IJNRD.ORG ISSN : 2456-4184

International Research Journal
IJNRD
Research Through Innovation

INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

Radio Mean Gd-distance Number Of Some Cycle Related Graphs

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Abstract

A Radio Mean Gd-distance labeling of a connected graph G is an injective map f from the vertex set V(G) to ℕ such that for two distinct vertices u and v of G, $d^{Gd}(u,v)+\left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 1+diam^{Gd}(G)$, where $d^{Gd}(u,v)$ denotes the Gd-distance between u and v and $diam^{Gd}(G)$ denotes the Gd-diameter of G. The Radio Mean Gd-distance number of f, $rmn^{Gd}(f)$ is the maximum label assigned to any vertex of G. The Radio Mean Gd-distance number of G, $rmn^{Gd}(G)$ is the minimum value f of G. In this paper we find the radio mean Gd-distance number of some cycle related graphs.

Keywords: Gd-distance, Radio Mean Gd-distance, Radio mean Gd-distance number.

Introduction

By a graph G = (V(G), E(G)) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.

The Gd-distance was introduced by V. Maheswari and M. Joice Mabel. If u and v are vertices of a connected graph G, Gd-length of a u-v path is defined as $d^{Gd}(u,v)=d(u,v)+\deg(u)+\deg(v)$. The Gd-radius, denoted by $r^{Gd}(G)=min\{e^{Gd}(v): v\in V(G)\}$. Similarly the Gd-diameter $d^{Gd}(G)=max\{e^{Gd}(v): v\in V(G)\}$. We observe that for any two vertices u and v of G we have d(u, v) $\leq d^{Gd}(u,v)$. The equality holds if and only if u, v are identical. If G is any connected graph, then the d^{Gd} distance is a metric on the set of vertices of G. We can check easily that for any non-trivial connected graph, $r^{Gd}(G)\leq d^{Gd}(G)\leq 2r^{Gd}(G)$.The lower bound is clear from the definition and the upper bound follows from the triangular inequality.

Radio mean labeling was introduced by R. Ponraj et al. [17,18]. A radio mean labeling is a one to one mapping f from V(G) to ℕ satisfying the condition $d(u,v)+\left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 1+diam(G)$ for every u, v ∈ V(G). The span of labeling f is the maximum integer that f maps to a vertex of G. The radio mean number of G, $rmn(G)$ is the lowest span taken over all radio mean labelings of the graph G.

In this paper, we introduced the concept of radio mean Gd-distance labeling of a graph G. Radio mean Gd-distance labeling is a function f from V(G) to ℕ satisfying the condition $d^{Gd}(u,v)+\left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 1+diam^{Gd}(G)$, where $diam^{Gd}(G)$ is the Gd-distance diameter of G. A Gd-distance radio labeling number of G is the maximum label assigned to any vertex of G. It is denoted by $rmn^{Gd}(G)$.

Radio labeling can be regarded as an extension of distance-two labeling which is motivated by the channel assignment problem introduced by W. K. Hale [6]. G. Chartrand et al.[2] introduced the concept of radio labeling of graph.Also G. Chartrand et al.[3] gave the upper bound for the radio number of path. The exact value for the radio number of path and cycle was given by Liu and Zhu [9]. However G. Chartrand et al.[2] obtained different values for them. They found the lower and upper bound for the radio number of cycle. Liu [8] gave the lower bound for the radio number of Tree. M. M. Rivera et al. [22] gave the radio number of $C_n\times C_n$, the Cartesian product of C_n. In [4] C. Fernandez et al.found the radio number for Complete graph, Star graph, Complete Bipartite graph, Wheel graph and Gear graph. M.T Rahim and I. Tomescu[19] investigated the radio number of helm graph. In this paper, we determine the radio mean Gd-distance number of some cycle related graphs.

2. Main Result

Theorem 2.1

The radio mean Gd-distance number of cycle graph

$$rmn^{Gd}(C_n) \leq \begin{cases} \frac{n}{2} + (n-2) & if\ n\ is\ even,\ \ n \geq 6 \\ \frac{n-1}{2} + (n-2) & if\ n\ is\ odd,\ \ n \geq 7 \end{cases}$$

Proof.

