n+1)+q-4p-4)/(p(6n+1)+q) – (6n-3)/(6n+1)

= (4q-4(6n+1)) / (6n+1)(p(6n+1)+q)

= (4q-4(6n+1))/(6n+1) X 1/(p(6n+1)+q)

Therefore,

-1 ≤ The difference (error) between case (4) and 6n-3/(6n+1) < 0

[since, q < 6n+1 implies, -4 ≤ (4q-4(6n+1))/(6n+1) < 0 & 0 < 1/(p(6n+1)+q) ≤ 1/7].

Here is also, approximation comes closer, when length of [x, y] increases.

So far our discussion yields following results, which are essential in proving theorems.

The probability of selecting a pair of two consecutive natural numbers randomly in [x, y] such that it belongs to $([n]_{6n+1}\cup[-n]_{6n+1})^c$ is almost (approximately) equal to the probability of selecting a pair of two consecutive natural numbers randomly in the infinite set N such that it belongs to $([n]_{6n+1}\cup[-n]_{6n+1})^c$. where [x, y] is the arbitrary closed interval, length of the closed interval is p(6n+1)+q+1. p is a natural number, q is a whole number such that 0 ≤ q < 6n+1, and n < x.

difference (error) between the probability of selecting a pair of two consecutive natural numbers randomly in [x, y] such that it belongs to $([n]_{6n+1}\cup[-n]_{6n+1})^c$ and the probability of selecting a pair of two consecutive natural numbers randomly in the infinite set N such that it belongs to $([n]_{6n+1}\cup[-n]_{6n+1})^c$ become smaller and smaller when length of the closed interval [x, y] increases. i.e approximation comes closer, when length of [x, y] increases.

Now

From the above result,

The probability of selecting a pair of two consecutive natural numbers randomly in [x, y] such that it belongs $([n]_{6n+1}\cup[-n]_{6n+1})^c$ is almost (approximately) equal to (6n-3)/(6n+1) ……………………………….……….(1)

and by the definition of I_n and J_{-n},

n < x implies, ($[x, y]\cap[n]_{6n+1}$) $\subset I_n$ and ($[x, y]\cap[-n]_{6n+1}$) $\subset J_{-n}$.

implies, ($[x, y]\cap[n]_{6n+1}$) $\subseteq$ ($[x, y]\cap I_n$) and ($[x, y]\cap[-n]_{6n+1}$) $\subseteq$ ($[x, y]\cap J_{-n}$).

But

$I_n \subset [n]_{6n+1}$ implies ($[x, y]\cap I_n$) $\subset$ ($[x, y]\cap[n]_{6n+1}$) and

$J_{-n} \subset [-n]_{6n+1}$ implies ($[x, y]\cap J_{-n}$) $\subset$ ($[x, y]\cap[-n]_{6n+1}$)

Therefore,

$[x, y]\cap[n]_{6n+1} = [x, y]\cap I_n$ and $[x, y]\cap[-n]_{6n+1} = [x, y]\cap J_{-n}$

Implies, $([x, y]\cap[n]_{6n+1})\cup([x, y]\cap[-n]_{6n+1}) = ([x, y]\cap I_n)\cup([x, y]\cap J_{-n})$

$[x, y]\cap([n]_{6n+1}\cup[-n]_{6n+1}) = [x, y]\cap(I_n\cup J_{-n})$

[since $(X\cap Y)\cup(X\cap Z) = X\cap(Y\cup Z)$]

Therefore, $[x, y]\cap([n]_{6n+1}\cup[-n]_{6n+1})^c = [x, y]\cap(I_n\cup J_{-n})^c$

Hence,

The event of selecting a pair of two consecutive natural numbers in [x, y] such that it belongs to $([n]_{6n+1}\cup[-n]_{6n+1})^c$ is nothing but event of selecting a pair of two consecutive natural numbers in [x, y] such that it belongs to $(I_n\cup J_{-n})^c$

Hence from (1), For n < x and (6n+1) < length of [x, y]

The probability of selecting a pair of two consecutive natural numbers randomly in [x, y] such that it belongs to $(I_n\cup J_{-n})^c$ is almost equal to (6n-3)/(6n+1).

i.e $P((I_n\cup J_{-n})^c) \approx (6n-3)/(6n+1)$.

where $P((I_n\cup J_{-n})^c)$ is the probability of selecting a pair of two consecutive natural numbers randomly in [x, y] such that it belongs to $(I_n\cup J_{-n})^c$.

