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APPLICATION OF DIFFERENTIAL TRANSFORM METHOD IN DEPENDENCE OF PRESSURE WITH ALTITUDE

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Abstract : In this paper, we bring out a comparative study between Differential Transform Method (DTM) and analytical method in dependence of pressure with altitude. It is shown that DTM has an advantage over the analytical method which takes very less time to solve the non linear Riccati equation. Also DTM is more effective and powerful technique.

INDEXTERMS - DIFFERENTIAL TRANSFORM METHOD(DTM), IN DEPENDENCE OF PRESSURE WITH ALTITUDE

INTRODUCTION AND PRELIMINARIES

The French mathematician Liouvillein 1841 provided Riccati equation was the one of the simplest non linear first order differential equation. This paper outlines a reliable comparison among the powerful methods that were recently developed. Differential Transform Method (DTM) was introduced by Zhou in 1986.The main advantage of this method is that it can be applied directly to nonlinear ordinary and partial differential equations without requiring linearization, discretization or perturbation and also it is able to limit the size of computational work while still accurately providing the series solution with fast converge rate. It has been studied and applied during the last decades widely.It was further extended to stochastic systems by using the Ito integral.DTM is used to find the solution of various kinds of Riccati differential equation such solution of first order , second order and system of Riccati equations [1], solving linear and non linear system of ordinary differential equations[2], solving system of differential equations [3], solution of Non-Linear Differential equations[4], Solution of Riccati equation with variable co-efficient[5], Quadratic Riccati Differential Equation [6], on the solutions of Nonlinear Higher Order Boundary value problems [7].

DIFFERENTIAL TRANSFORM METHOD

The transformation of the k th derivative of a function y(x) in one variable is defined as follows

$$Y(k) = \frac{1}{k!}\left[\frac{d^k(y(x))}{dx^k}\right]_{x=0} \qquad (1)$$

and the inverse transform of Y(k) is defined as

$$y(x) = \sum_{k=0}^{\infty} Y(k)x^k \qquad (2)$$

The following are the important theorems of the one dimensional differential transform method

Theorem 1: If $y(x) = m(x) \pm n(x)$, then $Y(k) = M(k) \pm N(k)$

Theorem 2: If $y(x) = \alpha m(x)$, then $Y(k) = \alpha M(k)$

Theorem 3: If $y(x) = \frac{dm(x)}{dx}$, then $Y(k) = (K+1)Y(k+1)$

Theorem 4: If $y(x) = m(x)n(x)$, then $Y(k) = \sum_{r=0}^{k} M(r)N(k-r)$

Theorem 5: If $y(x) = x^{l}$, then $Y(k) = \delta(k-l) = \begin{cases} 1, \text{if } k = l \\ 0, \text{if } k \neq l \end{cases}$

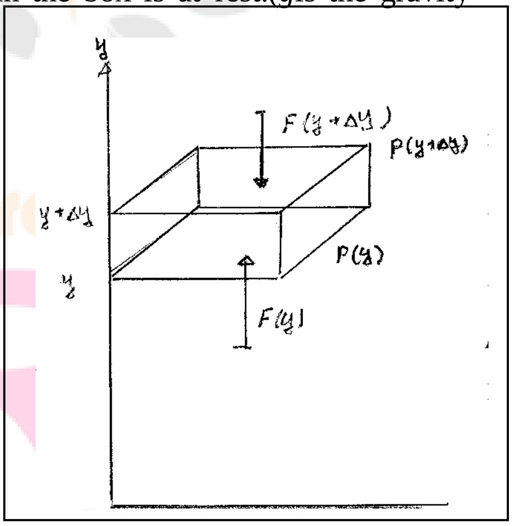
1. DEPENDENCE OF PRESSURE WITH ALTITUDE

We consider a rectangular horizontal section of the atmosphere. The area of the two end faces are *A*. The box is situated in the hight *y* over the ground. The height of the boxis *Δy*. The pressure on the upper and the lower side are $p(y+\Delta y)$ and $p(y)$ respectively.