Let V$(C_n) = \{v_1, v_2, \dots, v_n\}$ be the vertex set and
E$(C_n) = \{v_i v_{i+1}, v_n v_1\ ; 1 \leq i \leq n-1\}$ be the edge set

Then, $d^{Gd}(v_i, v_{i+1}) = 5, 1 \leq i \leq n,\ d^{Gd}\left(v_1, v_{\frac{n+2}{2}}\right) = \frac{n}{2} + 4$, if n is even, $n \geq 6$,

$d^{Gd}\left(v_1, v_{\frac{n+1}{2}}\right) = \frac{n-1}{2} + 4$, if n is odd, $n \geq 7$

So, $diam^{Gd}(C_n) = \begin{cases} \frac{n}{2} + 4 & if\ n\ is\ even,\ \ n \geq 6 \\ \frac{n-1}{2} + 4 & if\ n\ is\ odd,\ \ n \geq 7 \end{cases}$

Without loss of generality $f(v_1) < f(v_1) < \cdots < f(v_n)$
We shall check the radio mean Gd-distance condition
$d^{Gd}(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^{Gd}(G)$ for every pair of vertices (u,v); $u \neq v$

Case 1. If n is even, $n \geq 6$
Fix $f(v_1) = \frac{n}{2} - 1$, for (v_i, v_{i+1}) $1 \leq i \leq n-1$
$d^{Gd}(v_1, v_2) + \left\lceil \frac{f(v_1)+f(v_2)}{2} \right\rceil \geq 5 + \left\lceil \frac{\frac{n}{2}-1+f(v_2)}{2} \right\rceil \geq \frac{n}{2} + 4 + 1$
$\frac{n}{2} - 1 + f(v_2) \geq n-2$, therefore $f(v_2) = \frac{n}{2}$

for (v_2, v_3)
$d^{Gd}(v_2, v_3) + \left\lceil \frac{f(v_2)+f(v_3)}{2} \right\rceil \geq 5 + \left\lceil \frac{\frac{n}{2}+f(v_3)}{2} \right\rceil \geq \frac{n}{2} + 4 + 1$
$\frac{n}{2} + f(v_3) \geq n-2$, therefore $f(v_3) = \frac{n}{2} + 1$
$\therefore f(v_i) = \dfrac{n}{2} + i - 2, \quad 1 \leq i \leq n$

Case 2. If n is odd, $n \geq 7$
Fix $f(v_1) = \frac{n-1}{2} - 1$, for (v_i, v_{i+1}) $1 \leq i \leq n-1$
$d^{Gd}(v_1, v_2) + \left\lceil \frac{f(v_1)+f(v_2)}{2} \right\rceil \geq 5 + \left\lceil \frac{\frac{n-1}{2}-1+f(v_2)}{2} \right\rceil \geq \frac{n-1}{2} + 4 + 1$
$\frac{n-3}{2} + f(v_2) \geq n-3$, therefore $f(v_2) = \frac{n-1}{2}$

for (v_2, v_3)
$d^{Gd}(v_2, v_3) + \left\lceil \frac{f(v_2)+f(v_3)}{2} \right\rceil \geq 5 + \left\lceil \frac{\frac{n-1}{2}+f(v_3)}{2} \right\rceil \geq \frac{n-1}{2} + 4 + 1$
$\frac{n-1}{2} + f(v_3) \geq n-2$, therefore $f(v_3) = \frac{n-1}{2} + 1$
$\therefore f(v_i) = \dfrac{n-1}{2} + i - 2, \quad 1 \leq i \leq n$

Hence, $rmn^{Gd}(C_n) \leq \begin{cases} \frac{n}{2} + (n-2) & if\ n\ is\ even,\ \ n \geq 6 \\ \frac{n-1}{2} + (n-2) & if\ n\ is\ odd,\ \ n \geq 7 \end{cases}$

Note. $rmn^{Gd}(C_n) = n\ if\ 3 \leq n \leq 5$

Theorem 2.2

The radio mean Gd-distance number of wheel graph $rmn^{Gd}(W_{1,n}) \leq 2n - 4,\ \ n \geq 5$

Proof.