Similarly, by continuing the so far probability analysis for $([n]_{6n-1}\cup[-n]_{6n-1})^c$, we can show that,

$P((I_{-n}\cup J_n)^c) \approx (6n-5)/(6n-1)$.

Let m1 and m2 be two different nonzero integers, such that their absolute values are smaller than or equal to n. i.e |m1| ≤ n and |m2| ≤ n

$(I_{m1} \cup J_{-m1})^c$ is a complement of union of two residue classes of same modulo $(6|m1|\pm1)$, and $(I_{m2} \cup J_{-m2})^c$ is a complement of union of two residue classes of same modulo $(6|m2|\pm1)$.

Therefore, each one of $(I_{m1} \cup J_{-m1})^c$ and $(I_{m2} \cup J_{-m2})^c$ is a complement of two residue classes of different modulo. Hence, $(I_{m1} \cup J_{-m1})^c \cap (I_{m2} \cup J_{-m2})^c \neq \{\}$ i.e $(I_{m1} \cup J_{-m1})^c$ and $(I_{m2} \cup J_{-m2})^c$ are not disjoint. i.e event of selecting a pair of two consecutive natural numbers such that it belongs to both $(I_{m1} \cup J_{-m1})^c$ and $(I_{m2} \cup J_{-m2})^c$ is a possible event. Hence, the event of selecting a pair of two consecutive natural numbers such that it belongs to $(I_{m1} \cup J_{-m1})^c$ and the event of selecting a pair of two consecutive natural numbers such that it belongs to $(I_{m2} \cup J_{-m2})^c$ are independent events. i.e occurrence of one event does not affect the occurrence of other event.

Therefore, from the theory of probability,

$$P((I_{m1} \cup J_{-m1})^c \cap (I_{m2} \cup J_{-m2})^c) = P((I_{m1} \cup J_{-m1})^c) \ X \ P((I_{m2} \cup J_{-m2})^c).$$

[since X and Y are independent events, implies $P(X\cap Y)= P(X) \ X \ P(Y)$]

And also, it can be shown for any number of different nonzero integers, say m1, m2, m3, ……, mn. Such that their absolute values smaller than or equal to n. i.e |m1| ≤ n, |m2| ≤ n, |m3| ≤ n, …………… |mn| ≤ n.

As in above, each one of $(I_{m1} \cup J_{-m1})^c$, $(I_{m2} \cup J_{-m2})^c$, $(I_{m3} \cup J_{-m3})^c$, ……………., $(I_{mn} \cup J_{-mn})^c$ is a complement of two residue classes of different modulo. Hence,

$$(I_{m1} \cup J_{-m1})^c \cap (I_{m2} \cup J_{-m2})^c \cap (I_{m3} \cup J_{-m3})^c \cap \ldots\ldots \cap (I_{mn} \cup J_{-m n})^c \neq \{\}$$

i.e

$$\bigcap_{i=1}^{n} (I_{mi} \cup J_{-mi})^c \neq \{\}$$

Therefore, similar to above arguments, the following result is obvious.

$$P\left(\bigcap_{i=1}^{n}(I_{mi} \cup J_{-mi})^c\right) = P((I_{m1} \cup J_{-m1})^c) \text{ X } P((I_{m2} \cup J_{-m2})^c) \text{ X } P((I_{m3} \cup J_{-m3})^c) \text{ X } \ldots\ldots \text{ X } P((I_{mn} \cup J_{-mn})^c)$$

Hence, for $n < x$ and $6n+1 < y-x+1$ = length of $[x, y]$.

$$P\left(\bigcap_{i=1}^{n}(I_i \cup J_{-i})^c\right) = P((I_1 \cup J_{-1})^c) \text{ X } P((I_2 \cup J_{-2})^c) \text{ X } P((I_3 \cup J_{-3})^c) \text{ X } \ldots\ldots \text{ X } P((I_n \cup J_{-n})^c)$$

$$\approx 3/7 \text{ X } 9/13 \text{ X } 15/19 \text{ X } \ldots\ldots \text{ X } (6n-3)/(6n+1)$$

$$P\left(\bigcap_{i=1}^{n}(I_{-i} \cup J_i)^c\right) = P((I_{-1} \cup J_1)^c) \text{ X } P((I_{-2} \cup J_2)^c) \text{ X } P((I_{-3} \cup J_3)^c) \text{ X } \ldots\ldots \text{ X } P((I_{-n} \cup J_n)^c)$$

$$\approx 1/5 \text{ X } 7/11 \text{ X } 13/17 \text{ X } \ldots\ldots \text{ X } (6n-5)/(6n-1).$$

Now let us summarise all the above results.

