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FAIR ISOLATE DOMINATION NUMBER OF A GRAPH

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Abstract : A fair isolate dominating set in a graph G is a dominating set S such that all vertices not in S are dominated by the same number of vertices from S. ie) every two vertices not in S have the same number of neighbors in S and <S> has atleast one isolated vertex. The fair isolate domination number$\gamma_{0_{fd}}(G)$, of G is the minimum cardinality of a fair dominating set.In this paper we investigate the properties of the graphs for which the fair isolate dominating set of a graph. We determine the fair isolate domination number of path, cycle, complete graph, complete bipartite graph and some special graphs.

Index Terms - dominating set, fair dominating set, isolate dominating set, fair isolatedominating set, isolate domination number.

1. INTRODUCTION

Throughout this paper, we only consider a simple graph (a graph without loop or multiple edges). For a graph G, we denote (G) and (G) as vertex set and edge set of G, respectively. Two vertices u and v are said to be neighbor or adjacent (denoted by uv) if $uv \in (G)$. The set of all neighbors of u is denoted by (u) and it is called the open neighborhood of u, that is, $N(u)=\{x \in V(G): xu \in E(G)\}$. The closed neighborhood of u, denoted by $N[u]$, is given by $N[u]=N(u) \cup \{u\}$. An induced subgraph of G (simply, subgraph of G) is a graph with vertices less than or equal to (G) such that adjacencies and non-adjacencies of any two vertices in G are preserved. A subgraph of G induced by S is denoted by $\langle S \rangle$. The complement of a graph G, denoted by $\bar{G}$, is a graph of the same vertices as G such that any two vertices are adjacent if and only if they are not adjacent in G.
We say that a subset $S \subseteq (G)$ is a dominating set of G if for all vertex $x \in S$, there exists a vertex $v \in V(G) \backslash S$ such that $uv \in E(G)$. The minimum cardinality of the dominating set of G is called a domination number of G, denoted by (G) Haynes et al (1998). If a subgraph $\langle S \rangle$ induced by a dominating set S of G has an isolated vertex, then we say that S is an isolate dominating set of G. The minimum cardinality of an isolate dominating set is called isolate domination number, denoted by $\gamma_0(G)$. The concept of isolate domination was introduced by Hamid & Balamurugam (2013) and further studied by Arriola (2015).

A Fair dominating set in a graph G is a dominating set S such that all vertices not in S are dominated by the same number of vertices from S. ie) every two vertices not in S have the same number of neighbors in S. The fair domination number$\gamma_{fd}(G)$, of G is the minimum cardinality of a fair dominating set.The concepts of fair domination in graphs were introduced by Caro, Hansberg, and Henning [3].

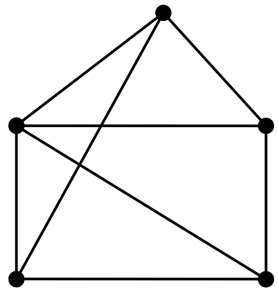
2. Preliminary Results

In this section, we study fair isolate domination number of graphs such as complete graph, cycles, complete bipartite graph, wheel graph and paths.

Definition: 2.1

A Dominating set $S\subseteq V(G)$ is a fair isolate dominating set in G if every two distinct vertices not in S have the same number of neighbours from S and <S> has atleast one isolated vertex.ie) For every two distinct vertices u and v from $V(G)-S$, $|N(u)\cap S| = |N(v)\cap S|$and <S> has atleast one isolated vertex. The fair isolate domination number is denoted by $\gamma_{0_{fd}}$.

Example: 2.2 v_1



$v_5 v_2$

G:

$v_4 v_3$

Figure 2.1

In this graph G,$S = \{v_1, v_3\}$,$V - S = \{v_2, v_4, v_5\}$

$N(v_2) = \{v_1, v_3, v_5\}$,$N(v_4) = \{v_1, v_3, v_5\}$ and $N(v_5) = \{v_1, v_2, v_3, v_4\}$

$N(v_2) \cap S = \{v_1, v_3\}$, $N(v_4) \cap S = \{v_1, v_3\}$.

Hence $\gamma_{0_{fd}} = 2$.

Theorem: 2.3

Let P_n,C_n, K_n be the path, cycle, complete graph of order n then the fair isolate domination number is

(i) $\gamma_{0_{fd}}(P_n) = \lceil\frac{n}{3}\rceil$.

(ii) $\gamma_{0_{fd}}(C_n) = \lceil\frac{n}{3}\rceil$.

(iii) $\gamma_{0_{fd}}(K_n) = 1$.

Theorem: 2.4

In a complete bipartite graph with $K_{m,n}$ vertices then the fair isolate domination number $\gamma_{0_{fd}}$ is min (K_m, K_n).