Let V$(W_{1,n}) = \{v_0, v_1, v_2, \dots, v_n\}$ be the vertex set, where v_0 be the apex vertex and
E$(W_{1,n}) = \{v_0 v_i, v_i v_{i+1}, v_n v_1\ ; 1 \leq i \leq n\}$ be the edge set
Then, $d^{Gd}(v_0, v_i) = n + 4,\ 1 \leq i \leq n;\ d^{Gd}(v_i, v_{i+1}) = 7, 1 \leq i \leq n,\ d^{Gd}(v_i, v_j) = 8, 1 \leq i, j \leq n,\ i \neq j$

So, $diam^{Gd}\left(W_{1,n}\right) = n+4$

Without loss of generality $f(v_0) < f(v_1) < \cdots < f(v_n)$
We shall check the radio mean Gd-distance condition
$d^{Gd}(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^{Gd}(G) = n+5$ for every pair of vertices (u,v); $u \neq v$

Case 1. Fix $f(v_0) = n-4$ for $(v_0, v_i), 1 \leq i \leq n$

$$d^{Gd}(v_0, v_1) + \left\lceil \frac{f(v_0)+f(v_1)}{2} \right\rceil \geq \text{n} + 4 + \left\lceil \frac{n-4+\ f(v_1)}{2} \right\rceil \geq n+5$$

$n-4+\ f(v_1) \geq 0$, therefore $f(v_1) = n-3$

Case 2. For any adjacent vertices $(v_i, v_{i+1}),\ 1 \leq i \leq n$

$$d^{Gd}(v_1, v_2) + \left\lceil \frac{f(v_1)+f(v_2)}{2} \right\rceil \geq 7 + \left\lceil \frac{n-3+f(v_2)}{2} \right\rceil \geq n+5$$

$n-3+f(v_2) \geq 2n-6$, therefore $f(v_2) = n-2$

Case 3. For any pair of non-adjacent vertices $(v_i, v_j),\ i \neq j\ , 1 \leq i,j \leq n$

$$d^{Gd}(v_1, v_3) + \left\lceil \frac{f(v_1)+f(v_3)}{2} \right\rceil \geq 8 + \left\lceil \frac{n-3+f(v_3)}{2} \right\rceil \geq n+5$$

$n-3+f(v_3) \geq 2n-8$, therefore $f(v_3) = n-1$
$\therefore f(v_i) = n+i-4,\ 1 \leq i \leq n$

Hence, $rmn^{Gd}(W_{1,n}) \leq\ 2n-4,\ \ n \geq 5$

Note. $rmn^{Gd}(W_{1,n}) = n+1\ \ if\ 1 \leq n \leq 4$

* The graph $C_n^{(t)}$ denoted the one point union of t copies of cycle C_n. The graph $C_3^{(t)}$or $K_3^{(t)}$ is called friendship graph

Theorem 2.3

The radio mean Gd-distance number of friendship graph $rmn^{Gd}(C_3^{(t)}) \leq 4t-4;\ t \geq 3$

Proof.

Let $V(C_3^{(t)}) = \{v_0, v_{1,}v_2, \dots, v_{t,}v_{t+1}, \dots, v_{2t}\}$ be the vertex set, where v_0 be the central vertex and $\mathrm{E}(C_3^{(t)}) = \{v_0v_i\ ; 1 \leq i \leq 2t\ and\ v_iv_{t+i}; 1 \leq i \leq t\}$ be the edge set
Then,$d^{Gd}(v_0, v_i) = 2t+3,\ d^{Gd}(v_{\mathrm{I}}, v_{t+i}) = 5; 1 \leq i \leq t,$

for non adjacent vertices $\left(v_i, v_j\right),\ d^{Gd}\left(v_i, v_j\right) = 6; 1 \leq i,j \leq 2t, i \neq j$

