IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

A SORT ON INTUITIONISTIC FUZZY - HYPERGROUPSAND MIMIC HYPERGROUPS

[1]S. Abirami

[1]Assistant Professor of Mathematics,
[1]PG & Research Department of Mathematics,
[1]Sri S. Ramasamy Naidu Memorial College, Sattur, 626 203, Tamil Nadu India.

Abstract : **This study developed a novel idea known as intuitionistic fuzzy hypergroups, which is a generalisation of conventional fuzzy hypergroups. This work enumerates intuitionistic fuzzy mimic hypergroups, intuitionistic fuzzy commutable mimic hypergroups, intuitionistic fuzzy hypergroups and examines their features using a few instances.**

Key words: Intuitionistic Fuzzy Hyperoperation (IFHO), Intuitionistic Fuzzy Hypergroup (IFHG), Intuitionistic Fuzzy Mimic Hypergroup (IFMHG), Intuitionistic Fuzzy Commutable Mimic Hypergroup (IFCMHG).

AMS subject classification : 08A72, 20N20, 20N25.

INTRODUCTION

At the eighth congress of Scandinavian mathematicians in 1934, F. Marty [6] described a hypergroup as a set with an associative and reproductive hyperoperation and examined their properties. This was the beginning of the concept of hyperstructure theory (Hyper compositional al- gebra). The quotient of a group by any, not necessarily normal, subgroup served as the motivating example. The traditional algebraic structure is naturally extended by algebraic hyperstructures. While the combination of two elements is a set in an algebraic hyperstructure, it is an element in a traditional algebraic structure. The theory of algebraic hyperstructures (or hypersystems) has currently developed into a well-established branch in algebraic theory as a result of widespread applications in numerous branches of mathematics and applied Science. Numerous algebraic hyperstructure appli- cations, particularly those from the last ten years, were recently presented by Corsini and Leoreanu, [3]. These applications include those for hyperstruc- tures in geometry, hypergraphs, binary relations, lattices, fuzzy and rough sets, automata, cryptography, codes, median algebras, relation algebras, artificial intelligence, and probabilities.

A helpful mathematical tool for characterising the behavior of systems that are too complex or poorly defined to enable exact mathematical analysis by conventional methods and tools is the notion of fuzzy sets, which Zadeh [11] introduced in 1965. In this regard, Rosenfeld [9] developed the idea of fuzzy groups and looked into its structural details.

The connections between fuzzy sets and algebraic hyperstructures have garnered a great deal of interest recently. A straight forward generalisation of fuzzy algebras (fuzzy groups, fuzzy rings, fuzzy modules, etc.) is the idea of fuzzy hyperstructures. Fuzzy hypergroups can be included in this strategy. As a generalisation of fuzzy set, Atanassov [2] introduced intuitionistic fuzzy set (IFS) in the year 1986. With the advent of the ideas of intuitionistic fuzzy hypergroups, the research of intuitionistic fuzzy algebraic hyperstructures has begun.

My goal in this work is to explain the idea of IFHO through examples and a discussion of some of their intriguing aspects. Additionally, several theo- rems relating to the characterization of these intuitionistic fuzzy hypergroups are established.

2 INTUITIONISTIC FUZZY HYPERGROUPS.

As a generalisation of fuzzy hypergroups and their attributes, intuitionistic fuzzy hypergroups were introduced in this section.

Definition 2.1.

An *Intuitionistic Fuzzy Hyperoperation (IFHO)* maps the ordered pairs of elements of the cartesian product X X to an intuitionistic fuzzy set of X . Thus, if we denote the collection of all IFS of X *by* IF (X), then an IFHO is the map $*: X \times X \to IF(X)$. Hence, if $*$ is an IFHO, then $a * b$ is an IFS, where $a, b \in X$ and is defined by $a * b = <x, \mu_{a*b}(x), \gamma_{a*b}(x)>$

(1) If $a, b, x \in X$, $B, C \in IF(X)$ then, the IFS's $a * B$, $B * a$ are defined as,

$$a * B = <x, \mu_{a*B}(x), \gamma_{a*B}(x)>$$

where $\mu_{a*B}(x) = \vee_{y\in X}[\mu_{a*y}(x) \wedge \mu_B(y)]$, $\gamma_{a*B}(x) = \wedge_{y\in X}[\gamma_{a*y}(x) \vee \gamma_B(y)]$.

Similarly, $B * a = <x, \mu_{B*a}(x), \gamma_{B*a}(x)>$

where $\mu_{B*a}(x) = \vee_{y\in X}[\mu_{y*a}(x) \wedge \mu_B(y)]$,

$\gamma_{B*a}(x) = \wedge_{y\in X}[\gamma_{y*a}(x) \vee \gamma_B(y)]$.

