IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

EVALUATION OF THE CREDIBILITY OF BLACK-SCHOLES OPTION PRICING MODEL DURING TURBULENT AND NORMAL DAYS

[1]Chukwudi A.Ugomma, [2]Nancy O Onuoha and [3]Comfort U. Nkem
[1]Department of Statistics, Imo State University, Owerri
[2,3]Department of Mathematics, Imo State University, Owerri

Corresponding Author: Chukwudi Anderson Ugomma

Abstract
This paper evaluates the performance of the Black Scholes Option Pricing Model (BSOPM) to the credibility of the price stock index options. For this reason, the theoretical values of the Lehman Brothers of 2008 were calculated under the Black Scholes Model. The data collected were divided into two different windows; the normal trading days and the turbulent days built around the bankruptcy of Lehman Brothers in 2008.The empirical result showed that the Black Scholes model performed differently in the normal days and turbulent days.

Keywords: Black-Scholes, Options, Bankruptcy, Pair wise t-test

1.0 Introduction

The use of commodity derivatives has a long history. Prokopczuk, et al (2010) described documented and published accounts of option trading in Aristotle's book dating back 322BC, proving that this type of financial transactions has been in practice informally since time. However several academic literatures have mainly focused on the dynamics of markets with formal focused trading Exchanges, and these works are mostly from developed countries with advanced and efficient markets. Some developing countries have started advancing their commodity markets. For example China added two commodity Exchanges in 2014, to reposition it for advancement in derivatives trading amongst other objectives. Most of these new Exchanges of developing countries are yet to meet expectations and studies investigating these markets are emerging. See for example, Hong (2017), Inani (2017), Gupta (2014) and many others have tested popular derivatives models on Chinese and Indian commodity markets.

The current global economic meltdown which started in late 2007 was as a result of a liquidity shortfall in the United State banking system. The immediate cause or trigger of the current crisis was the bursting of the United States housing bubble which peaked in approximately 2005-2006. Already-rising default rates on "subprime" and adjustable rate mortgages (ARM) began to increase quickly thereafter. An increase in loan packaging, marketing and incentives such as easy initial terms, and a long-term trend of rising housing prices had encouraged borrowers to take on difficult mortgages in the belief that they would be able to quickly refinance at more favorable terms. However, once interest rates began to rise and housing prices started to drop moderately in 2006-2007 in many parts of the U.S. refinancing became more difficult.

According to OECD paper (2010), the last financial crisis has been caused by two components such global macroeconomic policies affected by liquidity of banking and credit systems, and the other is a very low effective system of rules, which claimed acting as a second line defense which has a key factor for the crisis. (Angeli and Bonz, 2010). It is reasonable to argue that the financial crisis of 2008 has been caused by the distortions of incentives begun by previous monetary policies, in fact the financial sector created a new business model in order to take advantage of the incentives created during last two decades, see for example, IMF, (2008) and World Economic Outlook, (2008).

Several researchers have found more insights on the causes of the 2008 financial crisis and suggested some remedies, see for example, Monnet (2010); Troshkin, (2008); Cohen and Villemot (2008); Borgman, (2009); Cline, (2009); Reinhart and Rogoff (2018); Herring and Wachter, (2008); Walterskirchen, (2009), Sieczka, et al (2010); Aglietta, (2008) and Buiter, (2007); Shirley and Arturo (2021) and Owoloko and Okeke (2014).

Consequently, the above researchers have shown that the crash of 2008 was meant for a massive down turn to the financial market. Since the stock indices serve as underlying asset for equity options, there have been great effects on the option market as well. Meanwhile, the major activities on the 2008 financial turbulence already proved that the last financial crisis of 2008 had effect on prices, especially prices of financial products such as options. Options are financial contracts between two parties, the buyer and the seller, and they are linked to claim underlying assets such as stocks, options. The option buyer has to exercise the option on or before the expiration date, otherwise, the option expires automatically at the end of the expiration date. Hence, options are also known as contingent claims for a relatively unknown market to be tested where it requires a well-known model that is universally accepted for determining the price of derivatives contracts. Fortune (1996) describes the Black-Scholes option Pricing Model (BSOPM) as the best known option primary model. Black and Scholes (1973) published "The pricing of options and corporate inabilities where they introduced, the model that went on to become arguably, the most cited model for pricing derivatives of traded securities. The BSOPM is an equation that determines the future call price as a function of the present stock price which follows a random walk, with a constant mean and variance of the rate of return. Despite being conceived for European derivatives instruments, it is renowned for being conceived for being useable in pricing all derivatives contracts, from simple forwards to complex exotics. Most other effective models have either been modified or derived from its basic concepts, see for example, Macbeth and Marville (1979); Heston (1993); Kim and Mohammed (1997); Nagendra and Venkateswar (2014); Mckenzie and Subedar,(2017).

