



Breaking News from the World of Prime Numbers

*Khurshid A. Bhat**

*Principal, Department of School Education, Jammu and Kashmir (UT), India.

Abstract:

From the centuries, prime number conjectures/problems vestiges challenging task to the researchers, due to their distribution, due to their uniqueness, due to their special existence. Whenever we talk about prime numbers many conjectures wonder in our minds, whether it is Goldbach conjecture, Twin prime Conjecture, Prime Number Distribution etc. which endow with the open challenges to the researchers. Abundant work has been done on these conjectures but still we are far from the truth and couldn't settle yet. In 1742, modus operandi suggested by Christian Goldbach and Leonhard Euler registered in their letter is exploited to make a break through for Goldbach conjecture, using the concept introduced by these two great mathematician, a special approach is used to suggest some valuable arguments regarding the Goldbach conjecture and twin prime conjecture. The association perception used in this paper can help the mathematicians to proceed ahead in this area.

Key Words: Twin Prime, Goldbach Conjecture, Closest twin prime, Sieve of Eratosthenes, Historical letter.

Introduction:

A natural number is said to be prime if it is greater than 1 and can't be written as the product of two smaller natural numbers and the numbers greater than 1 that are not prime are called composite numbers. Prime numbers are called building blocks of arithmetic, as we can express all natural numbers as the product of prime numbers. Whenever we are talking about the prime numbers we cannot ignore the various conjectures linked to them, yet to solve/proved. In mathematics we have many conjectures like Riemann Hypothesis, Fermat Last Theorem, Collatz Conjecture, P Vs N-P problem, Hodge Conjecture and many other that are not solved/proved yet. In 1995 Prof. Andrew Wiles proved that the Fermat Last Theorem is not true for $n > 2$ [1]. Many researchers are working on these conjectures, still we are far from the proof/solutions, recently some interested results are published regarding few of the conjectures [2,3].

In this paper an association between two famous prime conjectures are confer, one is Twin Prime Conjecture and second is Goldbach Conjecture.

A twin prime is a prime that has a prime space of two. The conjecture associated to Twin prime numbers states that there are infinitely many primes of the form p and $p+2$.

Goldbach conjecture is one of the oldest unsolved problem in number theory. It states that every even natural number greater than 2 is the sum of two prime numbers. This is also known as Goldbach strong conjecture. One more is Goldbach weak conjecture which is also known as the odd Goldbach Conjecture, the ternary Goldbach problem or 3-prime problem states that every odd number greater than 5 can be expressed as the sum of three primes (A prime may be used more than once in the same sum).

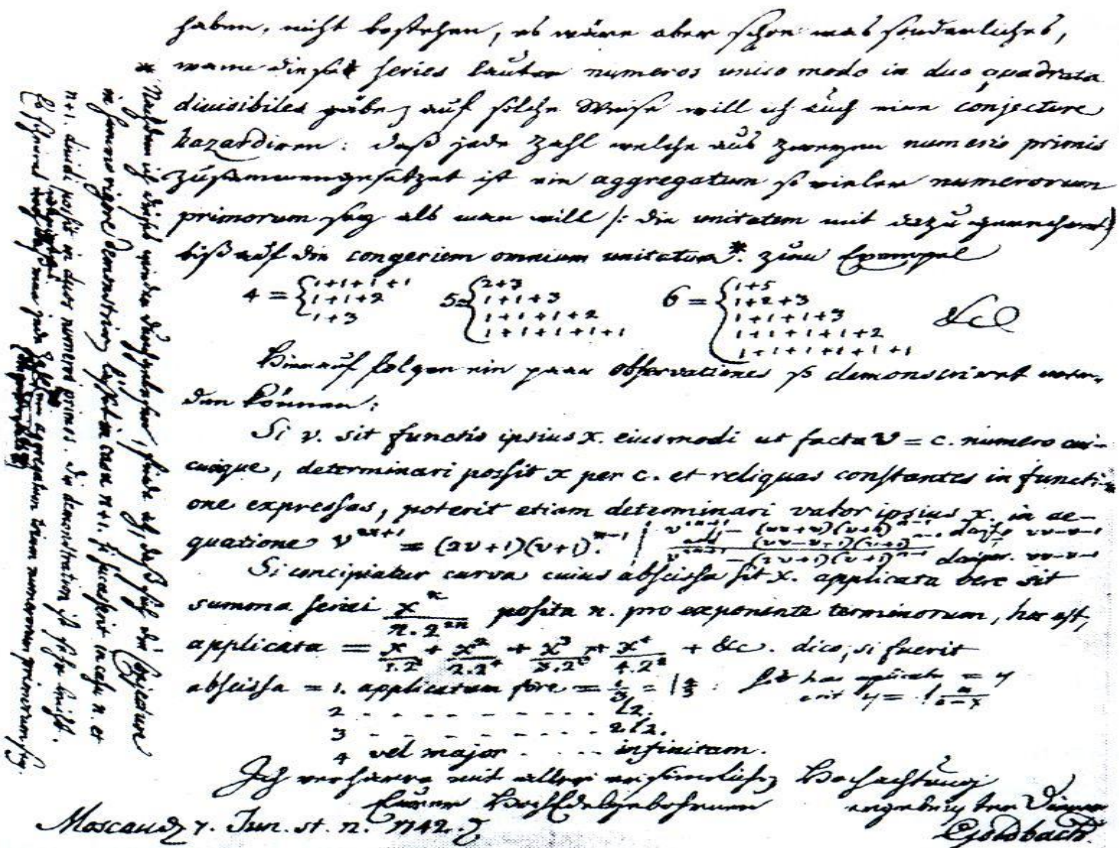
In 2013 Harald Helfgott make public a proof of Goldbach's weak conjecture but not published in peer-reviewed. He said that both the ternary Goldbach and binary or strong Goldbach had their origin in an exchange