(2) If $a, b, c, x \in X$ then, $(a * b) * c = <x, \mu_{(a*b)*c}(x), \gamma_{(a*b)*c}(x)>$

where $\mu_{(a*b)*c}(x) = \vee_{y\in X}[\mu_{y*c}(x) \wedge \mu_{a*b}(y)]$,

$\gamma_{(a*b)*c}(x) = \wedge_{y\in X}[\gamma_{y*c}(x) \vee \gamma_{a*b}(y)]$.

and $a * (b * c) = <x, \mu_{a*(b*c)}(x), \gamma_{a*(b*c)}(x)>$

where $\mu_{a*(b*c)}(x) = \vee_{y\in X}[\mu_{a*y}(x) \wedge \mu_{b*c}(y)]$,

$\gamma_{a*(b*c)}(x) = \wedge_{y\in X}[\gamma_{a*y}(x) \vee \gamma_{b*c}(y)]$.

(3) If $A, B \in IF(X)$, then an IFS $A * B$ is defined as,

$$A * B = <x, \mu_{A*B}(x), \gamma_{A*B}(x)>$$

where $\mu_{A*B}(x) = \vee_{y,z\in X}[\mu_{y*z}(x) \wedge \mu_A(y) \wedge \mu_B(z)]$,

$\gamma_{A*B}(x) = \wedge_{y,z\in X}[\gamma_{y*z}(x) \vee \gamma_A(y) \vee \gamma_B(z)]$

Definition 2.2.

An *IFHG* is a non-empty set X with an IFHO, whichsatisfies the following axioms,

(i) $(a * b) * c = a * (b * c)$ for all $a, b, c \in X$ (Associativity)

(ii) $a * 1_\sim = 1_\sim * a = 1_\sim$ for all $a \in X$ (Reproduction)

(iii) $(a * b) * c \cap a * (b * c) \ /= 0_\sim$ for all $a, b, c, \in X$ (Weak associativity)

where $1_\sim = \{< x, 1_-, 0_- > | x \in X\}$ and $0_\sim = \{< x, 0_-, 1_- > | x \in X\}$

If only (i) is valid, then an Intuitionistic Fuzzy hyperstructure $(X, *)$ is called an *Intuitionistic fuzzy semi-hypergroup*,

while, if only (ii) is valid, then Intuitionistic Fuzzy hyperstructure $(X, *)$is called an *Intuitionistic fuzzy quasi-hypergroup*.

Example 2.3.

If $X = \{a\}$, then any Intuitionistic fuzzy set of X definesan intuitionistic fuzzy semi-hypergroup structure on X.

Let $a * a = A = < x, \mu_A(x), \gamma_A(x) >$

where $\mu_A(x) = \frac{\alpha}{a}, \gamma_A(x) = \frac{\beta}{a}$ with $0 \leq \alpha + \beta \leq 1$.

Now, $A * a = < x, \mu_{A*a}(x), \gamma_{A*a}(x) >$

where $\mu_{A*a}(a) = \vee_{z\in X}[\mu_{z*a}(a) \wedge \mu_A(z)] = \mu_A(a) = \frac{\alpha}{a}$

Thus, $\mu_{A*a} = \mu_A$. Follows in similar manner, $\gamma_{A*a} = \gamma_A$

Then, $a * A = < x, \mu_{a*A}(x), \gamma_{a*A}(x) >$

where $\mu_{a*A}(a) = \vee_{z\in X}[\mu_{a*z}(a) \wedge \mu_A(z)] = \mu_A(a) = \frac{\alpha}{a}$

Thus, $\mu_{A*a} = \mu_A$. Follows in similar manner, $\gamma_{A*a} = \gamma_A$

Hence, $A * a = a * A = A$ and * is associative.

Remark 2.4.

If $X = \{a\}$, then an intuitionistic fuzzy set A defines anIFHG on X iff $A = 1_\sim$

Example 2.5.

