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THE STATISTICAL AND SQUEEZING PROPERTIES OF THREE LEVEL LASER DRIVEN BY COHERENT LIGHT

BY

G/MARIAM G/ANENIA
Physics department ,
Raya University, maychew, Ethiopia

Abstract

: In this thesis, we study the statistical and quantum properties of the light generated by a coherently driven three level laser in which three level atoms in a cavity coupled to a vacuum reservoir are pumped to upper level at a rate of r_a. Applying the quantum langevein equation for our system, we obtain the explicit form of atomic operators. Employing Q function we have calculated the mean photon number, variance of photon number, photon number distribution and quadrature variance. It is found that the mean and variance of photon number are greater for $a = 0$ than for $a = 1$. Furthermore, we determine the quadrature squeezing such as global and local quadrature squeezing for the cavity mode light. And 50% of maximum quadrature squeezing have been obtained from the system under consideration for $x = 0.5$ and $a = 0$.

IndexTerms - Component,formatting,style,styling,insert.

Quantum optics is a field of quantum physics that deals specifically with the interaction of radiation with matter. The quantum and statistical properties of the radiation generated by different quantum optical systems have been investigated since the earliest days that quantum optics was known. The quantum description of radiation is one of the central topics in quantum optics. This description requires the quantization of the radiation field. The quantization of the radiation field leads to the introduction of various possible quantum states of light such as the number, the coherent, chaotic and the squeezed states. Squeezed state is one of a non-classical features of light that has attracted a great deal of interest. In squeezed light the noise in one

quadrature is below the coherent state level at the expense of enhanced fluctuations in the other quadrature with the product of the uncertainty in the two quadrature satisfying the uncertainty relation. Squeezed light has potential application in low noise optical communication and weak signal detection. Quantum optics developed through the first half of the twentieth century more by understanding how photons

and matter interacted and interrelated . And also in 1953, the maser (micro amplifcation by stimulated emission of radiation) was developed which emitted coherent micro wave. After a time of being, laser (light amplification by stimulated emission of radiation) could be developed by different quantum optical system [1].There has been considerable interest in the analysis of the squeezing and the statistical properties of the light generated by the three level laser and coherently driven by cavity mode. A three level laser may be be defined as a quantum optical system in which three level atom in a cascade configuration,

Initially prepared in a coherent super position **of** the top and the bottom levels, are injected into a cavity coupled to a vacuum reservoir via a single port-mirror. When a three-level atom in a cascade configuration makes a transition from the top to the bottom level via the intermediate level, two photons are generated. If the two photons have same frequency, then the three level atom is called degenerate three-level atom otherwise it is called non degenerate[2]. The squeezing and statistical properties of the light produced by three-level atoms in which the crucial role is played by the coupling of the top and bottom levels. The coupling of the top and bottom levels is responsible for the interesting non-classical feature of the generated light. In general the atomic coherence can be induced in a three-level atom by coupling the levels between which direct transition is dipole for bidden by coherent light or by preparing the atom initially in coherent superposition

of these two levels [3]. The statistical properties of three level lasing were investigated theoretically. It is assumed that the three level medium was coherently excited by another laser with an arbitrary photon statistics. It was proved that under the specific conditions, the photon statistics of three level laser duplicate the photon statistics of the exciting laser. Generally the three level of a laser is a unique source of bright light [4]. The statistical and squeezing properties of light generated by three level lasers have been investigated by several authors [6]-[11]. Fesseha Kassahun [5] was found that a three level laser in either model generated squeezed light under certain conditions. It appears to be quite difficult to prepare the atoms in coherent super-positions of the top and bottom levels before they are injected in to the laser cavity. In addition,he studied that the atoms have decayed spontaneously before they are removed from the cavity. On the other hand, he study showed the degree of squeezing of the light generated by the three level laser, with the top and bottom levels coupled by coherent light, is relatively large when the mean photon number is relatively very small. A three level laser in which the top and bottom levels of the atoms injected into the cavity are coupled by a strong light has also been studied by different authors. Misrak Getahun [6] studied the squeezing and statistical properties of the light produced by a three-level laser whose cavity contains a parametric amplifier and with the cavity mode driven by coherent light and coupled to a squeezed vacuum reservoir. He obtained stochastic differential equations associated with the normal ordering using the pertinent master equation. Making use of the solutions of the resulting differential equations, he calculated the quadrature variances and squeezing spectrum. He also determined the mean and variance of the photon number and the