of letter between Prof. Euler and Prof. Goldbach in 1742.

Historical background

In about 200 BC the Greek Eratosthenes developed an algorithm for calculating primes called the Sieve of Eratosthenes, after that for long time prime number concepts remains untouched in the history of mathematics and is called Dark Age. In the beginning of 17th century Fermat proved a speculation of Albert Girard that every prime number of the form $4n + 1$ can be written in a unique way as the sum of two squares and was able to show how any number could be written as a sum of four squares.

In 18th century two great mathematicians were works a lot on several mathematical concepts and prime number concept was one of them. In 1742 Prof. Christan Goldbach write a letter to Prof. Leonhard Euler regarding the prime numbers and the two mathematicians offers two different ideas. The concept in this letter later becomes the “Goldbach conjecture” after the name of Prof. Christan Goldbach, which states that all even numbers can be written as sum of two primes, this conjecture is still unsolved.



Strong and Weak Goldbach Conjecture

This whole work is based on the letter written by Prof. Christan Goldbach to Prof. Leonhard Euler in 1742. In this letter Prof. Christan Goldbach proposed that the integers greater than 2 (two) are the sum of three primes, on return Prof. Euler answered that this would follow the statement that every even integer greater than 2 (two) is the sum of two primes. This was the communication of two great mathematicians. Understanding the pattern and the modulus operandi used in this letter, a unique association between twin prime and Goldbach conjecture is developed. The association is discussed as under:

For Even Number

Let ‘E’ be any even number, $E > 5$

$$E = T + C + 2 \quad (1)$$

$$T \in \{ T_1, T_2 \}$$

$\{ T_1, T_2 \}$ is the first closest twin prime set less than $(E - 2)$. In some cases it may be second or third closest twin prime set near $(E - 2)$. $T_1, T_2 < (E - 2)$.

$$a) E = (T + 2) + C$$

OR

$$b) E = T + (C + 2)$$

$$E = P_1 + P_2$$

For Odd Number

Let 'O' be any odd number, $O > 6$

$$O = E + 1 \quad (2)$$

$$O = T + C + 2 + 1$$

$$T \in \{ T_1, T_2 \}$$

$$O = P_1 + P_2 + P_3$$

$\{ T_1, T_2 \}$ is the first closest twin prime set less than $(E - 2)$. In some cases it may be second or third closest twin prime set near $(E - 2)$. $T_1, T_2 < (E - 2)$.

The claim is, using (1) any even number can be written as the sum of two prime numbers and using (2) any odd number can be written as the sum of three prime numbers.

Inference

- (1) and (2) provides the solution of strong and weak Goldbach Conjecture.
- Since there are infinite number of even/odd numbers and as per (1) and (2) every even/odd number is related to closest twin prime set, it means that there are infinite many twin prime numbers.

Since the matter is most sensitive, the two conjectures are very challenging, so I submit this claim to the world mathematicians with these two statements (a) and (b) and hope it will lead us to new paradigm in prime conjectures.

Reference:

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