



GENERATION OF PRIME NUMBERS

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Abstract : Prime Numbers - Generation a new method to generate all primes suggested.

IndexTerms - Prime Number-Generation-New Method

Introduction: Prime Numbers are Building Blocks to number system. So, the existing problems related to prime numbers attracts good interest among mathematician of all branches- not only number Theorists.

It was Pythagoras who bifurcated the natural numbers in to prime and composites some 2500 years ago. Till now many problem are remaind un solved regarding prime numbers.

Again it was Eratosthenes who firstly invented a very famous shieve to locate or identify primes and composites separately. There are a lot of improvements available is shieves.

Now in this paper we are suggesting a new methodology to generate primes very quickly using computers or even super computers. If this methodology is used to generate prime numbers without any limit quickly all prime numbers gets generated.

Para 1: **α Series**

Firstly we have to considered a new series by name α series in this series the first natural number 1 is get omitted because it is neither a prime nor a composite and unique. Then No 2, and all its multiplies such as $2 \times 2, 2 \times 3, 2 \times 4, 2 \times 5 \dots$ up to infinity is get omitted. Since except to there exist no other even prime numbers. Thirdly and similarly the number 3, and al its multiplies such as $3 \times 2, 3 \times 3, 3 \times 4 \dots$ Up to infinity its get omitted.

Off the remaining numbers which are not even, and not divisible by 3, We set two first numbers for two sequences by name α_1 sequence and α_2 sequence namely 5 and 7.

Now considered α_1 set. The first number is 5. $5+1=6, 5+3=8, 5+5=10 \dots$ all are even. $5+2=7, 5+4=9, 5+6=11$ all are odd. In this case 7 is Fn of sequence 2. 6,8,10 are even numbers 9 is

divisible by 3 so naturally the second number in α_1 sequence is 11. The different between the first term and second term is 6. Same manner and with the same argument we can generate an arithmetic progression by name α_1 series which first number is 5 and common difference is 6.

The series will be 5,11,17..... the similar manner we can produced another series by name α_2 Series which first number 7 and the common different as 6. The series will be 7,13,19,25,31.....

Now both series will consists of primes and composites.

Para 2:

Preparation of abstent term table

Considered α_1 series up to N=300

5 11 17 23 29 35 41 47 53 59 65 71 77 83 89 95 101 107 113 119 125 131 137
143 149 155 161 167 173 179 185 191 197 203 209 215 221 227 233 239 245
251 257 263 269 275 281 287 293 299

1) 5th order numbers (Numbers divisible by 5) are all composite

- 1) $5 \times 7 = 35$
- 2) $5 \times 13 = 65$
- 3) $5 \times 19 = 95$
- 4) $5 \times 25 = 125$
- 5) $5 \times 31 = 155$
- 6) $5 \times 37 = 185$
- 7) $5 \times 43 = 215$
- 8) $5 \times 49 = 245$
- 9) $5 \times 55 = 275$

2) 7th Order Numbers

- 1) $7 \times 11 = 77$
- 2) $7 \times 17 = 119$
- 3) $7 \times 23 = 161$
- 4) $7 \times 29 = 203$
- 5) $7 \times 35 = 245$
- 6) $7 \times 41 = 287$



3) 11th Order Numbers

1) $11 \times 13 = 143$

2) $11 \times 19 = 209$

3) $11 \times 25 = 275$

4) 13rd Order Numbers

1) $13 \times 17 = 221$

2) $13 \times 23 = 299$

Foot Note: The composites are generated as 5×7 , 7×11 , 11×13 ... considering the natural development of natural numbers and avoiding repetitions of maximum possible.

List down all the remaining numbers in $\alpha 1$ series They are

5, 11, 17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101, 107, 113, 131, 137, 149, 167, 173, 179, 191, 197, 227, 233, 239, 251, 257, 263, 269, 281, 293

All are Primes

Considered $\alpha 2$ series up to $N=300$

7 13 19 25 31 37 43 49 55 51 57 73 79 85 91 97 103 109 115 121 127 133 139
145 151 157 163 169 175 181 187 193 199 205 211 217 223 229 235 241 247
253 259 265 271 277 283 289 295.