In this paper, we intend to find out whether there is any remarkable differences in the reliability of the Black-Scholes model whether it is applied during 'normal' trading days or during 'financially turbulent' trading days using Lehman Brother's 2008 Bankruptcy period.

2.0 THE BLACK-SCHOLES MODEL

Proposition

If S is a stock price that follows an Itô process, then the value of an option f, of S is quantified as follows:

$$\frac{\partial f}{\partial t} + rs\frac{\partial f}{\partial S} + \frac{1}{2}\frac{\partial^2 f}{dS^2}\sigma^2 S^2 = rf \qquad (1)$$

Let S be a stock price that follows an Itô process and has a differential equation given as

$$dS = \mu S dt + \sigma S dz \qquad (2)$$

Supposing f is twice a differentiable function of the price of a call option or other derivative contingent on S using Itô lemma, hence, we have,

$$df = \left(\mu\frac{\partial f}{\partial S} + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial f}{\partial S}\sigma S dz dt \qquad (3)$$

We noticed that μ and σ are no longer contingent on t but are contingent on S then,(1) and (2) can also be written discretely over a time period Δt as

$$\Delta S = \mu \Delta S + \sigma S \Delta z \tag{4}$$

and

$$\Delta f = \left(\mu S \frac{\partial f}{\partial S} + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) \Delta t + \frac{\partial f}{\partial S} \sigma S \Delta z \tag{5}$$

Now S and f both follow the same Itô process. Thus, if we select a portfolio, we can eliminate the Itô process and effectively price an option. The portfolio we select will consist of short derivative and long $\frac{\partial f}{\partial S}$ shares of stock. It will become clear shortly why we select this portfolio.

Let π be the value of our portfolio and it is defined as:

$$\pi = -f + \frac{\partial f}{\partial S} S \tag{6}$$

Also, the discrete version of (6) over the time interval Δt can be written as:

$$\Delta \pi = -\Delta f + \frac{\partial f}{\partial S} \Delta S \tag{7}$$

$$\Delta \pi = -\left[\left(\mu S \frac{\partial f}{\partial S} + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) \Delta t + \frac{\partial f}{\partial S} \sigma S \Delta z \right] + \frac{\partial f}{dS} [\mu \Delta S + \sigma S \Delta z]$$

$$= -\frac{\partial f}{\partial S} \mu S \Delta t - \frac{\partial f}{\partial t} \Delta t - \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \Delta t - \frac{\partial f}{\partial S} \sigma S \Delta t + \frac{\partial f}{\partial S} \mu S \Delta t + \frac{\partial f}{\partial S} \sigma S \Delta z$$

Collecting like terms together, we obtain

$$\Delta \pi = -\frac{\partial f}{\partial t} \Delta t - \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \Delta t$$

By factoring out Δt

$$\Delta \pi = \left(-\frac{\partial f}{\partial t} - \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) \Delta t \tag{8}$$

Without a Δz term (stochastic variable), this portfolio is effectively riskless during the time period Δt. Since there are no arbitrage opportunities, security trading is continuous and all securities share the same short term constant interest rate, the portfolio we have created will earn instantaneous rates of return over the time period Δt.

Hence,

$$\Delta \pi = r \pi \Delta t \tag{9}$$

Substituting (5) and (7) into (9), we obtain

$$\left(-\frac{\partial f}{\partial t} - \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) \Delta t = \Delta \pi = \left(-f + \frac{\partial f}{\partial S} S \right) \Delta t$$

$$rf = -\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \tag{10}$$

3.0 MATERIALS AND METHODS

3.1 *Data Description*

The data collected for this study was a secondary type of data from Lehman Brothers stock prices of 2008 from February, 1st 2008 to October, 10th 2008. The data were divided into normal trading days and bankruptcy days.

