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Review on Non-Linear Gravitational Galaxy Clustering in the Expanding Universe: A Thermodynamic Approach

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Abstract

In this paper a thermodynamic approach for non linear gravitational galaxy clustering in the expanding universe has been evaluated and implemented. Different techniques for understanding the clustering of galaxies have also been discussed. A two-point correlation function along with the thermodynamic quantities such as temperature and pressure has been used for the study and research. A Thermodynamic Parameter ‘*b*’ measures the influence of two particle correlation function. The value of ‘*b*’ depends on the two-point correlation function which intern is a function of density n and temperature *T* of the system in a grand **canonical** ensemble. As it is valuable to understand the functional form of *b (n,T)* and derive its expression in terms of *n* and *T* from equation of state. In order to evaluate ‘*b*’ a partial differential equation is developed by using laws of thermodynamics in combination with equation of state taking gravitational interaction between particles into consideration. The solution of the equation satisfies a set of boundary conditions for correlation systems and provides new ways for understanding gravitational clustering in expanding universe.

***Key words**: distribution of matter, homogeneous, inhomogeneous, Gravitational clustering, linear, non-linear, two-point correlation function.*

1. Introduction

One of the most important problems of the cosmology is the quantitative description of the distribution of the galaxies in the universe and its physical understanding. We know that the distribution of matter throughout the space can be either homogeneous or inhomogeneous. With increase in volume, the average density decreases and this results homogeneity on larger scale. Inhomegeneity on smaller scales results due to greater density contrasts there. There are different evidences of the departure of homogeneity on scales of ~50 MPC. One of the evidence for the large scale structure is uniformity of cosmic ray background. One considers the early universe to have been completely homogeneous. During its expansion, instabilities arose to produce the structure we see now. The problem with this view is that we do not know any physical instability that arise naturally and are strong enough to do the job in standard cosmological models.

2. Techniques for Clustering of Galaxies:

There are different techniques for understanding the clustering of galaxies such as:

i) Percolation (Stauffer 1979, Grimmett 1989, Babul and Starkman 1992): this method describes the shape and connectivity of clustering in a quantitative way. We start with any galaxy and find its nearest neighbours with a distance say 'S'. Then find that galaxy's nearest neighbour within 'S' and so until we align on galaxy whose nearest neighbour is farther than 'S'. Repeat this with the other galaxies as the starting points until finding the largest number of galaxies in such a "percolation group".

ii) Minimal Spanning Tree (MST) (Pearson and Codes 1995): the method is to connect N galaxies by N-1 straight lines having the shortest total length. This forms a unique tree like structure with no closed circuit. All the possible tree structures are analysed and the structure which gives the minimum value to the sum of the lengths of the interacting lines is choosen. A mimimal spanning tree analysis of the zwicky catalog of observed galaxies shows it to be quite different from a poisson distribution (Barrow, Bhavsar and Sonoda 1985).

iii) Fractal description: this method is useful for characterizing galaxy packing. The fractal dimension describes how the number of areas needed to cover a volume depends on the size of area. The volume occupies by N-galaxies depends on how they are packed.

2.1 Correlation Function:

One of the methods for studying large-scale structure of the universe is correlation function. The correlation analysis was introduced by Neyman, Scott and Shane (1953), for two dimensions and later extended to three dimensions by Kiang and Saslaw (1969). The correlation analysis can be extended to higher order by solving BBGKY-hierarchy equations, but solving these equations is complicated process. To obtain a Correlation function, we have to start with the average density over a very large scale, and then we have to divide the area into many small cells of same size. We then have to find the difference between the density in each cell and the average net density. Multiplying the net densities of both cells in a pair of given separation. The result is the pair correlation as a function of distance. Similarly one can measure the three, four and higher galaxy correlation function of a sample. If there is a clump or cluster in the sample, a density excess in one part of cluster will tend to correlate positively with density excess of nearby parts. Beyond the scale of clump, correlation will tend to be negative as comparison occurs between regions of enhanced density. A uniform random distribution would give zero correlation. Density deficits within holes correlate positively. Thus the scale of clustering is characterized by scale of pair correlation function.

A homogenous system can also be clustered provided the clustering is same everywhere throughout the system. Two point correlation function is very helpful and informative measure of clustering, but since they are from a complete specification of clustering, it is directly related to the observation. For higher order correlations, the BBGKY hierarchy equations are not suitable as it becomes too complicated to handle the equations for high order clustering. This has been discussed by many workers like Saslaw (1972), Inagaki (1976), Fall and Saslaw (1968, 1985). We can measure high order clustering by concentrating on voids where there are no galaxies. An empty region is relatively simple to find and measure. If its size would have contained 'n' galaxies in a smooth distribution, then the hole is related to the 'n+1' galaxy correlation function. This correlation function describes the 'n' missing galaxies and the one galaxy defining the scale of the void. Comparison of void probability with observation provides a powerful test of gravitational clustering theories (Aarseh and Saslaw, 1982).