The Number which are composites are marked

1) 5th Order Number

1) $5 \times 5 = 25$

2) $5 \times 11 = 55$

3) $5 \times 17 = 85$

4) $5 \times 23 = 115$

5) $5 \times 29 = 145$

6) $5 \times 35 = 175$

7) $5 \times 41 = 205$

8) $5 \times 47 = 235$

9) $5 \times 53 = 265$

10) $5 \times 59 = 295$

2) 7th order Number

1) $7 \times 7 = 49$

2) $7 \times 13 = 91$

3) $7 \times 19 = 133$

4) $7 \times 25 = 175$

5) $7 \times 31 = 217$

6) $7 \times 37 = 259$

3) 11th Order Number

1) $11 \times 11 = 121$

2) $11 \times 17 = 187$

3) $11 \times 23 = 253$

4) 13th Order Numbers

1) $13 \times 13 = 169$

2) $13 \times 19 = 247$

5) 17th Order Numbers

1) $17 \times 17 = 289$

List down all the remaining numbers which are all by virtue primes

7, 13, 19, 31, 37, 43, 61, 67, 73, 79, 97, 103, 109, 127, 139, 151, 157, 163, 181, 193, 199, 211, 223, 229, 241, 271, 277, 283

Now all these numbers can be expressed by two functions

1) $C1 + Vr$ 2) $C2 + Vr$

Where $C1=5$; $C2=7$ which are constant V_0, V_1, V_2, \dots are multiples of six. So the function $C1 + Vr$ Generates the following numbers

1) $C1 + V_0 = 5 + 0 \times 6 = 5$

2) $C1 + V_1 = 5 + 1 \times 6 = 11$

3) $C1 + V_2 = 5 + 2 \times 6 = 17$

.....

4) $C1 + V_5 = 5 + 5 \times 6 = 35$

Similarly the function $C2 + Vr$ Generates the following numbers

1) $C2 + V_0 = 7 + 0 = 7$

2) $C2 + V_1 = 7 + 1 \times 6 = 13$

$$3) C2+V2 = 7+2 \times 6=19$$

$$4) C2+V3 = 7+3 \times 6=25 \dots\dots\dots$$

Again, the above said both functions generates both α_1 and α_2 Terms which includes both primes and composites.

Further considered the

5th, 7th, 11th,13th order.... terms in α_1 and α_2 Numbers they form the absent term tables of α_1 and α_2 .

α_1 Absent Term Table

1) 5th order 1 No.=35

This may be expressed as $5+V5 / 35-5$ $30 / 6=5$

Similarly $65-5 =60$. $60/6=10$

So the first row of Absent Term Table is

5, 10, 15, 20, 25.....

Likewise the 7th order term 1st Number is $77 / 77-5=72$. $72/6 =12$

So the second row will be

12, 19, 26, 33.....

Likewise other rows can be formed

Absent Term Table-1

1) 5 10 15 20 25

2) 12 19 26 33 40

3) 23 34 45 56 67

4) 36 49 62 75 88 and so on

Note: I we considered the difference between the first numbers of each row is

7, 11, 13.....

Similarly the horizontal wise different is also

5, 7, 11, 13.....

The difference of different will be

2, 4, 2, 4 alternatively which is property of α sequences.

Hence it will be easier to construct Absent Term Table to any extent.

Likewise the first term in α_2 composite order is 25, So $25-7=18$, $18/6=3$

So we can frame another function $C2+V2$ (Already Noted)

Where $C_2=7$

$V_0, V_1, V_2 \dots$ are multiples of six so

$$25-7=18 \quad 18/6=3$$

$$55-7=48 \quad 48/6=8$$

The first row of absent term table is

3 8 13 18 23.....