[x, y] be the closed interval, and X be any subset of N.

P(X) is the probability of selecting a pair of two consecutive natural numbers randomly in [x, y] such that it belongs to X.

For any natural number $n < x$. and $6n+1 < y - x + 1$ = length of $[x, y]$.

1) $P((I_n \cup J_{-n})^c) \approx (6n-3)/(6n+1)$.

2) $P((I_{-n} \cup J_n)^c) \approx (6n-5)/(6n-1)$.

3) $$P\left(\bigcap_{i=1}^{n}(I_i \cup J_{-i})^c\right) = P((I_1 \cup J_{-1})^c) \text{ X } P((I_2 \cup J_{-2})^c) \text{ X } P((I_3 \cup J_{-3})^c) \text{ X} \ldots\ldots \text{X } P((I_n \cup J_{-n})^c)$$

$$\approx 3/7 \text{ X } 9/13 \text{ X } 15/19 \text{ X } \ldots\ldots \text{ X } (6n-3)/(6n+1)$$

4) $P(\bigcap_{i=1.}^{n}(I_{-i} \cup J_i)^c) = P((I_{-1} \cup J_1)^c) \times P((I_{-2} \cup J_2)^c) \times P((I_{-3} \cup J_3)^c) \times \ldots \times P((I_{-n} \cup J_n)^c)$

$\approx 1/5 \quad X \quad 7/11 \quad X \quad 13/17 \quad X \quad \ldots\ldots \quad X \quad (6n-5)/(6n-1).$

……………………………………………(2)

Before go to the theorem, let us define a new subset of natural numbers.

$$G = \{ x \mid x \in N \text{ and } 6x-1, 6x+1, 6x+5 \ \& \ 6x+7 \text{ are all prime numbers}\}$$

In other word,

$$G = \{ x \mid x \in (A \cup B)^c \text{ and } x+1 \in (A \cup B)^c\}$$

i.e G is a subset of natural numbers such that x∈G implies, x and x+1 both belongs to $(A \cup B)^c$. in other word 6x-1, 6x+1, 6x+5 & 6x+7 are all prime numbers. For example 17∈G, since 17 and 18 both belongs to $(A \cup B)^c$. i.e 6(17)-1=101, 6(17)+1=103, 6(17)+5=107 and 6(17)+7=109 are all prime numbers. Similarly, we can show that 32, 137,247, ……….. are belongs to G. therefore,

G = {17, 32, 137, 247, 312, 347, 542, 577, 942, 1572, ……………………}

Lets go to the theorem.

THEOREM

$n([6b+1, (b-1)(6b+1)-1] \cap G)$

$\approx (b-2)(6b+1)(1/5)(3/7)(7/11)(9/13) \ldots\ldots\ldots ((P_r - 4)/P_r).$

Where P_r is the greatest prime number such that $P_r \le 6(b-1)+1$.

PROOF

In my third article "article 3 about theory of distribution prime numbers.",

I have proved the corollary,

$$[1, (b-1)(6b+1)] \cap (AUB)$$

$$= [1, (b-1)(6b+1)] \cap \Big(\big(\bigcup_{\substack{n=1 \ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})\big) \cup \big(\bigcup_{\substack{n=1 \ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)\big) \Big)$$

For b > 1, Let

$$C = \left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1}(I_n \cup J_{-n})\right) \cup \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1}(I_{-n} \cup J_n)\right)$$

[since if b=1, implies b-1=0, but $I_{\pm n}$ and $J_{\pm n}$ are defined only for natural number. i.e $I_{\pm(b-1)}$ and $J_{\pm(b-1)}$ are cannot be defined. Therefore C cannot be defined for b=1]