If $X = \{a, b\}$, then the four IFS of X defines an intu-itionistic fuzzy semi-hypergroup structure on X as follows,

$$a * a = A = < x, \mu_A(x), \gamma_A(x) >$$

where $\mu_A(x) = \frac{\alpha_1}{a} + \frac{\alpha_2}{b}$, $\gamma_A(x) = \frac{\beta_1}{a} + \frac{\beta_2}{b}$

$$a * b = B = < x, \mu_B(x), \gamma_B(x) >$$

where $\mu_B(x) = \frac{\alpha_3}{a} + \frac{\alpha_4}{b}$, $\gamma_B(x) = \frac{\beta_3}{a} + \frac{\beta_4}{b}$

$$b * a = C = < x, \mu_C(x), \gamma_C(x) >$$

where $\mu_C(x) = \frac{\alpha_5}{a} + \frac{\alpha_6}{b}$, $\gamma_C(x) = \frac{\beta_5}{a} + \frac{\beta_6}{b}$

$$b * b = D = < x, \mu_D(x), \gamma_D(x) >$$

where $\mu_D(x) = \frac{\alpha_7}{a} + \frac{\alpha_8}{b}$, $\gamma_D(x) = \frac{\beta_7}{a} + \frac{\beta_8}{b}$ with $0 \le \alpha_i + \beta_i \le 1$, i = 1 to 8

LHS $= B * a = <x, \mu_{B*a}(x), \gamma_{B*a}(x)>$

where $\mu_{B*a}(x) = \vee_{z\in X}[\mu_{Z*a}(x) \wedge \mu_B(z)] = [\mu_A(x) \wedge \mu_B(a)] \vee [\mu_C(x) \wedge \mu_B(b)]$

$$\mu_{B*a}(a) = [\mu_A(a) \wedge \mu_B(a)] \vee [\mu_C(a) \wedge \mu_B(b)] = [\alpha_1 \wedge \alpha_3] \vee [\alpha_5 \wedge \alpha_4]$$

If $\alpha_1 \le \alpha_3 \le \alpha_4 \le \alpha_5$, *then* $\mu_{B*a}(a) = \alpha_1 \vee \alpha_4 = \frac{\alpha_4}{a}$

$$\mu_{B*a}(b) = [\mu_A(b) \wedge \mu_B(a)] \vee [\mu_C(b) \wedge \mu_B(b)] = [\alpha_2 \wedge \alpha_3] \vee [\alpha_6 \wedge \alpha_4]$$

If $\alpha_2 \le \alpha_3 \le \alpha_4 \le \alpha_6$, *then* $\mu_{B*a}(b) = \alpha_2 \vee \alpha_4 = \frac{\alpha_4}{b}$

Hence, $\mu_{B*a}(x) = \frac{\alpha_4}{a} + \frac{\alpha_4}{b}$

and $\gamma_{B*a}(x) = \wedge_{z\in X}[\gamma_{Z*a}(x) \vee \gamma_B(z)] = [\gamma_A(x) \vee \gamma_B(a)] \wedge [\gamma_C(x) \vee \gamma_B(b)]$

$$\gamma_{B*a}(a) = [\gamma_A(a) \vee \gamma_B(a)] \wedge [\gamma_C(a) \vee \gamma_B(b)] = [\beta_1 \vee \beta_3] \wedge [\beta_5 \vee \beta_4]$$

If $\beta_1 \ge \beta_3 \ge \beta_4 \ge \beta_5$, *then* $\gamma_{B*a}(a) = \beta_1 \wedge \beta_5 = \frac{\beta_5}{a}$

$$\gamma_{B*a}(b) = [\gamma_A(b) \vee \gamma_B(a)] \wedge [\gamma_C(b) \vee \gamma_B(b)] = [\beta_2 \vee \beta_3] \wedge [\beta_6 \vee \beta_4]$$

If $\beta_2 \ge \beta_3 \ge \beta_4 \ge \beta_6$, *then* $\gamma_{B*a}(b) = \beta_2 \wedge \beta_4 = \frac{\beta_4}{b}$

Hence, $\gamma_{B*a}(x) = \frac{\beta_5}{a} + \frac{\beta_4}{b}$

LHS $= B * a = < x, \frac{\alpha_4}{a} + \frac{\alpha_4}{b}, \frac{\beta_5}{a} + \frac{\beta_4}{b} >$

RHS $= a * C = <x, \mu_{a*C}(x), \gamma_{a*C}(x)>$

where $\mu_{a*C}(x) = \vee_{z\in X}[\mu_{a*z}(x) \wedge \mu_C(z)] = [\mu_A(x) \wedge \mu_C(a)] \vee [\mu_B(x) \wedge \mu_C(b)]$

$$\mu_{a*C}(a) = [\mu_A(a) \wedge \mu_C(a)] \vee [\mu_B(a) \wedge \mu_C(b)] = [\alpha_1 \wedge \alpha_5] \vee [\alpha_3 \wedge \alpha_6]$$

If $\alpha_1 \le \alpha_3 \le \alpha_5 \le \alpha_6$, *then* $\mu_{a*C}(a) = \alpha_1 \vee \alpha_3 = \frac{\alpha_3}{a}$

$$\mu_{a*C}(b) = [\mu_A(b) \wedge \mu_C(a)] \vee [\mu_B(b) \wedge \mu_C(b)] = [\alpha_2 \wedge \alpha_5] \vee [\alpha_4 \wedge \alpha_6]$$