Similarly The Second Row will be

7 14 21 28 35

3rd Row will be

19 30 41 51

13th Row will be

27 40 51 62 73

So the Absent Term Table will be

3 8 13 18 23

7 14 21 28 35

19 30 41 52 63

27 40 51 62 73

47 64 81 98

59 78 97 116

87 110 133 156

103 128 153 170

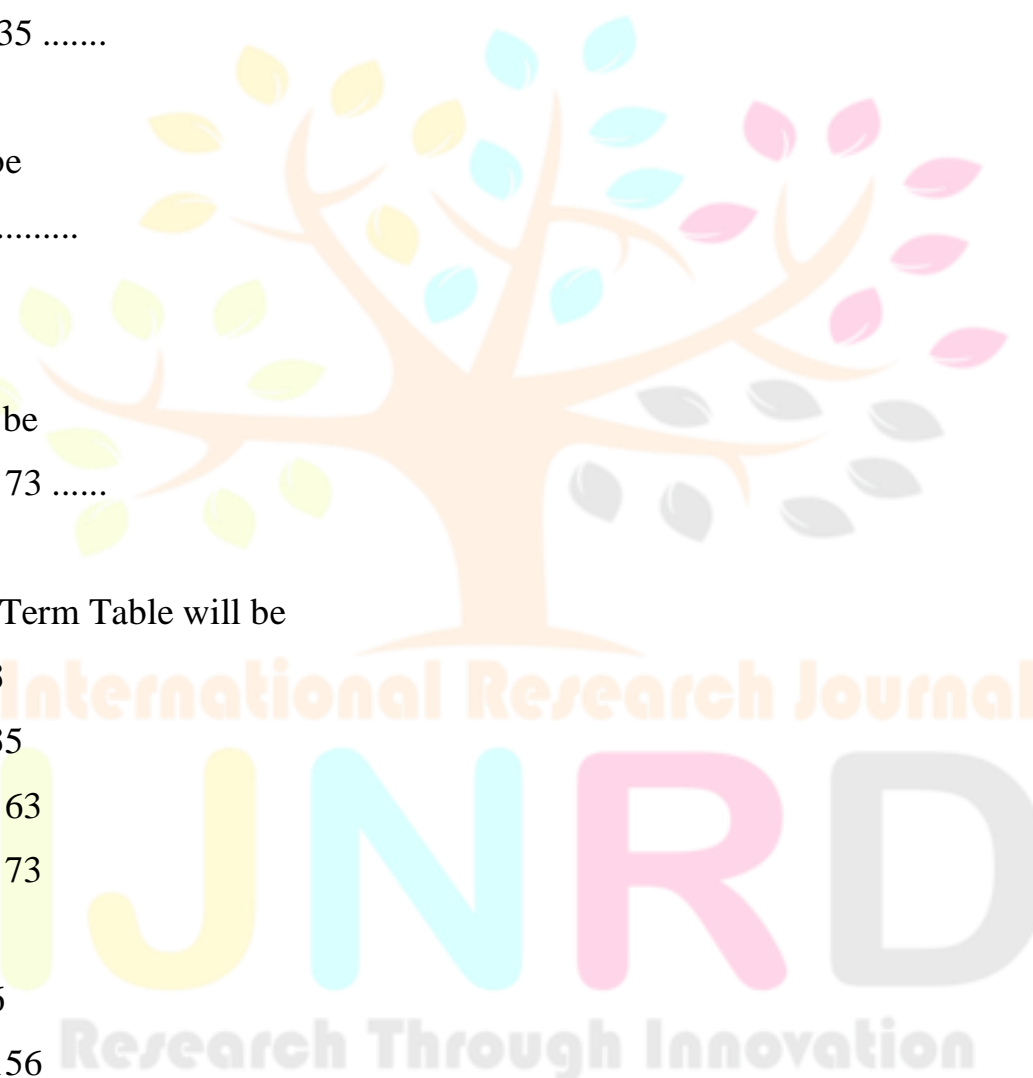
139 160 197 226

.....

Note: If we considered differences of 1st Number of each row will be

4, 8, 12..... That is multiples of 4 in to

1,2,3.....



The horizontal differences will be

5,7,11,13 which is an α sequence property

Proof to the differences of first number of each row is given separately in the last paragraph of this paper

Para 3:

Methodology to generate the α_1 Primes and α_2 Primes by using the above said Absent Term Table

Generation of α_1 Terms

Function 1: $C_1 + V_r$

$$1) 5 + 0.6 = 5$$

$$2) 5 + 1.6 = 11$$

$$3) 5 + 2.6 = 17$$

$$4) 5 + 3.6 = 23$$

$$5) 5 + 4.6 = 29$$

$$6) 5 + 5.6 = 35$$

$$7) \dots\dots\dots$$

Number 5 finds a face in α_1 Absent Term Table so it is a composite.

Similarly all α_1 Primes can get generated upto the limit to the need of a searcher omitting the values of V_r which finds a place in α_1 Absent Term 1 Table.