The above corollary becomes,

$$[1, (b-1)(6b+1)] \cap (A \cup B) = [1, (b-1)(6b+1)] \cap C.$$

Implies,

$$[1, (b-1)(6b+1)] \cap (A \cup B)^c = [1, (b-1)(6b+1)] \cap C^c.$$

[since $X \cap Y = X \cap Z$ implies $X \cap Y^c = X \cap Z^c$]

Implies,

$$[6b+1, (b-1)(6b+1)] \cap (A \cup B)^c = [6b+1, (b-1)(6b+1)] \cap C^c.$$

[since $Y \subset X$, $X \cap P = X \cap Q$ implies, $Y \cap P = Y \cap Q$].

Implies,

$$No([6b+1, (b-1)(6b+1)] \cap (A \cup B)^c) = No([6b+1, (b-1)(6b+1)] \cap C^c)$$

Where

$No([6b+1, (b-1)(6b+1)] \cap (A \cup B)^c)$ is the number of pairs of two consecutive natural numbers contained in $[6b+1, (b-1)(6b+1)] \cap (A \cup B)^c$

$No([6b+1, (b-1)(6b+1)] \cap C^c)$ is the number of pairs of two consecutive natural numbers contained in $[6b+1, (b-1)(6b+1)] \cap C^c$.

$No([6b+1, (b-1)(6b+1)])$ is the number of pairs of two consecutive natural numbers contained in [6b+1, (b-1)(6b+1)]

Let

$P(C^c)$ is the probability of selecting a pair of two consecutive natural numbers randomly in [6b+1, (b-1)(6b+1)] such that it belongs to C^c.

It is obvious that n([6b+1, (b-1)(6b+1)]) = (b-2)(6b+1)+1.

i.e length of [6b+1, (b-1)(6b+1)] = (b-2)(6b+1)+1.

It is also obvious that,

No([6b+1, (b-1)(6b+1)]) = length of [6b+1, (b-1)(6b+1)] – 1

= (b-2)(6b+1).

By the basic definition of probability,

$$P(C^c) = No([6b+1, (b-1)(6b+1)] \cap C^c) \div No([6b+1, (b-1)(6b+1)])$$

Hence,

$$P(C^c) = No([6b+1, (b-1)(6b+1)] \cap C^c) \div (b-2)(6b+1)$$

Transposing yields

$$No([6b+1, (b-1)(6b+1)] \cap C^c) = (b-2)(6b+1) \times P(C^c)$$

………………………………………….(3)

Next,

$$P(C^c) = P\Big(\Big(\Big(\bigcup_{\substack{n=1 \ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})\Big) \cup \Big(\bigcup_{\substack{n=1 \ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)\Big) \Big)^c \Big)$$

$$= P\Big(\Big(\bigcup_{\substack{n=1 \ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})\Big)^c \cap \Big(\bigcup_{\substack{n=1 \ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)\Big)^c \Big)$$

[since $P((X \cup Y)^c) = P(X^c \cap Y^c)$]

$$= P\Big(\Big(\bigcup_{\substack{n=1 \ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})\Big)^c \Big) \times P\Big(\Big(\bigcup_{\substack{n=1 \ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)\Big)^c \Big)$$

[since each one of

$$\left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n U J_{-n})\right)^c \quad \& \quad \left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} U J_n)\right)^c$$

is a complement of union of residue classes of different prime moduli. Hence they are not disjoint. i.e selecting a pair of two consecutive natural numbers such that it belongs to both complements is a possible event.

i.e event of selecting a pair of two consecutive natural numbers such that it belongs to

$$\left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n U J_{-n})\right)^c$$

And the event of selecting a pair of two consecutive natural numbers such that it belongs to

$$\left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} U J_n)\right)^c$$

are independent events.]

Here C is union of $I_{\pm n}$ and $J_{\pm n}$ such that $1 \le n \le b-1 < 6b+1$.

And $6n\pm1 \le 6(b-1)+1 < (b-2)(6b+1)+1$ = length of [6b+1, (b-1)(6b+1)]. Therefore, the results of (2) what we have found from above probability analysis obeys in the closed interval [6b+1, (b-1)(6b+1)] for any $n \le b-1$.