Proof: Let G be a complete bipartite graph with $K_{m,n}$ vertices. Take $S = \{v_1, v_2, v_3, \ldots \ldots v_m\}$. Then $V - S = \{u_1, u_2, u_3 \ldots\ldots u_n\}$. It is enough to prove that i)S is a dominating set and <S> has atleast one isolated vertex. ii) every two distinct vertices not in S have the same number of neighbors from S. i) Clearly S is a dominating setand m < n.$ii)$ $N(u_1)$=$\{v_1, v_2, v_3 \ldots\ldots v_m\}$ and $N(u_1)\cap S = \{v_1, v_2, v_3 \ldots\ldots v_m\}$, $N(u_2)$=$\{v_1, v_2, v_3 \ldots\ldots v_m\}$and $N(u_2)\cap S = \{v_1, v_2, v_3 \ldots\ldots v_m\}$,$N(u_3)$=$\{v_1, v_2, v_3 \ldots\ldots v_m\}$ and $N(u_3)\cap S = \{v_1, v_2, v_3 \ldots\ldots v_m\}$. Similarly, $N(u_n)$=$\{v_1, v_2, v_3 \ldots\ldots v_m\}$and $N(u_n)\cap S = \{v_1, v_2, v_3 \ldots\ldots v_m\}$. Therefore S is a fair isolatedominating set and hence$\gamma_{0_{fd}}$is min (K_m, K_n).

Theorem: 2.5

In a star graph with n vertices then the fair isolate domination number $\gamma_{0_{fd}}$ is 1.

Proof: Let G be a Star graph with n vertices. Take $S = \{v_1\}$(centre vertex) .Then $V - S = \{v_2, v_3, \ldots\ldots v_n\}$. To prove that i)every two vertices are nonadjacent in S. ii) S is a isolate dominating set and iii)every two distinct vertices not in S have the same number of neighbors from S.

i) It is clearly obivious.

ii) Clearly, S is a dominating set and the vertex of degree zero in S.Therefore S is a isolate dominating set.

iii) $N(v_2)$= $\{v_1\}$, $N(v_2)\cap S = \{v_1\}$ and $N(v_3)$=$\{v_1\}$, $N(v_3)\cap S = \{v_1\}$ and $N(v_4)$=$\{v_1\}$, $N(v_4)\cap S = \{v_1\}$, Similarly, $N(v_n)$= $\{v_1\}$ and $N(v_n)\cap S = \{v_1\}$.

Therefore S is a fair isolate dominating set and hence the fair isolate domination number $\gamma_{0_{fd}}$ is 1.

Theorem:2.6

In a cubic with n vertices then the fair isolate domination number $\gamma_{0_{fd}}$ is 1.

Proof: Let G be a cubic graph with n vertices. Take $S = \{v_2\}$ .Then $V - S = \{v_1, v_3, \ldots \ldots . v_n\}$. To prove that i)every two vertices are nonadjacent in S. ii) S is a isolate dominating set iii)every two distinct vertices not in S have the same number of neighbors from S.

i) It is clearly obivious.
ii) Clearly, S is a isolate dominating set.
iii) $N(v_1)$=$\{v_2\}$ and $N(v_1) \cap S = \{v_2\}$, $N(v_3)$=$\{v_2\}$and $N(v_3) \cap S = \{v_2\}$

$N(v_4)$= $\{v_2\}$ and $N(v_4) \cap S = \{v_2\}$, Similarly, $N(v_n)$= $\{v_2\}$ and $N(v_n) \cap S = \{v_2\}$.

Therefore S is a fair isolate dominating set. Hence fair isolate domination number $\gamma_{0_{fd}}$ is 1.

3. Main Results

In this section, we discuss some special graphs and illustrate these concepts through examples. Characterizations of the fair isolate domination and some exact values of its corresponding fair isolate domination number are also discussed in detailed.

Definition: 3.1

The wheel is a graph formed by connecting a single vertex to all vertices of a cycle.

Example: 3.2

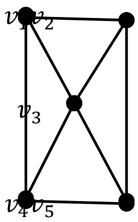


Figure 3.1

In this graph G, $S = \{v_3\}$ is a fair isolates dominating set and $\gamma_{0_{fd}}$ = 1.

Result : 3.3

(i) Let W_n = C_{n-1} + {v} be a wheel on n vertices then the fair isolate domination number $\gamma_{0_{fd}}$ is 1.
(ii) The fair isolate domination number of Friend ship graph F_n on n vertices is 1.

Definition: 3.4

A helm graph H_n is the graph obtained from n- wheel graph by adjoining a pendant edge at each node of the cycle. The helm graph H_n has 2n + 1 vertices and 3n edges.