If $\alpha_2 \le \alpha_4 \le \alpha_5 \le \alpha_6$, *then* $\mu_{a*C}(b) = \alpha_2 \vee \alpha_4 = \frac{\alpha_4}{b}$

Hence, $\mu_{a*C}(x) = \frac{\alpha_3}{a} + \frac{\alpha_4}{b}$

and $\gamma_{a*C}(x) = \wedge_{z\in X}[\gamma_{a*Z}(x) \vee \gamma_C(z)] = [\gamma_A(x) \vee \gamma_C(a)] \wedge [\gamma_B(x) \vee \gamma_C(b)]$

$$\gamma_{a*C}(a) = [\gamma_A(a) \vee \gamma_C(a)] \wedge [\gamma_B(a) \vee \gamma_C(b)] = [\beta_1 \vee \beta_5] \wedge [\beta_3 \vee \beta_6]$$

If $\beta_1 \ge \beta_3 \ge \beta_5 \ge \beta_6$, *then* $\gamma_{a*C}(a) = \beta_1 \wedge \beta_3 = \frac{\beta_3}{a}$

$$\gamma_{a*C}(b) = [\gamma_A(b) \vee \gamma_C(a)] \wedge [\gamma_B(b) \vee \gamma_C(b)] = [\beta_2 \vee \beta_5] \wedge [\beta_4 \vee \beta_6]$$

If $\beta_2 \ge \beta_4 \ge \beta_5 \ge \beta_6$, *then* $\gamma_{a*C}(b) = \beta_2 \wedge \beta_4 = \frac{\beta_4}{b}$

Hence, $\gamma_{a*C}(x) = \frac{\beta_3}{a} + \frac{\beta_4}{b}$

RHS $= a * C = < x, \frac{\alpha_3}{a} + \frac{\alpha_4}{b}, \frac{\beta_3}{a} + \frac{\beta_4}{b} >$

If $\alpha_4 = \alpha_3$ and $\beta_5 = \beta_3$, then * is associative and (X, *) is an Intuitionistic fuzzy semi-hypergroup.
If not, then * is not associative and hence (X, *) is not an intuitionistic fuzzy semi-hypergroup.

Remark 2.6.

If $X = \{a, b\}$, then the IFS defines an IFHG on X iff $A = 1_\sim$ and $D = 1_\sim$.

Example 2.7.

The following example distinguishes intuitionistic fuzzy hyperstructure from fuzzy hyperstructure. The four IFS's are defined as,

$$a * a = A = < x, \mu_A(x), \gamma_A(x) >$$

where $\mu_A(x) = \frac{0.4}{a} + \frac{0.03}{b}$, $\gamma_A(x) = \frac{0.5}{a} + \frac{0.6}{b}$

$$a * b = B = < x, \mu_B(x), \gamma_B(x) >$$

where $\mu_B(x) = \frac{0.02}{a} + \frac{0.7}{b}$, $\gamma_B(x) = \frac{0.5}{a} + \frac{0.3}{b}$

$$b * a = C = < x, \mu_C(x), \gamma_C(x) >$$

where $\mu_C(x) = \frac{0.02}{a} + \frac{0.4}{b}$, $\gamma_C(x) = \frac{0.04}{a} + \frac{0.1}{b}$

$$b * b = D = < x, \mu_D(x), \gamma_D(x) >$$

where $\mu_D(x) = \frac{0.8}{a} + \frac{0.07}{b}$, $\gamma_D(x) = \frac{0.2}{a} + \frac{0.5}{b}$

Here, associativity axiom does not satisfied. Since, membership value of both LHS and RHS of $(a * b) * a = a * (b * a) = \frac{0.02}{a} + \frac{0.4}{b}$ are equal but the non-membership of the intuitionistic fuzzy sets $(a * b) * a = \frac{0.3}{a} + \frac{0.3}{b}$ but $a * (b * a) = \frac{0.5}{a} + \frac{0.3}{b}$ are not equal.

Thus, $(a * b) * a \neq a * (b * a)$.

Lemma 2.8.