Therefore,

$$P(C^c) = P\left(\left(\bigcup_{\substack{n=1\ \&\\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n U J_{-n})\right)^c \right) X\ P\left(\left(\bigcup_{\substack{n=1\ \&\\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} U J_n)\right)^c \right)$$

$$= P\Big(\bigcap_{\substack{n=1 \ \& \\ 6n+1 \text{ a prime number.}}}^{b-1} (I_n \cup J_{-n})^c \Big) \ \text{X} \ P\Big(\bigcap_{\substack{n=1 \ \& \\ 6n-1 \text{ a prime number.}}}^{b-1} (I_{-n} \cup J_n)^c \Big)$$

[since complement of union of sets is equal to intersection of complements of sets]

$$\approx (3/7) \times (9/13) \times (15/19) \times \ldots\ldots\ldots \times ((6s-3)/(6s+1))$$

$$\text{X } (1/5) \times (7/11) \times (13/17) \times \ldots\ldots\ldots \times ((6r-5)/(6r-1))$$

[since from (2)]

Where 6s+1 is the greatest prime number in the form 6k+1 such that $6s+1 \leq 6(b-1)+1$, similarly 6r-1 is the greatest prime number in the form 6k-1 such that $6r-1 \leq 6(b-1)-1$. Note: each factor of $P(C^c)$ is in the form (P-4)/P. Where P is the prime number in the form $6k\pm1$ such that $P \leq 6(b-1)+1$. After rearrangement of all factors in ascending order,

$$P(C^c) \approx (1/5) \text{ X } (3/7) \text{ X } (7/11) \text{ X } (9/13) \text{ X } \ldots\ldots\ldots\ldots\ldots\ldots \text{ X } ((P_r - 4)/P_r).$$

Where P_r is the greatest prime number such that $P_r \leq 6(b-1)+1$.

Now (3) becomes,

$$No([6b+1, (b-1)(6b+1)] \cap C^c)$$

$$\approx (b-2)(6b+1)(1/5)(3/7)(7/11)(9/13) \ldots\ldots\ldots\ldots ((P_r - 4)/P_r).$$

But

$$No([6b+1, (b-1)(6b+1)] \cap C^c)$$

$$= No([6b+1, (b-1)(6b+1)] \cap (A \cup B)^c)$$

implies

$$No([6b+1, (b-1)(6b+1)] \cap (A \cup B)^c)$$

$$\approx (b-2)(6b+1)(1/5)(3/7)(7/11)(9/13) \ldots\ldots\ldots\ldots ((P_r - 4)/P_r).$$

i.e Number of pairs of two consecutive natural numbers contained in $[6b+1, (b-1)(6b+1)] \cap (A \cup B)^c$ is almost (approximately) equal to

$$(b-2)(6b+1)(1/5)(3/7)(7/11)(9/13) \ldots\ldots\ldots\ldots ((P_r - 4)/P_r).$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

From the definition of G, two consecutive natural number, say p and p+1 are belong to $(A\cup B)^c$ implies that p belongs to G. And p belongs to G implies that p and p+1 are belong to $(A\cup B)^c$. Similarly, p and p+1 are belong to [6b+1, (b-1)(6b+1)] implies that p belongs to [6b+1, (b-1)(6b+1)-1]. And p belongs to [6b+1, (b-1)(6b+1)-1] implies that p and p+1 are belong to [6b+1, (b-1)(6b+1)].

Therefore, p and p+1 are belong to $[6b+1, (b-1)(6b+1)]\cap(A\cup B)^c$ implies that p belongs to $[6b+1, (b-1)(6b+1)-1]\cap G$. And p belongs to $[6b+1, (b-1)(6b+1)-1]\cap G$ implies that p and p+1 are belong $[6b+1, (b-1)(6b+1)]\cap(A\cup B)^c$. Hence, number of pairs of two consecutive natural numbers contained in $[6b+1, (b-1)(6b+1)]\cap(A\cup B)^c$ is equal to number of natural numbers contained in $[6b+1, (b-1)(6b+1)-1]\cap G$. i.e

$$No([6b+1, (b-1)(6b+1)]\cap(A\cup B)^c) = n([6b+1, (b-1)(6b+1)-1]\cap G).$$

Therefore, (4) becomes,

$$n([6b+1, (b-1)(6b+1)-1]\cap G) \approx (b-2)(6b+1)(1/5)(3/7)(7/11)(9/13) \ldots\ldots ((P_r - 4)/P_r). \quad \ldots\ldots(5)$$

Hence the theorem is proved.