photon number distribution for the cavity mode employing the Q function. And almost perfect squeezing can be obtained for slightly more probability for the atoms to be in the bottom level and for large value of linear gain coefficient. Sintayehu [7] has a detailed analysis of the squeezing properties of the light produced by the degenerate three-level cascade laser coupled to an external coherent light via one of the coupler mirrors and vacuum reservoir in the other employing the stochastic differential equation associated with the normal ordering. He studied the squeezing properties and also calculated the mean photon number of the cavity radiation. The cavity radiation exhibits 98.3% squeezing under certain conditions

pertaining to the initial preparation of the superposition and strength of the coherentradiation. His results also showed that the mean photon number is found to be large where there is a better squeezing and the system under consideration could generate an intense squeezed light. Beyene [8] has studied the squeezing and statistical properties for degenerate three level laser and the superposition of light beams produced by pair of degenerate three level laser. He carried out the analysis applying the solutions of c-number Langevin equations associated with the normal ordering. These equations were obtained using the master equation obtained applying the linear approximation scheme with the help of Q function, he has calculated the mean photon number, the variance of photon number, the photon number distribution and the quadrature variances.

His results indicated that 47.9% quadrature squeezing has been obtained for A=3 and κ = 0.8 at steady state. It is observed that the degree of squeezing increases with the linear gain coefficient and the steady state mean photon number simple sum the mean photon numbers of the two light beams. The squeezing of the superposed light mode increases with the linear gain coefficient. His results indicated that 95.8%

squeezing have be found for A = 3 and κ = 0.8. Driba Demissie [9] has studied the quantum analysis of coherently driven three level laser. He carried out the analysis by applying the solution of c- number langevin equations associated with normal ordering. These equations were obtained applying the linear approximation scheme. Using of Q functions he calculated quadrature

variance, squeezing spectrum, the variance of the photon number and photon number distribution.

Dawit Hiluf and Fesseha Kassahun [10] have studied the statistical and squeezing properties odegenerate three level laser coupled to a squeezed vacuum reservoir. They carried out the analysis of quantum optical system using of the pertinent stochastic differential equations for the cavity mode variables associated with normal ordering. The solutions of the resulting equations are then used to calculate the quadrature variance the squeezing spectrum, the mean photon number and the variance of photon number distribution.

Mulugeta Melaku [11] has studied the quantum properties of the light emitted by a degenerate three level atom available in open cavity and driven by coherent light. And he also studied the interaction of the degenerate three level atoms with resonant coherent light in open cavity coupled to vacuum reservoir. From his thesis hederived the equation of evolutions of atomic and cavity mode operators by applying

the quantum langevin equation and the master equation. Using the steady state solution of the resulting equations he obtained photon statistics, quadrature squeezing and power spectrum. In this thesis, we consider the case in which N three level atoms in cascade configuration and available in a cavity coupled to a vacuum reservoir via a single port mirror are pumped from the lower level to the upper level at a rate of r_a. We first drive the equation of evolution of atomic operator applying the quantum langevin equation using of large approximation scheme. Employing the steady state solution of the resulting equation, we obtain the expectation value of the atomic operator. In addition using of Q function in anti normal order we calculate mean photon number, variance of photon number, photon number distribution and quadrature variance of cavity mode light. Finally we calculate quadrature squeezing such as global and local

quadrature squeezing for cavity mode light

2.The Three Level Laser

We consider here the case in which N three level atoms in cascade configuration are available in a closed cavity. We denote the top, middle and bottom levels of these atoms by $|a\rangle$, $|b\rangle$ and $|c\rangle$ respectively. In addition, we assume the cavity modes to be at resonance with the two transitions $|a\rangle \rightarrow |b\rangle$ and $|b\rangle \rightarrow |c\rangle$, with direct transition between levels $|a\rangle$ and $|c\rangle$ to be dipole forbidden as shown in Fig. (2.1)

In this chapter we seek to determine the operator dynamics for coherently driven three level atom in side a cavity, in which the atom is pumped by coherent light coupled to a vacuum reservoir. We first drive the equation of evolution of atomic operators applying the quantum langevin equation. Employing the steady state solution of the resulting equations, we obtain the expectation value of the atomic operators. The