Example: 3.5

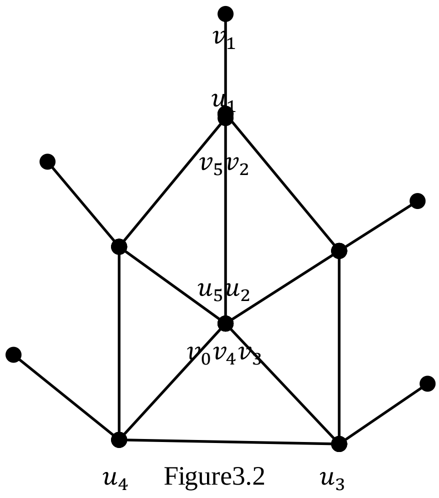


Figure3.2

In this graph G, $S = \{v_1, v_2, v_3, v_4, v_5, v_0\}$ is a fair isolate dominating set and γ_{0fd} = 6.

Definition: 3.6

A closed helmCH_n is the graph obtained by taking a helm $H_n$and adding edges between the pendant vertices to form a cycle.

Theorem: 3.7

The fair isolate domination number of Helm graph $H_n$and a closed helm graph CH_non 2n + 1 vertices is n +1.

Proof: Let G be ahelm graph with 2n + 1vertices. Take $S = \{v, v_1, v_2, v_3, \dots, v_n\}$($v$ −center vertex). Then $V - S$={$u_1, u_2, u_3, \dots, u_n$}.It is enough to prove that i) every two vertices are nonadjacent in S. ii) S is a isolate dominating set iii) every two distinct vertices not in S have the same number of neighbors from S.

i) It is clearly obvious.
ii) v is dominated by$u_1, u_2, u_3, \dots, u_n$, and the remaining vertices $v_1, v_2, v_3, \dots, v_n$are dominated by itself and the set S form a isolate set.Therefore S is a isolate dominating set.
iii) Now, $N(u_i)$= $\{v, v_i, u_{i+1}, u_n\}$ and $N(u_i) \cap S = \{v, v_i, u_{i+1}, u_n\}$, Therefore S is a fair isolate dominating set. Hence fair isolate independent domination number is n +1.Similarly, the same result is hold for closed helm graph.

Definition: 3.8

For M≥ 2, Jahangir graph $J_{n,m}$ is a graph of order nm + 1, consisting of a cycle of order nm with one vertex adjacent to exactly m vertices of $C_{n,m}$ at a distance n to each other.

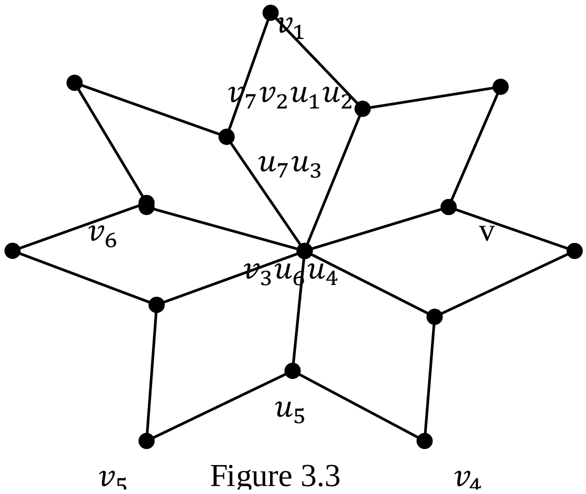


Figure 3.3

In this graph G, $S = \{v, v_1, v_2, v_3, v_4, v_5, v_{6,} v_7\}$ is a fair isolate dominating set and $\gamma_{0_{fd}}$ = 8.

Theorem: 3.10

In a Jahangir graph with $J_{2,n}$ vertices then the fair isolate domination number is n +1.

Proof:

Let G be a Jahangirgraph with $J_{2,n}$vertices. Take$S = \{u_1, w_1, w_2, w_3, w_4, w_5\}(u_1$ −center vertex). Then $V - S$={v_1 ,$v_{2,}v_{3,}v_4, v_5$}.

It is enough to prove that i) every two vertices are nonadjacent in S. ii) S is a isolate dominating set and iii) every two distinct vertices not in S have the same number of neighbors from S.

i)It is clearly obivious.

ii) Clearly, The vertex of degree zero in S.

Therefore S is a isolate dominating set.

iii) $N(v_1)$= $\{u_1, w_1, w_2\}$

$N(v_1) \cap S = \{u_1, w_1, w_2\}$

$N(v_2)$= $\{u_1, w_2, w_3\}$

$N(v_2) \cap S = \{u_1, w_2, w_3\}$

$N(v_3)$= $\{u_1, w_3, w_4\}$

$N(v_3) \cap S = \{u_1, w_3, w_4\}$

$N(v_4)$= $\{u_1, w_4, w_5\}$

$N(v_4) \cap S = \{u_1, w_4, w_5\}$

$N(v_5)$= $\{u_1, w_1, w_5\}$

$N(v_5) \cap S = \{u_1, w_1, w_5\}$

Therefore S is a fair isolate independent dominating set. And Hence fair isolate independent domination number is n+1.

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