After rearrangement of factors.

$$n([6b+1, (b-1)(6b+1)-1]\cap G)$$

$$\approx (b-2)(6b+1)(1/5)(3/7)(7/11)(9/13) \ldots\ldots ((P_r - 4)/P_r).$$

$$= (b-2)(3/5)(7/7)(9/11)(13/13)(15/17)(19/19)(25/23)(27/29)$$

$$(33/31)\ldots\ldots((P_r-4)/P_{r-1})((6b+1)/P_r).$$

In the rearrangement, first factor is (b-2), and last factor is $(6b+1)/P_r$. $P_r \le 6(b-1)+1$ implies $(6b+1)/P_r > 1$. Other factors are in the form $(P_m-4)/P_{m-1}$. Where P_m is m[th] prime in the form 6k±1 and P_{m-1} is (m-1)[th] prime in the form 6k±1. And $P_m \le 6(b-1)+1$. If P_{m-1} and P_m are twin prime numbers then $P_m - 2 = P_{m-1}$ implies, $P_m - 4 < P_{m-1}$. Otherwise $P_m - 4 \ge P_{m-1}$. Therefore, if P_{m-1} and P_m are twin prime numbers then the factor $(P_m - 4)/P_{m-1} < 1$. Otherwise $(P_m - 4)/P_{m-1} \ge 1$. But it is obvious from my fourth and fifth article, number of pairs of twin prime numbers is fewer than the number of other pairs two consecutive prime number. Therefore, in the rearrangement, only fewer factors are smaller than 1, other factors are greater than or equal to 1. Hence, when b increases,

$$(b-2)(3/5)(7/7)(9/11)(13/13)(15/17)(19/19)(25/23)(27/29)(33/31)\ldots\ldots((P_r-4)/P_{r-1})((6b+1)/P_r).$$

increases. for example

when b=7, $(7-2)(3/5)(7/7)(9/11)(13/13)\ldots\ldots(33/31)(43/37) = 2.7115$

when b=8, $(8-2)(3/5)(7/7)(9/11)(13/13)\ldots\ldots(39/41)(49/43) = 3.0348$

Hence, n([6b+1), (b-1)(6b+1)-1]∩G) > 3.0348 for arbitrary natural number b > 8. But, b is arbitrary natural number implies that there are infinite number of closed intervals in the form [6b+1, (b-1)(6b+1)-1]. Therefore,

n(G) is infinite. Hence G is infinite subset of set of natural numbers.

IMMEDIATE CONSEQUENCE OF ABOVE THEOREM

But in my second article IJNRD 2304175 "About theory of distribution of prime numbers.", I have shown that the natural number k determines the nature of natural numbers in the form 6k±1. i.e nature of natural numbers in the form 6k±1 depends on natural number k. Therefore, the natural numbers in [6b+1, (b-1)(6b+1)] determines the nature of natural numbers in the form 6k±1 contained in the closed interval

[6(6b+1)-1, 6(b-1)(6b+1)+1] = [36b+5, 6(b-1)(6b+1)+1].

Especially arbitrary natural number k belongs to $(A\cup B)^c$ implies 6k+1 and 6k-1 both are prime numbers, i.e twin prime numbers. By the definition G, p belongs to G implies, p and p+1 belongs to $(A\cup B)^c$ hence 6p-1, 6p+1, 6(p+1)-1=6p+5 and 6(p+1)+1=6p+7 are all prime numbers. Therefore, infiniteness of G implies that the pairs of two consecutive pairs of twin prime numbers in the form 6k-1 & 6k+1 and 6k+5 & 6k+7 are infinite.

i.e

pairs of two consecutive pairs of twin prime numbers in the form 6k-1 & 6k+1 and 6k+5 & 6k+7 are infinite.

And every natural number contained in [6b+1, (b-1)(6b+1)-1]∩G determines exactly one pair of two consecutive natural numbers in [6b+1, (b-1)(6b+1)]∩$(A\cup B)^c$. hence, every natural number contained in [6b+1, (b-1)(6b+1)-1]∩G determines exactly one pair of two consecutive pairs of twin prime numbers in the form 6p-1 & 6p+1 and 6p+5 & 6p+7 contained in the closed interval [36b+5, 6(b-1)(6b+1)+1].