For every $a, b \in X$ and $C \in IF(X)$, the following is true: $(a * b) * C = a * (b * C)$

Proof. Let $a, b \in X$, $C \in IF(X)$, $(a * b) * C = <x, \mu_{(a*b)*C}(x), \gamma_{(a*b)*C}(x)>$

where $\mu_{(a*b)*C}(x) = \vee_{y,z \in X}[\mu_{y*z}(x) \wedge \mu_{a*b}(y) \wedge \mu_C(z)]$ and

$\gamma_{(a*b)*C}(x) = \wedge_{y,z \in X}[\gamma_{y*z}(x) \vee \gamma_{a*b}(y) \vee \gamma_C(z)]$

$$\begin{aligned}
Now,\ \mu_{(a*b)*C}(x) &= \vee_{y,z \in X}[\mu_{y*z}(x) \wedge \mu_{a*b}(y) \wedge \mu_C(z)] \\
&= \vee_{z \in X}\{\vee_{y \in X}[\mu_{y*z}(x) \wedge \mu_{a*b}(y)] \wedge \mu_C(z)\} \\
&= \vee_{z \in X}\{\mu_{(a*b)*z}(x) \wedge \mu_C(z)\} \\
&= \vee_{z \in X}\{\mu_{a*(b*z)}(x) \wedge \mu_C(z)\} \\
&= \vee_{z \in X}\{\vee_{y \in X}[\mu_{a*y}(x) \wedge \mu_{b*z}(y)] \wedge \mu_C(z)\} \\
&= \vee_{y,z \in X}[\mu_{a*y}(x) \wedge \mu_{b*z}(y) \wedge \mu_C(z)] \\
&= \vee_{y \in X}\{\mu_{a*y}(x) \wedge \vee_{z \in X}[\mu_{b*z}(y) \wedge \mu_C(z)]\} \\
&= \vee_{y \in X}[\mu_{a*y}(x) \wedge \mu_{b*c}(y)]
\end{aligned}$$

$$\mu_{(a*b)*C}(x) = \mu_{a*(b*C)}(x)$$

$$\begin{aligned}
Then,\ \gamma_{(a*b)*C}(x) &= \wedge_{y,z \in X}[\gamma_{y*z}(x) \vee \gamma_{a*b}(y) \vee \gamma_C(z)] \\
&= \wedge_{z \in X}\{\wedge_{y \in X}[\gamma_{y*z}(x) \vee \gamma_{a*b}(y)] \vee \gamma_C(z)\} \\
\gamma_{(a*b)*C}(x) &= \wedge_{z \in X}\{\gamma_{(a*b)*z}(x) \vee \gamma_C(z)\} \\
&= \wedge_{z \in X}\{\gamma_{a*(b*z)}(x) \vee \gamma_C(z)\} \\
&= \wedge_{z \in X}\{\wedge_{y \in X}[\gamma_{a*y}(x) \vee \gamma_{b*z}(y)] \vee \gamma_C(z)\} \\
\gamma_{(a*b)*C}(x) &= \wedge_{y,z \in X}[\gamma_{a*y}(x) \vee \gamma_{b*z}(y) \vee \gamma_C(z)] \\
\gamma_{(a*b)*C}(x) &= \wedge_{y \in X}\{\gamma_{a*y}(x) \vee \wedge_{z \in X}[\gamma_{b*z}(y) \vee \gamma_C(z)]\} \\
&= \wedge_{y \in X}[\gamma_{a*y}(x) \vee \gamma_{b*C}(y)] \\
\gamma_{(a*b)*C}(x) &= \gamma_{a*(b*C)}(x)
\end{aligned}$$

Hence, $(a * b) * C = a * (b * C)$ □

Theorem 2.9.

If an intuitionistic fuzzy hyperstructure $(X, *)$ is endowed with the weak associativity, then $a * b \neq 0_{\sim}$ is valid for every $a, b \in X$.

Proof. Let us assume that $a * b = 0_{\sim}$ for some $a, b \in X$.

Now, $(a * b) * c = 0_{\sim} * c$ for some $c \in X$.

Then, $0_{\sim} * c = < x, \mu_{0_{\sim}*c}(x), \gamma_{0_{\sim}*c}(x) >$

where $\mu_{0_{\sim}*c}(x) = \vee_{y \in X}[\mu_{y*c}(x) \wedge \mu_{0_{\sim}}(y)] = \mu_{0_{\sim}}(x)$,

$\gamma_{0_{\sim}*c}(x) = \wedge_{y \in X}[\gamma_{y*c}(x) \vee \gamma_{0_{\sim}}(y)] = \gamma_{0_{\sim}}(x)$

Thus, $0_{\sim} * c = 0_{\sim} \implies (a * b) * c = 0_{\sim}$

Hence, the weak associativity is not valid in $(X, *)$

which is a contradiction to our assumption. □

Corollary 2.10.
$a * b \neq 0_\sim$ is valid for any pair of elements a, b in an intuitionistic fuzzy hypergroup X.

