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A Deteriorating Inventory Model For Infinite Time – horizon by Allowing Special Sales and Discount.

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ABSTRACT: This paper develops a deterministic inventory model to determine the optimal value of retained units with continuous replenishment. For disposing the excessive inventories, the inventory manager offers the special sales allowing discount. This model is under the consideration of deterioration of products and allowing shortages. This model deals with the selling cost and the returning cost to determine the expected total cost.

KEYWORDS: Inventory, Deterioration, Discount rate, Shortages, special sales.

INTRODUCTION

Inventory can be defined as collection of physical product which can be used to fulfil some future demand for that product. Any inventory system deals with input and output process. The process of input deals with supply of materials and process of output deals with demand of materials depending on situation over time.

Deterioration or decay of item in inventory is a general phenomenon that is observed for many products. It is commonly observed in food stuffs, fruits, green vegetables, fashion items which retain their fresh quality for some time before starting to decay.

Demand is often found to be affected by the price of the item, its quality, the time span for which it has remained in inventory etc. Many inventory models have been studied under the assumption that demand occurs at a uniform rate.

Many researchers established models in the area of deteriorating inventories.

Chen (2008) considered suitable return policies between each pair of neighbouring firms in which unsold units are returned phase by phase from the retailer to the upstream firms and both firm contributes the loss due to overstock.

Amutha and Chandrasekharam (2013) studied an inventory model for constant demand with shortages. Banerjee and Sharma (2010) developed an inventory model with pricing policy for which price and time depend on seasonal demand. A robust optimization approach to model supply and demand uncertainties in inventory systems have been discussed by Chu, Huang and Thiele (2019).

Chun (2003) studied under the condition of optimal pricing and ordering policies for perishable commodities. Rapolu and Kandpal (2017) have given an inventory model for deteriorating items with continuous demand under allowing delay in payments.

The present model has been developed under the assumption that the excessive inventories is returned by a certain amount and do away with special sales allowing discount. This model is under the consideration of deterioration of products and allowing shortages. The optimal value of retained units has been studied with continuous replenishment.

ASSUMPTIONS AND NOTATIONS:

i. R_D is the rate of demand of W_R units per unit time is known and constant during the cycle of operation.

ii. D_T is total demand in a period of length T.

iii. The ordering cost per unit is assumed as A.

iv. A constant stock level of Q units per replenishment increases the invntory level at the initial of each period to the order-level of S units.

v. Shortages if any, are completely backlogged.

vi. The process begins with W units, out of which only $W_P(\geq S)$ amounts of units with hold after returning or selling the rest.

vii. C is the unit cost, the stock holding cost C_h per unit per unit time, the selling cost C_{SL}per unit, $C_d$is the discount cost per unit per unit time, the shortage cost C_S per unit per unit time, the replenishment cost C_R per order, the returning cost C_{RT} per unit are known and constant during the period under consideration.

viii. The parameter θ is a constant fraction of the inventory deteriorates per unit time.

ix. The parameter α of the inventory is a fraction that is returned and the rest $1-\alpha$ is sold by discount rate i.e. $0 \leq \alpha \leq 1$.

MODEL FORMULATION:

In this chapter we contemplate if nothings are returned or sold then stocks will be exhausted due to demand and deterioration. The variation on the variation level $Q_T(t)$ with respect to time,$0 < t < T$, can be described by the following differential equation,

$$\frac{dQ_T(t)}{dt} + \theta Q_T(T) + W_R = 0\,;\, 0 < t < T \qquad \ldots (1)$$

Using the boundary condition $Q_T(0) = W$ and $Q_T(T) = 0$ respectively, equation(1) gives the solution

$$Q_T(T) = \left(W + \frac{W_R}{\theta}\right) e^{-\theta t} - \frac{W_R}{\theta} \qquad \ldots (2)$$

And

$$T = \frac{1}{\theta} log\left(1 + \frac{\theta W}{W_R}\right) \qquad \cdots (3)$$

When W_P units will be sold with discount in time t_1 due to demand and deterioration of units in inventory. If it considers $Q_1(t)$ as the inventory level of the system at time $t, (0 \leq t \leq t_1)$, the concerning system during t_1 is given by the differential equation

$$\frac{dQ_1(t)}{dt} + \theta Q_1(t) + W_R = 0\,;\, 0 < t < \; t_1 \qquad \cdots (4)$$

Using the boundary condition $Q_1(0) = W_P$ and $Q_1(t_1) = 0$ respectively, equation (4) gives the solution

$$Q_1(t) = \left(W_P + \frac{W_R}{\theta}\right) e^{-\theta t} - \frac{W_R}{\theta} \quad ; 0 < t < \; t_1 \qquad \ldots (5)$$

and

$$t_1 = \frac{1}{\theta} log\left(1 + \frac{\theta W_P}{W_R}\right) \qquad \ldots (6)$$

Number of units that deteriorate is

$$D(W_P) = W_P - W_R t_1$$

$$= W_P - \frac{W_R}{\theta}\, log\left(1 + \frac{\theta W_P}{W_R}\right) \qquad \ldots (7)$$

Total amount carried in inventory during $[0, t_1]$is

$$I_1(W_P) = \int_0^{t_1} Q_1(t)dt = \frac{D(W_P)}{\theta} \qquad \ldots (8)$$

The total discount cost over the retained units during $[0, t_1]$ is

$$C_d = dC_{SL} W_P t_1 \qquad \cdots (9)$$

Where d is the discount factor, $0 < d \leq 1$.