Generation of α_2 Terms

Function 2: $C_2 + V_r$

$$1) 7 + 0.6 = 7$$

$$2) 7 + 1.6 = 13$$

$$3) 7 + 2.6 = 19$$

$$4) 7 + 3.6 = 25 \dots\dots\dots$$

No 3 Finds a face in the α_2 Absent Term Table 2, So it produces a composite.

Likewise all the α_2 Primes can get generated. Omitting the values of V_r Which finds a phase in α_2 Absent Term Table 2.

Para 4:

Arranging in Ascending order

- 1) 2,3 are primes which finds no phase in α_1 sequence.
- 2) α_1 Primes are 5, 11, 17,.....
- 3) α_2 Primes are 7, 13, 19,.....

So the ascending order of Primes will be

2, 3, 5, 7, 11, 13, 17, 19,.....

α Series Properties

If we amalgamate α_1 Series and α_2 Series we will get a series of α like

5, 7, 11, 13, 17, 19, 23, 25,.....

The difference will be

2, 4, 2, 4,

Proof to the first numbers of α_2 Absent Term Table of Each Row

1) The first terms of each row of the table is

3 7 19 27 47 59 87 103 139

d 4 12 8 20 12 28 16 36

1st Order

4 8 12 16

d2 4 4 4 4 Again $4/2=2$

2nd Order

12 20 28 36

d2 8 8 8 $8/2=4$

Thus the α sequence order is maintained at the level of d2

Part-2

Generation of Twin Primes

Twin Primes are a special prime of primes whos difference to the first and second primes is always true. Such pair of twin primes exists upto infinity. Nobody knows what is the last pair o twin primes.

In this para is suggest a new methodology to generate twin primes up to infinity.

Methodology

Considered the α_1 and α_2 series separately ,

$\alpha_1 - 5 \ 11 \ 17 \ 23 \ 29 \ 35 \ 41 \ 47 \ 53 \dots$

$\alpha_2 - 7 \ 13 \ 19 \ 25 \ 31 \ 37 \ 43 \ 49 \ 5 \dots$

Again we can generate α_1 composites and α_1 composites as soon here under:

α_1 composites:

Table 1:

1) $5 \times 7, 11.. \quad 35 \ 65 \ 95 \ 125 \ 155 \ 185 \ 215 \ 245 \ 275 \ 305 \dots$

d30

2) $7 \times 11 \Rightarrow 77 \ 119 \ 161 \ 203 \ 245 \ 287 \ 329$

3) $11 \times 13 \Rightarrow 143 \ 209 \ 275 \ 341$

4) $13 \times 17 \Rightarrow 221 \ 299 \ 373$

5) $17 \times 19 \Rightarrow 323$

Table 2:

α_2 composites

1) $5 \times 5 \Rightarrow 25 \ 55 \ 85 \ 115 \ 145 \ 175 \ 205 \ 235 \ 265 \ 295 \ 325 \dots$

2) $7 \times 7 \Rightarrow 49 \ 91 \ 133 \ 175 \ 217 \ 301$

3) $11 \times 11 \Rightarrow 121 \ 187 \ 253 \ 319$

4) $13 \times 13 \Rightarrow 169 \ 241 \ 355$

5) $17 \times 17 \Rightarrow 289 \ 397$

6) $19 \times 19 \Rightarrow 361$

Logic:

1) Firstly the twin tree may be generated up to the given number. The general notation is α_1 α_2 difference between these to is always 2

In practice,

α_1	α_2
1) 5	7
2) 11	13
3) 17	19



4) 23	25
5) 29	31
6) 35	37
7) 41	43
8) 47	49
9) 53	55
10) 59	61
11) 65	67
12) 71	73
13) 83	85
14) 89	91
15) 95	97
16) 101	103
17) 107	109

- 1) Remove all composites number in $\alpha 1$ column and $\alpha 2$ column
- 2) Sometimes either of the number in a pair may be prime even then they do not represent twin prime they may be removed
- 3) By virtue the remaining number are twin primes. We can generate such pair twin prime up to the limit that we require.

CONCLUSION

Just a new methodology is suggested to generate all Primes quickly both manually are using system.

There is no test numbers one number after another this is may be considered as an advanced shive model from Earstosthenis shive.

Again the twin primes also get generated in the methodology described above.

References:

- 1) Elementary Number Theory by David M Burton PP56
- 2) Ibid PP 59