It is clear that $diam^{Gd}\left(C_3^{(t)}\right) = 2t+3$

Without loss of generality, $f(v_0) < f(v_1) < \cdots < f(v_t) < f(v_{t+1}) < \cdots < f(v_{2t})$
We shall check the radio mean Gd-distance condition
$d^{Gd}(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^{Gd}(G) = 2t+4$ for every pair of vertices (u,v); $u \neq v$

Case 1. Fix $f(v_0) = 2t-4$, for $(v_0, v_i),\ 1 \leq i \leq 2t$

$$d^{Gd}(v_0, v_1) + \left\lceil \frac{f(v_0)+f(v_1)}{2} \right\rceil \geq 2\text{t} + 3 + \left\lceil \frac{2t-4+f(v_1)}{2} \right\rceil \geq 2t+4$$

$2t-4+f(v_1) \geq 0$, therefore $f(v_1) = 2t-3$

Case 2. For any non adjacent vertices (v_i, v_j) where $|i-j| > 1$

$$d^{Gd}(v_1, v_2) + \left\lceil \frac{f(v_1)+f(v_2)}{2} \right\rceil \geq 6 + \left\lceil \frac{2t-3+f(v_2)}{2} \right\rceil \geq 2t+4$$

$2t-3+f(v_2) \geq 4t-6$, therefore $f(v_2) = 2t-2$
$\therefore f(v_i) = 2t+i-4,\ 1 \leq i \leq t$

Case 3. For both $v_i\ and\ v_j$ are adjacent, $1 \leq i,j \leq 2t$ where $|i-j| = t$

$$d^{Gd}(v_1, v_{t+1}) + \left\lceil \frac{f(v_1)+f(v_{t+1})}{2} \right\rceil \geq 5 + \left\lceil \frac{2t-3+f(v_{t+1})}{2} \right\rceil \geq 2t+4$$

$2t-3+f(v_{t+1}) \geq 4(t-1)$, therefore $f(v_{t+1}) = 3t-3$
$\therefore f(v_{t+i}) = 3t+i-4\ 1 \leq i \leq t$

Hence $rmn^{Gd}(C_3^{(t)}) \leq 4t-4;\ t \geq 3$

Theorem 2.4

The radio mean Gd-distance number of fan graph $rmn^{Gd}(F_n) \leq 2n - 3,\ n \geq 4$

Proof.

let V$(F_n) = \{v_{0,}\ v_{1,}, \dots, v_n\}$ be the vertex set and E$(F_n) = \{v_0 v_i\,, v_i v_{i+1};\ 1 \leq i \leq n\}$ be the edge set

Then, $d^{Gd}(v_0, v_1) = d^{Gd}(v_0, v_n) = n + 3,\ d^{Gd}(v_0, v_i) = n + 4, 2 \leq i \leq n - 1,$

$d^{Gd}(v_1, v_2) = d^{Gd}(v_{n-1}, v_n) = d^{Gd}(v_1, v_n) = 6,\ d^{Gd}(v_i, v_{i+1}) = 7, 2 \leq i \leq n - 2$

So, $diam^{Gd}(F_n) = n + 4$

Without loss of generality $f(v_0) < f(v_1) < \cdots < f(v_n)$

We shall check the radio mean Gd-distance condition

$d^{Gd}(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^{Gd}(G) = n + 5$ for every pair of vertices (u,v); $u \neq v$

Fix $f(v_0) = n - 3$, for $(v_0, v_i),\ i = 1, n$

$$d^{Gd}(v_0, v_1) + \left\lceil \frac{f(v_0) + f(v_1)}{2} \right\rceil \geq \mathrm{n} + 3 + \left\lceil \frac{n - 3 + f(v_1)}{2} \right\rceil \geq n + 5$$

$n - 3 + f(v_1) \geq 2$, therefore $f(v_1) = n - 2$

For (v_1, v_2),

$$d^{Gd}(v_1, v_2) + \left\lceil \frac{f(v_1) + f(v_2)}{2} \right\rceil \geq 6 + \left\lceil \frac{n - 2 + f(v_2)}{2} \right\rceil \geq n + 5$$