The total cost of the inventory system during the period T is

$$K(W_P) = C_{RT}(W - W_P)(1 - \alpha) + C_{SL}(W - W_P)\alpha + dC_{SL} W_P t_1 + C\big(D(W_P)\big) + \; C_h I_1(P) + (T - t_1)K_1$$

$$= (W - W_P)\left[\frac{W_R C_{RT}(1-\alpha) + W_R C_{SL}\alpha + K_1}{W_R}\right] + dC_{SL}\frac{{W_P}^2}{W_R} \quad + \left(C + \frac{C_h}{\theta}\right)\left[W_P - \frac{W_R}{\theta}\log\left(1 + \frac{\theta W_P}{W_R}\right)\right]$$

For optimal value of W_P,

$$\frac{dK(W_P)}{dW_P} = 0$$

$$\Rightarrow W_P = \frac{[(K_1 + RC_{RT}(1-\alpha) + RC_{SL}\alpha) - 2dC_{SL}]W_R}{[W_R(\theta C + C_h) + 2dC_{SL}\,\theta - \{K_1 + RC_{RT}(1-\alpha) + RC_{SL}\alpha\}\theta]}$$

$$\cdots (10)$$

and

$$\frac{d^2K(W_P)}{d{W_P}^2} \geq 0.$$

By considering the minimality of $K(W_P)$ at equation (10), during time $(T - t_1)$ K_1 is the average total cost per unit time of the optimal order-level for deteriorating items.

Suppose $Q^*(t)$ is the inventory position of the system at time t, $0 \leq t \leq t_3$, where $t_3$is the scheduling period of the order-level lot -size system operating at time t_1, then

$$\frac{dQ^*(t)}{dt} + \theta Q^*(t) + W_R = 0\,;\, 0 < t < t_2 \qquad \ldots\ldots\ldots (11)$$

$$\frac{dQ^*(t)}{dt} + W_R = 0\,;\, t_2 < t < t_3$$

Using the boundary condition $Q^*(0) = S$ and $Q^*(t_2) = 0$ respectively, equation (11) gives

$$Q^*(t) = \frac{W_R}{\theta}\left(e^{\theta(t_2 - t)} - 1\right);\; 0 < t < t_2 \qquad \ldots (12)$$

$$= W_R(t_2 - t_3);\, t_2 < t < t_3\,.$$

Where

$$S = \frac{W_R}{\theta}\left(e^{\theta t_2} - 1\right) \qquad \ldots (13)$$

The total deteriorating unit is ,

$$D(t_2) = S - \text{Demand during time } t_2$$

$$= \frac{W_R}{\theta}\left(e^{\theta t_2} - 1\right) - W_R t_2 \qquad \cdots (14)$$

Average number of units carried in the inventory per unit time is

$$I_1(t_2, t_3) = \frac{1}{t_3}\int_0^{t_2} Q^*(t)dt$$

$$= \frac{D(t_2)}{\theta t_3} \qquad \ldots (15)$$

Average of shortages quantity per unit is

$$I_2(t_2,t_3) = \frac{1}{t_3}\int_0^{t_2}[-Q^*(t)]dt = \frac{W_R(t_3-t_2)^2}{2t_3} \qquad \dots(16)$$

The average cost per unit time of the inventory level during time t_3 is

$$K_1(t_2,t_3) = C\frac{D(t_2)}{t_3} + C_h I_1(t_2,t_3) + C_S I_2(t_2,t_3) + \frac{C_R}{t_3}$$

For minimization cost we set,

$$\frac{\partial K_1}{\partial t_2} = 0$$

$$\Rightarrow \left(C + \frac{C_h}{\theta}\right)\left(e^{\theta t_2} - 1\right) - C_S W_R(t_3 - t_2) = 0 \qquad \cdots(17)$$

$$\frac{\partial K_1}{\partial t_3} = 0$$

$$\Rightarrow K_1(t_2,t_3) - C_S W_R(t_3 - t_2) = 0 \qquad \cdots(18)$$

This gives

$$t_2 = \frac{1}{\theta}log\left(\frac{K_1}{C + \frac{C_h}{\theta}} + 1\right)$$

and

$$t_3 = \frac{1}{\theta}log\left(\frac{K_1}{C + \frac{C_h}{\theta}} + 1\right) + \left(\frac{K_1 + \left(C + \frac{C_h}{\theta}\right)}{C_S W_R}\right)$$

Therefore the optimal order level is

$$S_0 = \frac{W_R}{\theta}\left(e^{\theta t_2(0)} - 1\right) \qquad \dots(19)$$

Also substituting the value of $t_2 = t_2(0)$ and $t_3 = t_3(0)$ in eqn.(19), the minimum total cost per unit time of the inventory system as

$$K_1 = (C_h + \theta C)W_R S_0$$

The optimal value of W_P using above value of K_1 becomes

$$W_P = \frac{[((C_h + \theta C)W_R S_0 + RC_{RT}(1-\alpha) + RC_{SL}\alpha) - 2dC_{SL}]W_R}{[W_R(\theta C + C_h) + 2dC_{SL}\,\theta - \{(C_h + \theta C)W_R S_0 + RC_{RT}(1-\alpha) + RC_{SL}\alpha\}\theta]}$$

CONCLUSION: We have developed an infinite horizon inventory model with uniform rate of deterioration. The optimal value of retained units has been studied under special sales allowing discount with continuous replenishment.

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