2.2 Two-Point Correlation Function:

The two-point correlation satisfies a simple power law $\xi = \left[\frac{r}{r_0}\right]^{-1.8}$ upto a distance r=20MPC given by Totsuji and Kihara in 1969. The same power law is also obeyed by cluster-cluster correlation function. The correlation functions for different clusters of galaxies have been measured by a number of workers like Bahcall and Soneria (1983), Cappi and Maurogordato (1992) and others. There are several reasons why it is interesting to measure two point correlation function ξ accurately on large scales. First, in the large scale linear regime of the standard gravitational instability, the correlation function presumes its shape from around the time of recombination, providing a direct clue to the nature of primodial fluctuations. Second the anisotropy of the red shift correlation function in the linear regime yield an estimate of the cosmological density parameter Ω (Kaiser 1987, Hamilton, 1992). Thus, two-point correlation function $\varepsilon(r)$ can be defined in terms of probability of finding a particle (galaxy) in certain volume element. Correlation could grow naturally from gravitational clustering from the initial conditions of galaxy formation sitting in any galaxy, one would find that the number of galaxies in a small volume element *dv* within a radius *r* is not $\bar{n}\ dv$.

A statistical approach to two-point correlation function for nonlinear galaxy clustering has been developed by Saslaw and Hamilton (1984) with the help of gravitational thermodynamics. In gravitation thermodynamics, the value of Parameter *b*, the ratio of gravitational correlation energy (*w*) to twice kinetic energy ($b = \frac{-W}{2K}$) measures the influence of two point correlation function $\varepsilon(r)$. Many attempts were made (Ahmad 1996, Ahmad and Masood 2000) to understand the functional form of *b*. A partial differential equation is developed using laws of thermodynamics (I^{st} and 2^{nd}) in combination with equation of a State. By integrating the developed equation along its characteristics (Ahmed 1996) gives required solution in terms of *n* and *T*. The dependence of *b* on *n* and *T* can only occur in the combination (nT^{-3}). This value is traced on the basis of entropy in an equilibrium state and does not depend on the path by which the system reaches that state and so the entropy must be total differential.

3. Development of differential equation for two point correlation function:

Consider an infinite system of *N* single particles (galaxies) distributed homogeneously in a volume *V* having internal energy *'U'* and pressure *'P'*. The quantity *'b'* which measures the influence of gravitational correlation energy *(W)* is related to the two point correlation function ε (*n*, *t*, *r*) (Saslaw and Hamilton 1984) as given below;

$$b(n,t) = \frac{-w}{2K} = \frac{2\pi G m^2 \bar{n}}{3T} \int \varepsilon(\bar{n}, t, r) r dr \qquad (1)$$

Here $\bar{n} = \frac{\bar{N}}{V}$ is the average number density. The kinetic energy '*K*' of the peculiar motion is related to temperature *T* by:

$$\boldsymbol{K} = \frac{3}{2}\mathrm{NT} = \frac{1}{2}\sum_{i=1}^{N}(\mathbf{M})\mathrm{V}_i^2 \qquad (2)$$

The internal energy *'U'* and pressure *'P'* are related to two-point correlation function (Saslaw 1985) by:

$$U = \frac{3}{2}\mathrm{NT}(1 - 2\mathrm{b} \qquad (3)$$

$$P = \frac{\mathrm{NT}}{\mathrm{V}}\ (1\text{-b}) \qquad (4)$$

Where the value *'b'* which measures the correlation is already given by equation (1)

The thermodynamic parameters *P*, *V*, *T* are related by:

P= *P* (*V*, *T)* and *U*= *U* (*V*, *T)*

For fixed value of the number of particles *N*, the internal energy *U* is related to *V* and *T* by

$$\mathrm{dU} = \left[\frac{\partial \mathrm{U}}{\partial \mathrm{V}}\right]\mathrm{dV} + \left[\frac{\partial \mathrm{U}}{\partial \mathrm{T}}\right]\mathrm{dT} \qquad (5)$$

Combining the above equation with first and second law of thermodynamics and applying the condition that entropy is a perfect differential, leads us to (Lavdav and Lifshitz =1981)

$$\left[\frac{\partial \mathrm{U}}{\partial \mathrm{V}}\right]_{T,N} = T\left[\frac{\partial \mathrm{P}}{\partial \mathrm{T}}\right] - P \qquad (6)$$

Above equation in combination with equation of state derives a partial differential equation in terms of density and temperature of the system; From equation (3) and (4), we get:

$$\left[\frac{\partial \mathrm{P}}{\partial \mathrm{T}}\right] = \frac{N}{V}\left[(1-b) - T\frac{\partial \mathrm{b}}{\partial \mathrm{T}}\right] \quad (7)$$

$$\left[\frac{\partial \mathrm{U}}{\partial \mathrm{V}}\right] = \frac{3}{2}NT * -2\left[\frac{\partial \mathrm{b}}{\partial \mathrm{V}}\right] \quad (8)$$