3.2 Methods of Data Analysis

3.2.1 *The Black-Scholes Formula*

The Black-Scholes formula used in this study is given as

$$C = S_t N(d_1) - Ke^{-rT} N(d_2) \tag{11}$$

where

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + 0.5\sigma^2 T\right)}{V\sqrt{T}}, t = 1, 2, \ldots, n \tag{12}$$

$$d_2 = d_1 - V\sqrt{T} \tag{13}$$

T = Time to Expiration Days

V = the volatility and it is given as

$$V = \sqrt{\frac{T}{N}\sum \frac{(x_i - \bar{x})^2}{n(n-1)}}, \tag{14}$$

$$x_i = \left(\frac{S_t}{S_t - 1}\right), i = 1, 2, \ldots, n \tag{15}$$

$$\bar{x} = \frac{1}{n}\left(\frac{S_t}{S_t - 1}\right) \tag{16}$$

and

$$r = \frac{1}{T}\ln\left(\frac{K}{S_t}\right) \tag{17}$$

3.2.2 The Pair wise t- Test

In this paper, we used this test to compare two population means where two samples can be paired as one observation. That is, “before – and – after” observation on the same subject.

Here, we assumed the null hypothesis of no significant difference between the means of the market prices of the stock and Black-Scholes prices of the same stock at 5% level of significance.

The pair wise t-test used in this study is given as

$$t = \frac{\bar{d}}{SE(\bar{d})} \tag{18}$$

where

$$\bar{d} = \frac{\sum d_i}{n} \tag{19}$$

$$SE\left(\bar{d}\right)=\frac{S_d}{\sqrt{n}} \tag{20}$$

and

$$S_d=\sqrt{\frac{n\sum d_i^2-\left(\sum d_i\right)^2}{n\left(n-1\right)}} \tag{21}$$

Note that Equation (18) follows a t-distribution with $n-1$degrees of freedom (Nwobi, 2003).

4.0 Empirical Evidence

4.1 Descriptive Statistics for Normal Trading Days

Table 1 displays the descriptive of 176 normal trading days. From the statistics, we observed that that data shows normality since it skewed towards the right by proving positive skewness. Since volatility affects the call options, we further observed that there is low volatility of about 0% in the 176 normal trading days of options implying low risk of investing in the Lehman's brothers stocks during the non-turbulence days of the 2008 financial crisis.

Table 1 Summary descriptive statistics for the normal trading days

Trading days	Mean	Variance	Standard Dev	Skewness	Kurtosis	Volatility
176	0.0419	0.0072	0.08472	4.2808	19.7386	0.0032

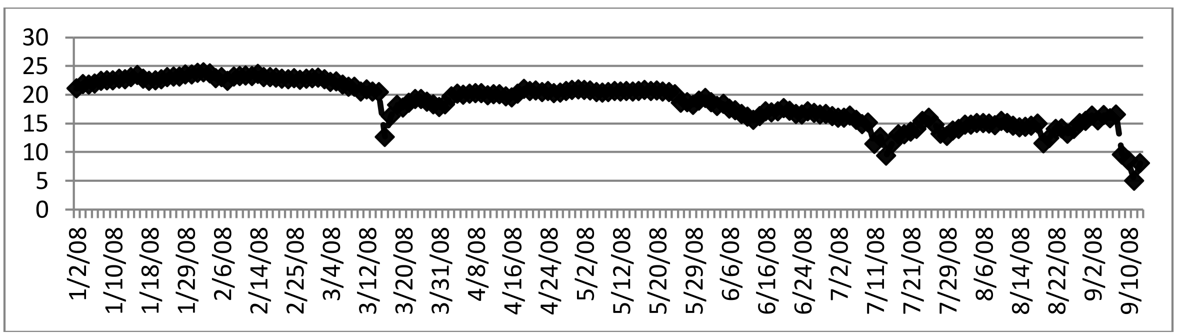

Fig 1 Plot of the underlying price during the normal Trading days

Figure 1 is a plot of the normal trading days of Lehman Brothers. From the plot, we observed that there are fluctuations during the trading days. The plot shows that the stocks maintained steady price from the 2nd of January until it depreciated on the 12th of March, 2008 increased again from 20th March, 2008, and maintained fairly steady price until 11th July, 2008 when we observed a lot price which subsequently increased and decreased intermittently until it declines towards the bankruptcy period on 10th of September, 2008.

4.2 Paired T-Test for Normal Trading period

$$H_0:\mu=0$$
$$H_1:\mu\neq 0$$

Table 2The output of the paired t-test

	Trading days	Mean	Standard Deviation	Standard Error	T-value	P-value	Decision
Underlying price	176	19	4	0	1.00	0.319	Accept
Black-Scholes call	176	-96423	1279193	96423			
Difference	176	96442	1279193	96423			

From Table 2, we observed that the T-value is 1.00 and the P-value is 0.319. Since the P-value (0.319) is greater than the 0.05, we do not reject the null hypothesis of no significant difference; hence, we conclude that there is no difference between the underlying (actual) price and the theoretical (Black-Scholes) price of the stocks of Lehman Brothers during the normal trading days in 2008.

