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ON $\delta \hat{g}$ CONTINUOUS FUNCTIONS IN GRILL DELTA TOPOLOGICAL SPACES

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Abstract : The aim of this paper is to introduce and study the concepts of new class of functions called $\zeta\delta\hat{g}$ - continuous functions, contra $\zeta\delta\hat{g}$ - continuous functions and $\zeta\delta\hat{g}$ - irresolute functions using $\zeta\delta\hat{g}$ - closed sets.

IndexTerms - $\zeta\delta\hat{g}$ - continuous and $\zeta\delta\hat{g}$ - irresolute functions.

1.INTRODUCTION

Norman Levine [9] initiated the idea of continuous function in 1970. T. Noiri [11] introduced δ - continuous in 1980. The generalised continuous (briefly g - continuous) was studied by K. Balachandran [2] in 1991. Further many others contributed their research towards continuity. R. Sudha. et.al [12], defined and analysed δg - continuous, δg∗ - continuous respectively. In this paper a new class of functions called $\zeta\delta\hat{g}$ - continuous functions, contra $\zeta\delta\hat{g}$ - continuous functions, and $\zeta\delta\hat{g}$ - irresolute functions using $\zeta\delta\hat{g}$ - closed sets are defined and their basic properties and relationship with other existing continuous functions are investigated.

2.PRELIMINARIES

Definition 2.1

A set is δ open if it is the union of regular open sets. The complement of δ open is called δ closed. Alternatively, a set $A \subseteq (X,\tau)$ is called δ closed if A= cl_δ(A) where cl_δ(A) = {x ∈ X/ int(cl (U)) ∩ A ≠ ϕ,U ∈ τ and x ∈ U}

Definition 2.2

A subset A of (X, τ) is called

(i) ω closed set [14] if $cl(A) \subseteq U$ U whenever $A \subseteq U$ and U is semi open in (X,τ)

(ii) δg closed [6] if $\delta\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)

(iii) δg^* closed [13] if $\delta\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g- open in (X,τ)

(iv) $\zeta\omega$ closed set [4] if $\varphi(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition : 2.3

Let (X,τ) be a topological space and ζ be a grill on X. Then a subset A of X is said to be a $\zeta\delta\hat{g}$ - closed [5] if $\varphi_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $\zeta\omega$ open in (X, τ, ζ).

Definition : 2.4

A function $f:(X,\tau) \to (Y,\sigma)$ is said to be

(i) δ -continuous [12] if $f^{-1}(V)$ is δ closed of (X, τ) for every closed set V of (Y,σ).

(ii) r- continuous [1] if $f^{-1}(V)$ is r closed of (X,τ) for every closed set V of (Y,σ).

(iii) ω -continuous [14] if $f^{-1}(V)$ is ω closed of (X,τ) for every closed set V of (Y,σ).

(iv) $\zeta\omega$ -continuous [4] if $f^{-1}(V)$ is $\zeta\omega$ closed of (X,τ) for every closed set V of (Y,σ).

Definition : 2.5

A function $f:(X,\tau)\to(Y,\sigma)$ is said to be

(i) contra - continuous [7] if $f^{-1}(V)$ is closed in (X,τ) for every open set V in (Y,σ).

(ii) contra sg - continuous [8] if $f^{-1}(V)$ is sg - closed in (X,τ) for every open set V in (Y,σ).

(iii) contra δgb - continuous [3] if $f^{-1}(V)$ is δgb - closed in (X,τ) for every open set V in (Y,σ).

(iv) contra $\delta\hat{g}$ - continuous [9] if $f^{-1}(V)$ is $\delta\hat{g}$ - closed in (X,τ) for every open set V in (Y,σ).

Definition 2.6

A function $f:(X,\tau,\zeta)\to(Y,\sigma,\lambda)$ is called $\zeta\omega$ - irresolute [4] if $f^{-1}(V)$ is $\zeta\omega$ - open set in (X,τ,ζ) for every $\zeta\omega$ - open set V in (Y,σ,λ).