Hence the theorem can be restated as

n([6b+1, (b-1)(6b+1)-1]∩G)

= No. of pairs of two consecutive pairs of twin prime numbers in the form 6k-1 & 6k+1 and 6k+5 & 6k+7 contained in the closed interval [36b+5, 6(b-1)(6b+1)+1].

$$\approx (b-2)(6b+1)(1/5)(3/7)(7/11)(9/13) \ldots\ldots\ldots\ldots (P_r-4)/P_r.$$

REMARK

when b=2 the closed interval in the form [6b+1, (b-1)(6b+1)-1] is [13, 12], Which is incorrect notation of closed interval. And also though if we consider the interval as [12, 13].

The length of [12, 13] = 2 < 6(2-1)+1 = 6(b-1)+1. i.e (2) does not obey for [12, 13].

However, for [12, 13] the result is obvious. i.e

n([12, 13] ∩ G) = (2-2) X 13 X (1/5)(3/7) = 0

i.e [12, 13] contains no natural number that belongs to G

EXAMPLES

When b=3, The closed interval in the form [6b+1, (b-1)(6b+1)-1] is [19, 37]. Here is 1 natural number 32 belongs to [19, 37]∩G. i.e 32 & 33 belongs to [19, 38]∩$(A\cup B)^c$.

but (3-2) x 19 x (1/5) x (3/7) x (7/11) x (9/13) = 0.717 which is nearly.

When b=4, The closed interval is [25, 74]. here is also 32 belongs to [25, 74]∩G. but

(4-2) x 25 x (1/5) x (3/7) x (7/11) x (9/13) x (13/17) x (15/19) = 1.139 which is nearly.

When b=5, The closed interval is [31, 123]. here is also 32 belongs to [31, 123]∩G. but,

(5-2) x 31 x (1/5) x (3/7) x 7/11) x (9/13) x (13/17) x (15/19) x (19/23) = 1.751 which is nearly.

When b=6, the closed interval is [37, 184]. Here is 137 belongs to [37, 184]∩G. but,

(6-2)(37)(1/5)(3/7)(7/11)(9/13)(13/17)

(15/19)(19/23)(25/29)(27/31) =2.09 which is nearly

When b=7, the closed interval is [43, 257]. Here there are two natural numbers 137 and 247 are belong to [43, 257]∩G. but

(7-2)(43))(1/5)(3/7)(7/11)(9/13)(13/17)

(15/19)(19/23)(25/29)(27/31)(33/37) = 2.711. which is nearly.

When b=8, the closed interval is [49, 343]. Here there are three natural numbers 137, 247 & 312 are belong to [49, 343]∩G. but,

(8-2)(49)(1/5)(3/7)(7/11)(9/13)………………..(39/43) = 3.0348. which is nearly

When b=16. The closed interval is [97, 1454] here there are 7 natural numbers 137, 247, 312, 347, 542, 577 & 942 are belong to [97, 1454]∩G. but,

(16-2)(97)(1/5)(3/7)(7/11)(9/13)……..(85/89) = 7.475

When b 17. The closed interval is [103, 1647] here there are 8 natural numbers 137, 247, 312, 347, 542, 577, 942 & 1572 are belong to [103, 1647]∩G. but,

(17-2)(103)(1/5)(3/7)(7/11)(9/13)……..(93/97)=8.154.

See, the approximation comes closer, when interval length increases.

CONCLUTION

My name is **A. GABRIEL** a distance educated post graduate in mathematics. The thesis what we discussed above is myself realized one. Here I have submitted my completed concepts only. I am continuing my research about **THEORY OF DISTRIBUTION OF PRIME NUMBERS** by analyzing numbers which can be expressed in form 6ab±a±b, and which cannot be expressed in the form 6ab±a±b. i.e by analyzing the sets A, B, A^c, B^c, A∪B, $(A∪B)^c$, A∩B, and $(A∩B)^c$, where A and B are as defined above and the set of Natural numbers as universal set. then I conclude.

By

A. GABRIEL

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