3. THE INTUITIONISTIC MIMIC FUZZY HYPERGROUP

The IMFHG is introduced in this section because of to the characteristicsof intuitionistic hypergroups that relate to the relationship between IHO andthe induced IFHO with the empty set.

Definition 3.1.

If X is non-empty set with a IFHO , then the two new induced IFHO's and can be defined as follows,

$a/b = \langle x, \mu_{(a/b)}(x), \gamma_{(a/b)}(x)\rangle$ for every $a, b, x \in X$

where $\mu_{(a/b)}(x) = \mu_{x*b}(a)$ and $\gamma_{(a/b)}(x) = \gamma_{x*b}(a)$ $b\backslash a = \langle x, \mu_{(b\backslash a)}(x),$
$\gamma_{(b\backslash a)}(x)\rangle$ for every $a, b, x \in X$

where $\mu_{(b\backslash a)}(x) = \mu_{b*x}(a)$ and $\gamma_{(b\backslash a)}(x) = \gamma_{b*x}(a)$

The two new induced IFHO's were named as *intuitionistic fuzzy right division* and *intuitionistic fuzzy left division*.

Theorem 3.2.

For any pair of elements a, b *in* an IFHG X , then $a/b \neq \mathbf{O}_\sim$ and $a\backslash b \neq \mathbf{O}_\sim$ is valid.

Proof. Let a, b be any pair of elements in an IFHG X

To Prove: $a/b \neq 0_\sim$ and $a\backslash b \neq 0_\sim$.

By Reproductive axiom, we have

$1_\sim * b = 1_\sim$ is valid for every $b \in X$,

$(1_\sim * b)\ (a) = 1_\sim(a)$ is true for any $a \in X$.

Now, $(1_\sim * b)\ (a) = \langle a, \mu_{1_\sim * b}(a), \gamma_{1_\sim * b}(a)\rangle$

where $\mu_{1_\sim * b}(a) = \vee_{y\in X}[\mu_{y*b}(a) \wedge \mu_{1_\sim}(y)]$ and

$\gamma_{b*1_\sim}(a) = \wedge_{y\in X}[\gamma_{y*b}(a) \vee \gamma_{1_\sim}(y)]$

there exists $y \in X$ such that $\mu_{y*b}(a) = 1$ and $\gamma_{y*b}(a) = 0$.

Hence, $a/b \neq 0_\sim$, Similarly $a\backslash b \neq 0_\sim$ for all $a, x, y \in X$. □

The following new definition is introduced as a result of the aforementioned theorem.

Definition 3.3.

If $* : X \times X \to IF(X)$ is an IFHO, then X is called an IMFHG

(Intuitionistic *Fuzzy* $_M$ *-Hypergroup*), if the following two axioms are valid,

(i) $(a * b) * c = a * (b * c)$ for all $a, b, c, \in X$ (Associativity)

(ii) $a/b \neq 0_\sim$ and $a\backslash b \neq 0_\sim$ for all $a, b, \in X$

(iii) $(a * b) * c \cap a * (b * c) \neq 0_\sim$ for all $a, b, c, \in X$ (Weak Associativity)

where $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$

If only (ii) is valid, then X is called an *intuitionistic fuzzy* $_M$ *-quasi hypergroup*, while, if the weak associativity is valid instead of (i), then X iscalled an *intuitionistic fuzzy* $_{MHv}$ *-group*.

Definition 3.4.

An intuitionistic *fuzzy* $_M$ *-hypergroup* X will be called an *intuitionistic commutable fuzzy* $_M$ *-hypergroup* if $a * 1_\sim = 1_\sim * a$ for any $a \in X$.

Theorem 3.5.

In an intuitionistic fuzzy$_M$ -hypergroup X, it holds that $1_\sim * a \neq 0_\sim$ and $a * 1_\sim \neq 0_\sim$ for every $a, x \in X$.

Proof. Let X be an intuitionistic fuzzy$_M$ -hypergroup.

Thus, $x/a \neq 0_\sim$ for every $a, x \in X$.