3. $\zeta\delta\hat{g}$ - CONTINUOUS FUNCTIONS

Definition 3.1

A function $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ is said to be $\zeta\delta\hat{g}$ - continuous if $f^{-1}(V)$ is $\zeta\delta\hat{g}$ - closed in (X,τ^{δ},ζ) for every closed set V in (Y,σ).

Theorem 3.2.

If a map $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ is δ - continuous then it is $\zeta\delta\hat{g}$ - continuous but not conversely.

Proof: Let $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ be δ - continuous. Let V be any closed set in Y. Then inverse image $f^{-1}(V)$ is δ - closed in X. Since every δ - closed set is $\zeta\delta\hat{g}$ - closed, $f^{-1}(V)$ is $\zeta\delta\hat{g}$ - closed in X. Therefore f is $\zeta\delta\hat{g}$ - continuous.

Example 3.3

Let $X=Y=\{a,b,c\}$ with topologies $\tau=\{\phi,\{a\},\{b,c\},X\}$ and $\sigma=\{\phi,\{a,c\},Y\}$ and grill $\zeta=\{\{a\},\{c\},\{a,b\},\{b,c\},\{a,c\},X\}$. Then the $\zeta\delta\hat{g}$ - closed sets are $\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},X\}$. Let $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ be the identity map. Then the inverse image of every closed set in Y is $\zeta\delta\hat{g}$ - closed in X. Hence f is $\zeta\delta\hat{g}$ - continuous but not δ - continuous.

Theorem 3.4

If a map $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ is regular - continuous then it is $\zeta\delta\hat{g}$ - continuous but not conversely.

Proof: Let $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ be regular - continuous. Let V be any closed set in Y. Then inverse image $f^{-1}(V)$ is regular - closed in X. Since every regular - closed set is $\zeta\delta\hat{g}$ - closed, $f^{-1}(V)$ is $\zeta\delta\hat{g}$ - closed in X. Therefore f is $\zeta\delta\hat{g}$ - continuous.

Example 3.5

The converse is false as shown in example 3.3.

Theorem 3.6

If $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ is $\zeta\delta\hat{g}$ - continuous then it is ζg - continuous but not conversely.

Proof: Let V be any closed set in Y. Since V is $\zeta\delta\hat{g}$ - continuous then $f^{-1}(V)$ is $\zeta\delta\hat{g}$ - closed in X. Since every $\zeta\delta\hat{g}$ - closed set is ζg - closed, then $f^{-1}(V)$ is $\zeta\delta\hat{g}$ - closed in X. Hence f is ζg - continuous.

Example 3.7

Let $X = Y = \{a,b,c\}$ with topologies $\tau = \{\phi,\{a\},\{b\},\{a,b\},\{b,c\},X\}$, $\sigma = \{\phi,\{a\},\{b,c\},Y\}$ and grill $\zeta = \{X,\{a\},\{c\},\{a,b\},\{b,c\},\{a,c\}\}$, $\zeta\delta\hat{g}$ - closed set are $\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},X\}$, ζg - closed sets are $\{\phi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},X\}$. Let $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ is defined by $f(a)=c, f(b)=b, f(c)=a$ then f is ζg - continuous but not $\zeta\delta\hat{g}$ - continuous as the inverse image of a closed set $\{a\}$ in Y is $\{c\}$ is not $\zeta\delta\hat{g}$ - closed in X.

Remark 3.8

The concept of continuous, ω - continuous, $\zeta\omega$ - continuous, δg^{*} - continuous, δg - continuous are independent of $\zeta\delta\hat{g}$ - continuous as it follows from example 3.3.