4.3 Descriptive Statistics for bankruptcy Trading Days

Table 3 depicts the descriptive of 39 bankruptcy trading period of Lehman Brothers stock prices. From the result, we observed that the data showed normality also like the normal trading period because of its positive skweness. The volatility is somehow higher than that of the normal trading implying more trading risk during the financial crisis.

Table 4.3 Summary descriptive statistics for the bankruptcy trading days

Trading days	Mean	Variance	Standard Dev	Skewness	Kurtosis	Volatility
39	0.4133	0.2886	0.5375	2.4253	7.2791	0.0462

We also display the plot of the underlying prices during the bankruptcy period in order to visualize properly the price movement of the Lehman Brothers.

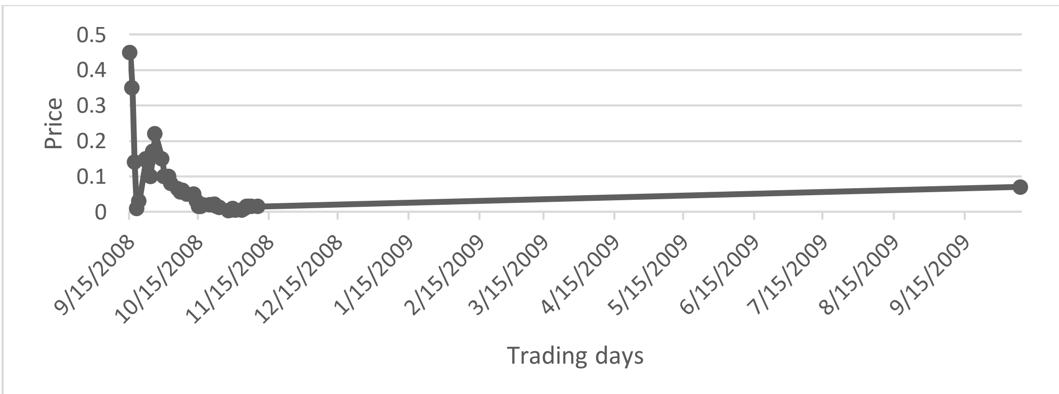

Fig 2 Plot of the underlying price during the bankruptcy Trading days

From the plot in Figure 2 showed a steady decline of the stock prices from 15th September 2008 to 15th September 2009. During this period, the stock prices maintained very depreciating movement thereby resulting high volatility of the stocks, hence, there is more risk in investing during these turbulence days in the financial market.

4.4 Paired T-Test for bankruptcy Trading period

$$H_0 : \mu = 0$$

$$H_1 : \mu \neq 0$$

Table 4TheOutput of the Paired T-Test

	Trading days	**Mean**	**Standard Deviation**	**Standard Error**	**T-value**	**P-value**	**Decision**
Underlying price	39	0.0742	0.0152	0	2.88	0.006	Reject
Black-Scholes call	39	0.0196	0.1449	96423			
Difference	39	0.0547	0.1184	96423			

From the result in Table 4, we observed that the critical value is 2.88 and the P-value is 0.006. Since the p-value (0.006) is less than the 0.05, we reject the null hypothesis of no significant difference andconclude that there is significant difference between the underlying (actual) price and the theoretical (Black-Scholes) price of the stocks of Lehman Brothers during the bankruptcy trading days in 2008/2009.

5.0 Conclusion

In this paper we evaluated the credibility of the Black Scholes option pricing model during normal and turbulent trading days of Lehman Brothers bankruptcy in 2008 and the result showed that the Black Scholes Model was credible during the normal trading days but was not credible during financial turbulent days.

REFERENCES

Aglietta, M (2008): Understanding the Structural Credit Crisis, CEP II, Research Centre, Issue 275

Angeli and Bonz (2010). Changes in the Creditability of the Black-Scholes Option Pricing Model Due to Financial Turbulences, P. 57.

Black, F. and Scholes,M(1973);The pricing of Options and Corporate Inabilities. *Journal of Political Economy 81 (May/June).637-659*.