It follows that, there exists $y \in X$ such that

$$<y, \mu_{x/a}(y), \gamma_{x/a}(y)> \neq 0_\sim$$

where, $\mu_{x/a}(y) = \mu_{y*a}(x)$ and $\gamma_{x/a}(y) = \gamma_{y*a}(x)$

Thus, $\mu_{y*a}(x) \neq 0_\sim$ and $\gamma_{y*a}(x) \neq 0_\sim$ – – – – – – – – (1)

Since, $1_\sim * a = <x, \mu_{1_\sim *a}(x), \gamma_{1_\sim *a}(x)>$

where $\mu_{1_\sim *a}(x) = \vee_{y\in X}[\mu_{y*a}(x) \wedge \mu_{1_\sim}(y)]$ and

$\gamma_{1_\sim *a}(x) = \wedge_{y\in X}[\gamma_{y*a}(x) \vee \gamma_{1_\sim}(y)]$

Using (1), either $\mu_{1_\sim *a}(x) \neq 0_\sim$ or $\gamma_{1_\sim *a}(x) \neq 0_\sim$

Hence, $1_\sim * a \neq 0_\sim$ also $a * 1_\sim \neq 0_\sim$ for every $a, x \in X$ □

Theorem 3.6.

In an intuitionistic fuzzy$_M$ -hypergroup X, it holds that $a * b \neq 0_\sim$ forevery $a, b \in X$.

Proof. Suppose that there are $a, b \in X$ such that $a * b = 0_\sim$

Then, $(a * b) * 1_\sim = 0_\sim * 1_\sim = 0_\sim$

Using Lemma 2.8, $(a * b) * 1_\sim = a * (b * 1_\sim) = 0_\sim$But $a * (b * 1_\sim) =$

$<x, \mu_{a*(b*1_\sim)}(x), \gamma_{a*(b*1_\sim)}(x)>$

where $\mu_{a*(b*1_\sim)}(x) = \vee_{y\in X}[\mu_{a*y}(x) \wedge \mu_{b*1_\sim}(y)]$ and

$\gamma_{a*(b*1_\sim)}(x) = \wedge_{y\in X}[\gamma_{a*y}(x) \vee \gamma_{b*1_\sim}(y)]$

Using Proposition 3.5, $\mu_{b*1_\sim}(y) \neq 0_\sim$ for all $b, y \in X$

Thus, $\mu_{a*y}(x) = 0_\sim$ for all $a, x \in X$.

Hence, $x/y = 0_\sim$ or $a\backslash x = 0_\sim$for all $a, x, y \in X$.

which is a contradiction to intuitionistic fuzzy$_M$ -hypergroup.

Hence $a * b \neq 0_\sim$ for every $a, b \in X$. □

Definition 3.7.

An intuitionistic *fuzzy $_M$ -hypergroup* X will be called an *intuitionistic commutable fuzzy $_M$ -hypergroup* if $a * 1_\sim = 1_\sim * a$ for any $a \in X$.

Example 3.8.

As in the example 2.5, the following are the four intuitionistic fuzzy sets defined as,

$a * a = A = < x, \mu_A(x), \gamma_A(x) >$

where $\mu_A(x) = \frac{\alpha_1}{a} + \frac{\alpha_2}{b}, \gamma_A(x) = \frac{\beta_1}{a} + \frac{\beta_2}{b}$

$a * b = B = < x, \mu_B(x), \gamma_B(x) >$

where $\mu_B(x) = \frac{\alpha_3}{a} + \frac{\alpha_4}{b}, \gamma_B(x) = \frac{\beta_3}{a} + \frac{\beta_4}{b}$

$b * a = C = < x, \mu_C(x), \gamma_C(x) >$

where $\mu_C(x) = \frac{\alpha_5}{a} + \frac{\alpha_6}{b}, \gamma_C(x) = \frac{\beta_5}{a} + \frac{\beta_6}{b}$

$b * b = D = < x, \mu_D(x), \gamma_D(x) >$

where $\mu_D(x) = \frac{\alpha_7}{a} + \frac{\alpha_8}{b}, \gamma_D(x) = \frac{\beta_7}{a} + \frac{\beta_8}{b}$ with $0 \leq \alpha_i + \beta_i \leq 1$, i = 1 to 8

Now, $a * 1_\sim = <x, \mu_{a*1_\sim}(x), \gamma_{a*1_\sim}(x)>$

where $\mu_{a*1_\sim}(x) = \vee_{y\in X}[\mu_{a*y}(x) \wedge \mu_{1_\sim}(y)]$,

$\gamma_{a*1_\sim}(x) = \wedge_{y\in X}[\gamma_{a*y}(x) \vee \gamma_{1_\sim}(y)]$ same as for $1_\sim * a$.