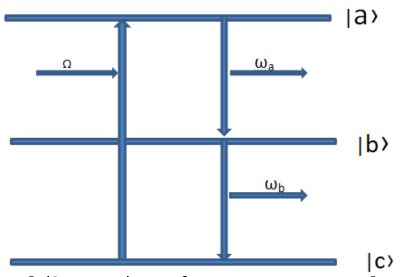

Fig. 2.1: The transition between $|a\rangle \rightarrow |b\rangle$ and $|b\rangle \rightarrow |c\rangle$ at frequency w_a and w_b are taken at

resonant with the cavity. coupling of the top and bottom levels of three level atom by coherent light treated

classically can be described by the Hamiltonian [12].

$$\hat{H} = \frac{i\Omega}{2}[\hat{\sigma}_c^{\dagger} - \hat{\sigma}_c], \tag{2.1}$$

where

$$\Omega = 2\varepsilon g, \tag{2.2}$$

$$\hat{\sigma}_c = |c\rangle\langle a|, \tag{2.3}$$

with g being the coupling constant between the coherent light and three level atom. Moreover, the interaction of three level atom with the cavity modes can be described at resonance by the Hamiltonian

$$\hat{H} = ig[\hat{\sigma}_a^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{\sigma}_a + \hat{\sigma}_b^{\dagger}\hat{b} - \hat{b}^{\dagger}\hat{\sigma}_b], \tag{2.4}$$

where $\hat{a}$ and $\hat{b}$ are the cavity mode operators,

$$\hat{\sigma}_a = |b\rangle\langle a|, \quad (2.5)$$

$$\hat{\sigma}_b = |c\rangle\langle b|. \quad (2.6)$$

are lowering atomic operators. On account of Eq. (2.1) and Eq. (2.4) the total Hamiltonian describing the interaction of a three level atom with the coherent light and cavity modes a and b has the form

$$\hat{H} = \frac{i\Omega}{2}[\hat{\sigma}_c^{\dagger} - \hat{\sigma}_c] + ig[\hat{\sigma}_a^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{\sigma}_a + \hat{\sigma}_b^{\dagger}\hat{b} - \hat{b}^{\dagger}\hat{\sigma}_b]. \quad (2.7)$$

We assume that the laser cavity is coupled to a vacuum reservoir via a single port mirror. Moreover, we carry out our calculation by putting the noise operators associated with the vacuum reservoir in normal order. Thus the noise operators will not have any effect on the dynamics of the cavity mode operators. We can therefore drop the noise operators and write the quantum langevin equations for the operators $\hat{a}$ and $\hat{b}$ as

$$\frac{d}{dt}\hat{a} = -i[\hat{a}, \hat{H}] - \frac{\kappa}{2}\hat{a}, \quad (2.8)$$

in which κ is the cavity damping constant. Then using Eq. (2.7), we easily find

$$\frac{d}{dt}\hat{a} = -\frac{\kappa}{2}\hat{a} - g\hat{\sigma}_a, \quad (2.10)$$

$$\frac{d}{dt}\hat{b} = -\frac{\kappa}{2}\hat{b} - g\hat{\sigma}_b, \quad (2.11)$$

2.1 Operator Dynamics

. In this section we seek to determine the time evolution for the expectation values of the atomic operators. To this end, using the relation

$$\frac{d}{dt}\hat{A} = -i[\hat{A}, \hat{H}], \quad (2.12)$$

along with Eq. (2.7), one can readily establish that

$$\frac{d}{dt}\hat{\sigma}_a = g(\hat{\eta}_b - \hat{\eta}_a)\hat{a} + g\hat{b}^{\dagger}\hat{\sigma}_c + \frac{\Omega}{2}\hat{\sigma}_b^{\dagger}. \quad (2.13)$$

where

$$\hat{\eta}_a = |a\rangle\langle a|, \quad (2.14)$$

$$\hat{\eta}_b = |b\rangle\langle b|, \quad (2.15)$$

$$\hat{\eta}_c = |c\rangle\langle c|, \quad (2.16)$$

Following the same procedure, we can easily find that

$$\frac{d}{dt}\hat{\sigma}_b = \frac{\Omega}{2}(\hat{\eta}_c - \hat{\eta}_b) + g\,\hat{\sigma}_a^\dagger \hat{a} - g\,\hat{a}^\dagger \hat{\sigma}_c \hat{b}, \tag{2.17}$$