Borgman, R.H, (2009): Prudent Investing? The Credit Crisis of August, 2007 Mainsail II, siv,Lite and State Crash Investment Pool, Academy of Economics Studies-Bucharest Romania., Vol II, 645 – 666

Buiter, W (2007): Lessons from the 2007 Financial Crisis, CEPR Policy Insight.n. Vol 18, 1- 17.

Cline, A.R, (2009): Losers and the Sub-Prime Mortgage Crisis, Poverty and Public, Policy Studies Organization, 1 (2).

Cohen, Dand Villemot, S (2008): Self Fulfilling and Self Enforcing Debt Crisis, Cepr. Discussion Paper 6718.

Fortune P. (1998): Anomalies in Option Pricing: The Black Scholes Model Revisited. New England Economic Revernt, Retrieved from https:///www.bostonfed.org/-/media/documents/neer/neer2966.pdt.

Fortune, P. (1996) “Anormalies in Option Pricing; the Black-Scholes Model Revisited” New England Economic Review.

Gupta, S.(2014):Effectiveness of the Black and Scholes Model in pricing Nifty call options. *The International Journal of Business & management, 7(2), 149-158.*

Herring, R and Wachter, S (2008): Bubbles in Real Estate Markets. Zell/Lurie Centre Working Paper 402, Wharton School, Samuel Zell and Robert Lurie Real Estate Centre, University of Pennsylvania.

Heston, S.,(1993). A Closed Form Solution for Options with Stochastic Volatility with Applications to Bounds and Currency Options. The Review of Financial Studies.

Hong, Q. (2017): Study of Agricultural Product Options Pricing. IOP Conf. Series: Materials Science and Engineering 242. Retrieved from http://iopscience.iop.org/article/10.1088/757-899X/242/1/012094/pdf.

Inani, S.(2017):Price Discovery and Efficiency of Indian Agricultural Commodity Futures Market: An Empirical Investigation. *Journal of Quantitative, Economic, 34, 45 – 47.*

International Monetary Fund, World Economic Outlook 2008, available at http://www.imf.org/external/pubs/ft/weo/2008/pdf/textpdf

Kim, H., Jong, C.R., and Mohammed, F.K (1997): An Empirical Investigation of Put Option Pricing. A specification test of at-the-money option implied volatility. *Journal of Financial and Strategic decision, 10(2), 75 – 83.*

Macbeth, J.D. and Marville, L.J. (1979): An Empirical Examination of the Black-Scholes Call Option Pricing Model. *The Journal of Finance Vol. 34 No 5, 1173 – 1186.*

Mckenzie, G.D.&Subedar Z.(2017): An Empirical Investigation of the Black-Scholes model. Evidence from the Australian Stock Exchange.The Australian Accounting Business and Finance Journal 1(4), 71 – 82.

Monnet, C, (2010): Lets Make It Clear:How Central Counterparties Save(d) the day Business, Review Q1 of the Federal Researve Bank of Philadelphia, available at http://www.philadelphiafed.org/research-and-data/publications/business-review/2010/q1/brq110-central-counterparties.pdf

Nagendra, R. and Venkateswar, S. (2014): Validating Black-Scholes model in Pricing Indian Stock call options. *Journal of Applied Finance and Banking, 4(3). 89 – 101.*

Nwobi, F.N.,(2003):Statistics 2: Introductory Statistics, Supreme Publishers, Owerri, Nigeria.

OECD (2010) available at http://www.oecd.org/dataoecd/47/26/4194872pdf

Owoloko, E.A and Okeke, M.C (2014): British Journal of Applied Science and Technology, 4(29)

Reinhart, C.M and Rogoff, K.S (2008): Is the 2007 U.S Sub-Prime Financial Crises so Different? An International Historical Comparison, NBER Working Papers, National Bureau of Economic Research, Inc.

Shirley, R and Arturo, C (2021): On the Performance of Bonds Option Pricing Formulas during the Sub-Prime and Covid-19 Crises. *The Journal of Corporate Accounting and Finance*.

Sieczka, P, Sornette, D and Holyst, J (2010): The Lehman Brothers Effect and Bankruptcy Cascades, Quantitative Finance Paper.

Troshkin, M (2008): Facts and Myths about the Financial Crisis of 2008 (technical Notes), Working Papers, Federal Reserve Bank of Minneapolis.

Walterskirchen, E (2009): The Bursting of the Real Estate Bubble: More than a Trigger for the Financial Crisis, WIFO Monatsberichla (monthly Reports), WIFO, 82(1), 934 – 950.