The density of the air in the height*y*is

The force on a flat piece with area is *F=pA*, where *p* is the pressure on the flat.

We then express that the difference in the force on the upper and the lower side is equal to the gravitational force on the air in the box, assuming that the air in the box is at rest.(*g*is the gravity acceleration)



$p(y)A-p(y+\Delta y)A=m_{air}g=\rho(y)V_{air}g=\rho(y)A\Delta yg$

So $p(y)A-p(y+\Delta y)A=\rho(y)A\Delta yg$

Divding by $A\Delta y$:

$\frac{p(y+\Delta y)-p(y)}{\Delta y} = -\rho(y)g$, and replacing $\frac{p(y+\Delta y)-p(y)}{\Delta y}$ by $\frac{dp}{dy}$ we find (3)

$\frac{dp}{dy} = -\rho(y)g$

To solve this differential equation we need to know another relation between This can how ever be obtained by:

1. The equation of state for ideal gasses: $PV = n_M RT$ (n_M is the number of moles)
2. Definition of the mole mass M : $m = n_M M \Leftrightarrow n_M = \frac{m}{M}$
3. The definition of density ; $\rho = \frac{m}{V} \Leftrightarrow m = \rho V$

Insertion of the two equation in (1) in the equation of state gives (4)

$PV = n_M RT = \frac{m}{M} RT = \frac{\rho V}{M} RT \Rightarrow \rho = \frac{M}{RT} P$

This expression for the density is then used in (3).

$$\frac{dp}{dy} = -\frac{Mg}{RT} p \qquad (4)$$

It is well known that the temperature decreases roughly by one centigrade for every 200 meters increase i altitude over the ground but Initially, we shall assume that the temperature is constant up through the atmosphere.

The differential equation(1.3) has the well known solution:

$$P(y) = p_0 \, e^{\frac{Mg}{RT}}(y) \qquad (5)$$

Using the known values: M_{air} =29 g/mol, g = 9.82 m/s^2, R= 8.31 J/(mol K) and T = 273 K,wefind:

$p(y)=p_0 e^{-1.2610^{-4} y}$

Where should be measured in meters

This results in a pressuredrop of 1.3% per100$m$and a drop of12% per1000 m.

Next we shall look at the solution to the differential equation, where we take into account that the temperature drops linearly 1^0C per200 m increase in altitude.

We put the temperature on the ground at 20 ^0C = 293 K. The temperature in the altitude y then becomes

$T = T(y)=293–y/200$. Then the differential equation becomes:

$$\frac{dp}{dy} = -\frac{Mg}{R(293-\frac{y}{200})} p$$

The equation is solved in the us always by separating the variables and integrating

$$\int_{p0}^{p} \frac{dp}{p} = -\frac{Mg}{R} \int_0^y \frac{1}{293-\frac{y}{200}} dy$$

$$\int_{p0}^{p} \frac{dp}{p} = \frac{Mg}{293} \int_0^y \frac{1}{1-\beta y} dy \quad \text{where } \beta = \frac{1}{293.200}$$

$$\text{In}\left(\frac{p}{p0}\right) = \frac{Mg}{293R\beta} \text{In}(1-\beta y)$$

$$\Rightarrow p = p_0(1-\beta y)^{\frac{Mg}{293R\beta}}$$

When M_{air} =29 g/mol, g = 9.82 m/s^2, R= 8.31 J/(mol K) and T = 273,$p_0 = 1, y = 1$

P = 0.889

Although it looks rather different from (1.5), it turns out that it only causes a deviation from (1.5)ofabout0.1 – 0.2 %.

Using DTM , **P= 0.882**

CONCLUSION:

In this paper, we compared the solution of Riccati equation by DTM.The result of DTM and Exact solution are in strong agreement with each other. DTM is reliable and powerful technique. We believe that the efficiency of DTM gives it much wider suitability which needs to be excavated further.

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