Then, $\mu_{a*1_\sim}(a) = \alpha_1 \vee \alpha_3$, $\mu_{a*1_\sim}(b) = \alpha_2 \vee \alpha_4$, $\mu_{1_\sim *a}(a) = \alpha_1 \vee \alpha_5$,

$\mu_{1_\sim *a}(b) = \alpha_2 \vee \alpha_6$ and$\gamma_{a*1_\sim}(a) = \beta_1 \wedge \beta_3$, $\gamma_{a*1_\sim}(b) = \beta_2 \wedge \beta_4$,

$\gamma_{1_\sim *a}(a) = \beta_1 \wedge \beta_5$, $\gamma_{1_\sim *a}(a) = \beta_2 \wedge \beta_6$

If $\alpha_1 = \max\{\alpha_3, \alpha_5\}$ and $\alpha_2 = \max\{\alpha_4, \alpha_6\}$ also $\beta_1 = \min\{\beta_3, \beta_5\}$ and $\beta_2 = \min\{\beta_4, \beta_6\}$ then $a * 1_\sim = 1_\sim * a$.

Thus, $(X,*)$ is an intuitionistic commutable fuzzy$_M$ – hypergroup. If not, then $(X, *)$ is not an *intuitionistic commutable fuzzy $_M$ -hypergroup.*

Definition 3.9.

An intuitionistic *fuzzy semihypergroup* X will be called an pseudo intuitionistic fuzzy$_M$ – hypergroup if $a * 1_\sim \neq 0_\sim$ and $1_\sim * a \neq 0_\sim$ for any $a \in X$.

Example 3.10.

Consider the above example 3.8, in that we have $1_{\sim} * a = <x, \mu_{1_{\sim}*a}(x), \gamma_{1_{\sim}*a}(x)>$

where $\mu_{1_{\sim}*a}(x) = \vee_{y \in X}[\mu_{y*a}(x) \wedge \mu_{1_{\sim}}(y)]$,

$\gamma_{1_{\sim}*a}(x) = \wedge_{y \in X}[\gamma_{y*a}(x) \vee \gamma_{1_{\sim}}(y)]$

where $\mu_{1_{\sim}*a}(a) = \alpha_1 \vee \alpha_5$, $\mu_{1_{\sim}*a}(b) = \alpha_2 \vee \alpha_6$ and

$\gamma_{1_{\sim}*a}(a) = \beta_1 \wedge \beta_5$, $\gamma_{1_{\sim}*a}(a) = \beta_2 \wedge \beta_6$.

If $\alpha_1 = \max\{\alpha_1, \alpha_5\} \neq 0_{-}$ and $\alpha_2 = \max\{\alpha_2, \alpha_6\} \neq 0$ also $\beta_1 = \min\{\beta_1, \beta_5\} \neq 1_{-}$ and $\beta_2 = \min\{\beta_2, \beta_6\} \neq 1_{-}$ same as for $1_{\sim} * a$.

Thus, $(X, *)$ is an *pseudo intuitionistic fuzzy $_M$ -hypergroup. If not, then $(X, *)$ is not a pseudo intuitionistic fuzzy $_M$ -hypergroup.*

CONCLUSION

In this paper, the concept of IFHG's, intuitionistic *fuzzy $_M$ -hypergroup* their examples with some of its properties were introduced. Also, some char- acterization theorems are proved. At last, pseudo intuitionistic *fuzzy $_M$ -hypergroup* were introduced.

ACKNOWLEDGMENT

The author is very thankful to the referees' insightful comments and rec-ommendations for strengthening the paper.

REFERENCES

[1] R. Ameri, Fuzzy transposition hypergroups, Ital. J. Pure Appl. Math. No. 18, 167-174, 2005.
[2] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and system, 20(1986), no.1, 87-96.
[3] P. Corsini, V. Leoreanu, Applications of Hyperstructures Theory, Kluwer Academic Publishers, 2003.
[4] B. Davvaz, Fuzzy H $_V$ -groups, Fuzzy sets and systems 101, 1999, 191- 195.
[5] J. Klir George and Bo Yuan, Fuzzy sets and Fuzzy logic theory and applications, EEE, Phi- 2007.
[6] F. Marty, Sur un generalisation de la notion de groupe Congres des Mathematiciens Scand., Stockholm 1934, pp. 45-49.
[7] C. G. Massouros, On Certain Fundamental Properties of Hypergroups and Fuzzy Hypergroups-Mimic Fuzzy Hypergroups, International Jour- nal of Risk Theory, Vol 2(no.2), 2012.
[8] C.G. Massouros and G.G. Massouros, On 2-element Fuzzy and Mimic Hypergroups, Numerical Analysis and Applied Mathematics, AIP Conf. Proc. 1479, 2213-2216, 2012.
[9] A. Rosenfeld, Fuzzy Groups, J. Math. Anal. Appl. 35, 1971,pp.512-517.
[10] M. K. Sen, Fuzzy hypersemigroups, Soft Computing 12, 2008, pp.891-900.
[11] L.A. Zadeh, Fuzzy sets,Information and control, 8(1965), 338-353.