$n - 2 + f(v_2) \geq 2n - 4$, therefore $f(v_2) = n - 1$

For any adjacent vertices $(v_i, v_{j=i+1}),\ 2 \leq i \leq n - 2$

$$d^{Gd}(v_2, v_3) + \left\lceil \frac{f(v_2) + f(v_3)}{2} \right\rceil \geq 7 + \left\lceil \frac{n - 1 + f(v_3)}{2} \right\rceil \geq n + 5$$

$n - 1 + f(v_3) \geq 2n - 6$, therefore $f(v_3) = n$

$\therefore f(v_i) = n + i - 3, 1 \leq i \leq n$

Hence $rmn^{Gd}(F_n) \leq 2n - 3 \quad n \geq 4$

Note. $rmn^{Gd}(F_n) \leq n + 1 \quad n = 3$

Theorem 2.5

The radio mean Gd-distance number of gear graph $rmn^{Gd}(G_n) \leq 3n - 5,\ n \geq 6$

Proof.

Let V$(G_n) = \{v_{0,}\ v_{1,} v_2, \dots, v_n\}$ and $\{u_{1,} u_2, \dots, u_n\}$ be the vertex set, where v_0 be the central vertex and E$(G_n) = \{v_0 v_i\,, v_i u_i, u_i v_{i+1}\,;1 \leq i \leq n\}$ be the edge set

Then, $d^{Gd}(v_0, v_i) = d^{Gd}(v_0, u_i) = n + 4,\ d^{Gd}(v_i, u_i) = 6;\ 1 \leq i \leq n, d^{Gd}(v_i, v_j) = 8; 1 \leq i, j \leq n, i \neq j,$ $d^{Gd}(v_i, u_j) = 8;\ 1 \leq i, j \leq n, i \neq j$

It is clear that $diam^{Gd}(G_n) = n + 4$

Without loss of generality $f(v_0) < f(v_1) < f(v_2) < \cdots < f(v_n) < f(u_1) < \cdots < f(u_n)$

We shall check the radio mean Gd-distance condition

$d^{Gd}(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^{Gd}(G) = n + 5$ for every pair of vertices (u,v); $u \neq v$

Case 1. Fix $f(v_0) = n - 5$, for adjacent vertices $(v_0, v_i)\ 1 \leq i \leq n$

$$d^{Gd}(v_0, v_1) + \left\lceil \frac{f(v_0) + f(v_1)}{2} \right\rceil \geq \mathrm{n} + 4 + \left\lceil \frac{n - 5 + f(v_1)}{2} \right\rceil \geq n + 5$$

$n - 5 + f(v_1) \geq 0$, therefore $f(v_1) = n - 4$

Case 2. For non adjacent vertices $(v_i, v_j)\ 1 \leq i, j \leq n,\ i \neq j$

$$d^{Gd}(v_1, v_2) + \left\lceil \frac{f(v_1) + f(v_2)}{2} \right\rceil \geq 8 + \left\lceil \frac{n - 4 + f(v_2)}{2} \right\rceil \geq n + 5$$

$n - 4 + f(v_2) \geq 2n - 8$, therefore $f(v_2) = n - 3$

$\therefore f(v_i) = n + i - 5,\ 1 \leq i \leq n$

Case 3. For adjacent vertices $(v_i, u_i)\ 1 \leq i \leq n$

$$d^{Gd}(v_1, u_1) + \left\lceil \frac{f(v_1) + f(u_1)}{2} \right\rceil \geq 6 + \left\lceil \frac{n - 4 + f(u_1)}{2} \right\rceil \geq n + 5$$

$n - 4 + f(u_1) \geq 2n - 4$, therefore $f(u_1) = 2n - 4$

Case 4. For non adjacent vertices $(v_i, u_j)\ 1 \leq i, j \leq n, i \neq j$

$$d^{Gd}(v_1,u_2)+\left\lceil\frac{f(v_1)+f(u_2)}{2}\right\rceil \geq 8+\left\lceil\frac{n-4+f(u_2)}{2}\right\rceil \geq n+5$$

$n-4+f(u_2) \geq 2n-8$, therefore $f(u_2)=2n-3$
$\therefore f(u_i)=2n+i-5,$

Hence $rmn^{Gd}(G_n) \leq 3n-5, n \geq 6$

Note. $rmn^{Gd}(G_n)= n+5,\ n=4,5$

Theorem 2.6

The radio mean Gd-distance number of helm graph $rmn^{Gd}(H_n) \leq 3n-5,\ n \geq 7$

Proof.