Remark 3.9

From the above discussions and the known result we have the following implications

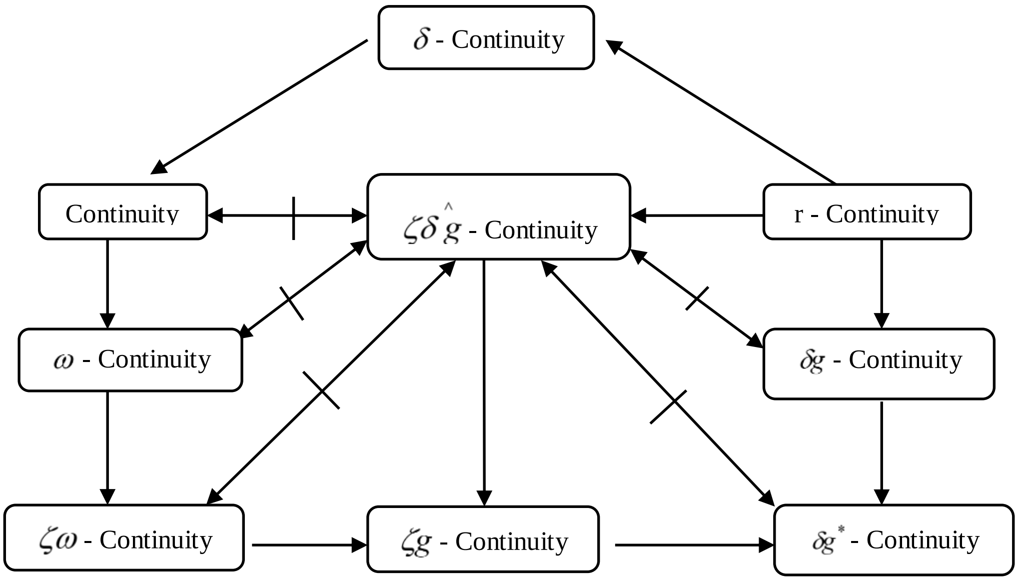

Figure 3.10

Theorem 3.11

For any function $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ the following statements are equivalent.

(i) f is $\zeta\delta\hat{g}$ - continuous.

(ii) The inverse image of each open set in Y is $\zeta\delta\hat{g}$ - open in X.

Proof: Assume that $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ be $\zeta\delta\hat{g}$ - continuous. Let P be open in Y. Then P^{c} is closed in Y. Since f is $\zeta\delta\hat{g}$ - continuous, $f^{-1}(P^{c})$ is $\zeta\delta\hat{g}$ - closed in X. But $f^{-1}(P^{c}) = X - f^{-1}(P)$. Thus $X - f^{-1}(P)$ is $\zeta\delta\hat{g}$ - closed in X and so $f^{-1}(P)$ is $\zeta\delta\hat{g}$ - open in X. Therefore $(i)\Rightarrow(ii)$.

Conversely assume that the inverse image of each open set in Y is $\zeta\delta\hat{g}$ - open in X. Let Q be any closed set in Y. Then Q^{c} is open in Y. By assumption, $f^{-1}(Q^{c})$ is $\zeta\delta\hat{g}$ - open in X. But $f^{-1}(Q^{c}) = X - f^{-1}(Q)$. Thus X - $f^{-1}(Q)$ is $\zeta\delta\hat{g}$ - open in X and so $f^{-1}(Q)$ is $\zeta\delta\hat{g}$ - closed in X. Therefore f is $\zeta\delta\hat{g}$ - continuous. Hence $(ii)\Rightarrow(i)$. Thus *(i)* and *(ii)* are equivalent.

4. CONTRA $\zeta\delta\hat{g}$ - CONTINUOUS FUNCTIONS

In this section, the concept of contra $\zeta\delta\hat{g}$ - continuous functions in grill delta topological spaces have been characterized.

Definition 4.1

A function $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ is called contra $\zeta\delta\hat{g}$ - continuous if $f^{-1}(V)$ is $\zeta\delta\hat{g}$ - closed in (X,τ^{δ},ζ) for every open set V in (Y,σ).

Remark 4.2

The concept of $\zeta\delta\hat{g}$ - continuity and contra $\zeta\delta\hat{g}$ - continuity are independent as shown in the following example.