Substituting equations (4), (7) and (8) in equation (6) **,** we get

$$-3NT\left[\frac{\partial \mathrm{b}}{\partial \mathrm{V}}\right] = T\left[\frac{N}{V}(1-b) - T\left(\frac{\partial \mathrm{b}}{\partial \mathrm{T}}\right)\frac{N}{V}\right] - \frac{NT}{V}(1-b)$$

$$-3NT\left[\frac{\partial \mathrm{b}}{\partial \mathrm{V}}\right] = \frac{NT}{V}(1-b) - T^2\left(\frac{\partial \mathrm{b}}{\partial \mathrm{T}}\right)\frac{N}{V} - \frac{NT}{V}(1-b)$$

$$-3NT\left[\frac{\partial \mathrm{b}}{\partial \mathrm{V}}\right] = -T^2\left(\frac{\partial \mathrm{b}}{\partial \mathrm{T}}\right)\frac{N}{V}$$

$$3\left[\frac{\partial \mathrm{b}}{\partial \mathrm{V}}\right] = \frac{T}{V}\left(\frac{\partial \mathrm{b}}{\partial \mathrm{T}}\right)$$

$$3\left[\frac{\partial \mathrm{b}}{\partial \mathrm{n}}\right]\left[\frac{\partial \mathrm{n}}{\partial \mathrm{V}}\right] = \frac{T}{V}\left(\frac{\partial \mathrm{b}}{\partial \mathrm{T}}\right)$$

We know $\bar{n} = \frac{N}{V}$ thus

$$\left[\frac{\partial \mathrm{n}}{\partial \mathrm{V}}\right] = -\frac{N}{V^2} = -\frac{\bar{n}}{V}$$

$$3\left[\frac{\partial \mathrm{b}}{\partial \mathrm{n}}\right]\left(\frac{-N}{V^2}\right) = \frac{T}{V}\left(\frac{\partial \mathrm{b}}{\partial \mathrm{T}}\right)$$

$$3n\left[\frac{\partial \mathrm{b}}{\partial \mathrm{n}}\right]_T + T\left[\frac{\partial \mathrm{b}}{\partial \mathrm{T}}\right]_n = 0 \quad (9)$$

Which is a required partial differential equation in terms of number density $\bar{n}$ and Temperature *T*.? As the two point correlation function ξ(r) is related to *'b'* (equation-1) will also dependent on the value of *n* and *T* in an expanding universe. Also gravity is binary interaction depends directly on the two particle correlation function and not on higher order correlation.

3.1 Functional form of b(n, T):

The value of *b* depends on the form of two point correlation function ξ (r) which in turn is function of density *n* and Temperature *T* of the system. Thus it is valuable to understand the functional form *b (n, T)* and derive its expression in terms of *n* and *T* from first principle in understanding the non-linear nature of the problem. The functional form of *'b'* has been discussed by number of workers like Saslaw and Hamilton(1984); Sheth (1995a); Saslaw and Fang (1996); Ahmad (1996) and Ahmad Saslaw and Bhat (2002). The partial differential equation can be solved by integrating along its Characteristic solution (Ahmad 1996).

$$\frac{\mathrm{dn}}{3\mathrm{n}} = \frac{\mathrm{dt}}{\mathrm{T}} \quad (10)$$

And admits

$$\mathrm{b(n, T)= f\,(nT^{-3})} \quad (11)$$

As its solution; equation (11) gives different particular solution for *b (n,T)* in terms of nT^{-3}. But we are in need of solution, which satisfies certain boundary condition for clustering under gravitationally various boundary conditions which are based on many physical conditions;

(1) The value of b should be positive in gravitational clustering of galaxies which means when:

$$n\to 0,\ b\to 0 \text{ and } T\to 0\ b\to 1$$

$$n\to\infty,\ \varepsilon\to \text{large } T\to\infty\ b\to o$$

(2) At low temperature and high densities, more strongly bound clusters form such clusters tend to contain more and more galaxies and tend to be close virial equilibrium, suggesting

$$nT^{-3}\to\infty,\ \varepsilon\to \text{large value, therefore } b\to 1$$

$$nT^{-3}\to o,\ \varepsilon\to o \text{ , therefore } b\to o$$

The simplest form of b which has these properties is

$$b=\frac{b_0 nT^{-3}}{1+b_0 nT^{-3}}$$

Where b_0 is a positive dimensionless constant.

3. Conclusion

Many theories have been presented by a number of scholars for understanding Gravitational Galaxy clustering in an expanding universe. In this paper, the two point correlation function has been used for understanding non linear gravitation galaxy clustering in an expanding universe. In gravitational thermodynamics, the value of parameter b, the ratio of gravitational correlation energy (w) to the twice kinetic energy (b=-w/2k) measures the influence of the two point correlation function ε (r). Partial differential equation developed indicates the dependence of two point correlation function (ε) on $\bar{n}$, T. The functional form of ε has been obtained from various physical boundary conditions. The solution of developed equations indicates the dependence of b on the specific combination $\bar{n}T^{-3}$ and therefore, clarifies the earlier result of Saslaw Hamilton (1984). A theoretical analysis of a thermodynamic model along with the correlation function used will lead to a new direction for understanding a large scale structure of the Universe.

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