Let V$(H_n)=\{v_{0,}\ v_{1,}v_2,\dots,v_n,u_{1,}u_2,\dots,u_n\}$ be the vertex set, where v_0 be the central vertex and E$(H_n)=\{v_0v_i\,,v_iu_i,v_iv_{i+1}\,;1 \leq i \leq n\}$ be the edge set

Then, $d^{Gd}(v_0,v_i)=$ n $+5; 1 \leq i \leq n,\ \ d^{Gd}(v_0,u_i)=n+3; 1 \leq i \leq n,\ \ d^{Gd}(v_i,u_i)=6; 1 \leq i \leq n,$ $d^{Gd}(v_i,v_j)=10;\ 1 \leq i,j \leq n,\ i \neq j,\ \ d^{Gd}(u_i,u_j)=6;\ 1 \leq i,j \leq n, i \neq j,\ \ d^{Gd}(v_i,v_{i+1})=9; 1 \leq i \leq n-1,\ d^{Gd}(u_i,u_{i+1})=5; 1 \leq i \leq n-1$
It is clear that $diam^{Gd}(H_n)=n+5$
Without loss of generality $f(v_0)<\ f(v_1)\ <\ f(v_2)<\cdots<f(v_n)<f(u_1)<\cdots<f(u_n)$
We shall check the radio mean Gd-distance condition
$d^{Gd}(u,v)+\left\lceil\frac{f(u)+f(v)}{2}\right\rceil \geq 1+diam^{Gd}(G)=n+6$ for every pair of vertices (u,v); $u \neq v$

Case 1. Fix $f(v_0)=n-5$, for adjacent vertices $(v_0,v_i)\ \ 1 \leq i \leq n$

$$d^{Gd}(v_0,v_1)+\left\lceil\frac{f(v_0)+f(v_1)}{2}\right\rceil \geq \mathrm{n}+5+\left\lceil\frac{n-5+f(v_1)}{2}\right\rceil \geq n+6$$

$n-5+f(v_1) \geq 0$, therefore $f(v_1)=n-4$

Case 2. For adjacent vertices $(v_{\mathrm{i}},v_{i+1})\ \ 1 \leq i \leq n-1$

$$d^{Gd}(v_1,v_2)+\left\lceil\frac{f(v_1)+f(v_2)}{2}\right\rceil \geq 9+\left\lceil\frac{n-4+f(v_2)}{2}\right\rceil \geq n+6$$

$n-4+f(v_2) \geq 0$, therefore $f(v_2)=n-3$
$\therefore f(v_i)=n+i-5; 1 \leq i \leq n$

Case 3. For adjacent vertices $(u_{\mathrm{i}},v_i)\ \ 1 \leq i \leq n$

$$d^{Gd}(v_1,u_1)+\left\lceil\frac{f(v_1)+f(u_1)}{2}\right\rceil \geq 6+\left\lceil\frac{n-4+f(u_1)}{2}\right\rceil \geq n+6$$

$n-4+f(u_1) \geq 2n-1$, therefore $f(u_1)=2n-4$

Case 4. For non adjacent vertices $(u_{\mathrm{i}},u_{i+1})\ \ 1 \leq i \leq n-1$

$$d^{Gd}(u_1,u_2)+\left\lceil\frac{f(u_1)+f(u_2)}{2}\right\rceil \geq 5+\left\lceil\frac{2n-4+f(u_2)}{2}\right\rceil \geq n+6$$

$2n-4+f(u_2) \geq 0$, therefore $f(u_2)=2n-3$
$\therefore f(u_i)=2n+i-5; 1 \leq i \leq n$

Hence, $rmn^{Gd}(H_n) \leq 3n-5,\ n \geq 7$

Note. $rmn^{Gd}(H_n)=\begin{cases} n+5\,; & n=3{,}4 \\ 3n-4\,; & n=5{,}6 \end{cases}$

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