Example 4..3

Let $X=Y=\{a,b,c\}$ with topologies $\tau=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},X\}$, $\sigma=\{\phi,\{a\},\{a,b\},Y\}$ and $\zeta=\{\{a\},\{c\},\{a,b\},\{b,c\},\{a,c\},X\}$. Let $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ be the identity function. Clearly f is contra $\zeta\delta\hat{g}$ - continuous but not $\zeta\delta\hat{g}$ - continuous. Since $f^{-1}\{c\}=\{c\}$ is not $\zeta\delta\hat{g}$ - closed in (X,τ^{δ},ζ). Therefore f is not $\zeta\delta\hat{g}$ - continuous.

Example 4.4

Let $X=Y=\{a,b,c\}$ with topologies $\tau=\{\phi,\{b\},\{a,b\},X\}$, $\sigma=\{\phi,\{b\},\{b,c\},Y\}$ and grill $\zeta=\{\{b\},\{b,c\},\{a,c\},X\}$. Let $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ be the identity function. Clearly f is $\zeta\delta\hat{g}$ - continuous but not contra $\zeta\delta\hat{g}$ - continuous because $f^{-1}\{b\}=\{b\}$ is not $\zeta\delta\hat{g}$ - closed in (X,τ^{δ},ζ) where $\{b\}$ is open in Y.

Remark 4.5

The concept of contra - continuous and contra $\zeta\delta\hat{g}$ - continuous are independent as shown in the following example.

Example 4.6

Let $X=Y=\{a,b,c\}$ with topologies $\tau=\{\phi,\{c\},\{a,c\},\{b,c\},X\}$, $\sigma=\{\phi,\{a\},\{b\},\{a,b\},Y\}$ and $\zeta=\{\{a\},\{c\},\{a,b\},\{b,c\},\{a,c\},X\}$. Let $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ be the identity function. Clearly f is contra - continuous but not contra $\zeta\delta\hat{g}$ - continuous. Since $f^{-1}\{a\}=\{a\}$ is not $\zeta\delta\hat{g}$ - closed in (X,τ^{δ},ζ) where $\{a\}$ is open in (Y,σ).

Example 4.7

Let $X=Y=\{a,b,c\}$ with topologies $\tau=\{\phi,\{c\},\{b,c\},X\}$, $\sigma=\{\phi,\{a,c\},Y\}$ with grill $\zeta=\{\{b\},\{c\},\{b,c\},\{a,c\},X\}$. Let $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma)$ be the identity function. Clearly f is contra $\zeta\delta\hat{g}$ - continuous but not contra - continuous because for the open set $\{a,c\}$ in (Y,σ), $f^{-1}\{a,c\}=\{a,c\}$ is not closed in (X,τ^{δ},ζ).

Remark 4.8

The composition of two contra $\zeta\delta\hat{g}$ - continuous functions need not be contra $\zeta\delta\hat{g}$ -continuous as shown in the following example.

Example 4.9

Let $X=Y=Z=\{a,b,c\}$ with topologies $\tau=\{\phi,\{c\},\{a,b\},X\}$, $\sigma=\{\phi,\{b\},\{c\},\{b,c\},Y\}$ $\eta=\{\phi,\{a\},\{b\},\{a,b\},\{a,c\},Z\}$ with $\zeta=\{\{a\},\{c\},\{a,b\},\{b,c\},\{a,c\},X\}$. Let $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma^{\delta},\lambda)$ and $g:(Y,\sigma^{\delta},\lambda)\to(Z,\eta^{\delta},\mu)$ be two identity functions. Then both f and g are contra $\zeta\delta\hat{g}$ - continuous but $g\circ f:(X,\tau^{\delta},\zeta)\to(Z,\eta^{\delta},\mu)$ is not contra $\zeta\delta\hat{g}$ -continuous. Since $(g\circ f)^{-1}\{a\}=\{a\}$ is not $\zeta\delta\hat{g}$ - closed in (X,τ^{δ},ζ) where $\{a\}$ is open in (Z,η^{δ},μ).

Theorem 4.10

If $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma^{\delta},\lambda)$ is $\mathcal{G}\delta\hat{g}$ - irresolute and $g:(Y,\sigma^{\delta},\lambda)\to(Z,\eta^{\delta},\mu)$ is contra $\mathcal{G}\delta\hat{g}$ - continuous functions. Then $g\circ f:(X,\tau^{\delta},\zeta)\to(Z,\eta^{\delta},\mu)$ is contra $\mathcal{G}\delta\hat{g}$ - continuous.

Proof: Let V be open in (Z,η^{δ},μ). Since g is contra $\mathcal{G}\delta\hat{g}$ - continuous, $g^{-1}(V)$ is $\mathcal{G}\delta\hat{g}$ - closed in $(Y,\sigma^{\delta},\lambda)$. Since f is $\mathcal{G}\delta\hat{g}$ - irresolute, $f^{-1}(g^{-1}(V))$ is $\mathcal{G}\delta\hat{g}$ - closed in (X,τ^{δ},ζ). That is $(g\circ f)^{-1}(V)$ is $\mathcal{G}\delta\hat{g}$ - closed in (X,τ^{δ},ζ). Hence $(g\circ f)$ is contra $\mathcal{G}\delta\hat{g}$ - continuous.

5. $\mathcal{G}\delta\hat{g}$ - IRRESOLUTE FUNCTIONS

In this section, $\mathcal{G}\delta\hat{g}$ - irresolute function in grill delta topological spaces is introduced and some of their properties are investigated.

Definition 5.1

A function $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma^{\delta},\lambda)$ is said to be $\mathcal{G}\delta\hat{g}$ - irresolute if the inverse image of every $\mathcal{G}\delta\hat{g}$ - closed set in $(Y,\sigma^{\delta},\lambda)$ is $\mathcal{G}\delta\hat{g}$ - closed set in (X,τ^{δ},ζ).

Theorem 5.2

A map $f:(X,\tau^{\delta},\zeta)\to(Y,\sigma^{\delta},\lambda)$ is $\mathcal{G}\delta\hat{g}$ - irresolute iff the inverse image of every $\mathcal{G}\delta\hat{g}$ - open in $(Y,\sigma^{\delta},\lambda)$ is $\mathcal{G}\delta\hat{g}$ - open in (X,τ^{δ},ζ).

Proof: Assume f is $\mathcal{G}\delta\hat{g}$ - irresolute. Let B be any $\mathcal{G}\delta\hat{g}$ - open in $(Y,\sigma^{\delta},\lambda)$. Then B^{c} is $\mathcal{G}\delta\hat{g}$ - closed in $(Y,\sigma^{\delta},\lambda)$. Since f is $\mathcal{G}\delta\hat{g}$ - irresolute, $f^{-1}(B)$ is $\mathcal{G}\delta\hat{g}$ - closed in (X,τ^{δ},ζ). But $f^{-1}(B^{c})= X-f^{-1}(B)$ and so $f^{-1}(B)$ is $\mathcal{G}\delta\hat{g}$ - open set in X. Hence the inverse image of every $\mathcal{G}\delta\hat{g}$ - open in $(Y,\sigma^{\delta},\lambda)$ is $\mathcal{G}\delta\hat{g}$ - open in (X,τ^{δ},ζ).

Conversely, assume that the inverse image of every $\mathcal{G}\delta\hat{g}$ - open set in $(Y,\sigma^{\delta},\lambda)$ is $\mathcal{G}\delta\hat{g}$ - open in X. Let B be any closed set in $(Y,\sigma^{\delta},\lambda)$. Then B^{c} is $\mathcal{G}\delta\hat{g}$ - open in $(Y,\sigma^{\delta},\lambda)$. By assumption $f^{-1}(B^{c})$ is $\mathcal{G}\delta\hat{g}$ - open set in (X,τ^{δ},ζ). But $f^{-1}(B^{c})=X-f^{-1}(B)$ and so $f^{-1}(B)$ is $\mathcal{G}\delta\hat{g}$ - closed set in X. Therefore f is $\mathcal{G}\delta\hat{g}$ - irresolute.

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