$$\frac{d}{dt}\hat{\sigma}_c = \frac{\Omega}{2}(\hat{\eta}_a - \hat{\eta}_c) + g(\hat{\sigma}_b \hat{a} - \hat{\sigma}_a \hat{b}), \tag{2.18}$$

$$\frac{d}{dt}\hat{\eta}_a = \frac{\Omega}{2}(\hat{\sigma}_c^\dagger + \hat{\sigma}_c) + g(\hat{\sigma}_a^\dagger \hat{a} + \hat{a}^\dagger \hat{\sigma}_a), \tag{2.19}$$

$$\frac{d}{dt}\hat{\eta}_b = -g(\hat{\sigma}_a^\dagger \hat{a} + \hat{a}^\dagger \hat{\sigma}_a) + g(\hat{\sigma}_b^\dagger \hat{b} + \hat{b}^\dagger \hat{\sigma}_b), \tag{2.20}$$

$$\frac{d}{dt}\hat{\eta}_c = -\frac{\Omega}{2}(\hat{\sigma}_c + \hat{\sigma}_c^\dagger) - g(\hat{\sigma}_b^\dagger \hat{b} + \hat{b}^\dagger \hat{\sigma}_b) \tag{2.21}$$

The probability of finding an atom in the top, middle and bottom levels respectively are defined by $\hat{\eta}_a$, $\hat{\eta}_b$ and $\hat{\eta}_c$ respectively. We observe that Eqs. (2.13), (2.17), (2.18), (2.19), (2.20) and Eq. (2.21) are non liner differential equations. Hence it is not possible to obtain the exact time dependent solutions of these equations. Thus applying the large time approximation schemes, we obtain from Eq. (2.10) and Eq. (2.11) the approximately valid relation

$$\hat{a}(t) = -\frac{2g}{\kappa}\hat{\sigma}_a. \tag{2.22}$$

$$\hat{b}(t) = -\frac{2g}{\kappa}\hat{\sigma}_b. \tag{2.23}$$

Now inserting Eq. (2.22) and Eq. (2.23) into Eq. (2.13), (2.17), (2.18), (2.19), (2.20) and (2.21), we have

$$\frac{d}{dt}\hat{\sigma}_a = -\gamma_c\,\hat{\sigma}_a + \frac{\Omega}{2}\hat{\sigma}_b^\dagger, \tag{2.24}$$

$$\frac{d}{dt}\hat{\sigma}_b = -\frac{\gamma_c}{2}\hat{\sigma}_b - \frac{\Omega}{2}\hat{\sigma}_a^\dagger, \tag{2.25}$$

$$\frac{d}{dt}\hat{\sigma}_c = -\frac{\gamma_c}{2}\hat{\sigma}_c + \frac{\Omega}{2}(\hat{\eta}_a - \hat{\eta}_c), \tag{2.26}$$

$$\frac{d}{dt}\hat{\eta}_a = -\gamma_c\,\hat{\eta}_a + \frac{\Omega}{2}(\hat{\sigma}_c + \hat{\sigma}_c^\dagger), \tag{2.27}$$

$$\frac{d}{dt}\hat{\eta}_b = -\gamma_c\,\hat{\eta}_b + \gamma_c\,\hat{\eta}_a, \tag{2.28}$$

$$-\hat{\eta}_c = \gamma_c\,\hat{\eta}_b - \frac{\Omega}{2}\sigma_c()\hat{\sigma}_c^{\dagger} + \hat{} \tag{2.29}$$

$$\gamma_c = \frac{4g^2}{\kappa}. \tag{2.30}$$

The parameter defined by Eq. (2.30) is called the stimulated emission decay constant.

The three level atoms available in the cavity are pumped from the bottom level to the top level by means of electron bombardment . The pumping process must surely affect the dynamics of η_a and $\hat{\eta}_c$. If r_a represents the rate at which a single atom is pumped from the bottom to the top level then $\hat{\eta}_a$ increases at the rate of $r_a\ \hat{\eta}_c$ and $\hat{\eta}_c$ decrease at the same rate of $r_a\ \hat{\eta}_c$ [5]. Incorporating the effect of the pumping process, we can write Eqs. (2.27) and Eq. (2.29) as

$$\frac{d}{dt}\hat{\eta}_a = -(\gamma_c\hat{\eta}_a + \frac{\Omega}{2}\sigma_c^{\dagger}) + r_a\ \hat{\eta}_c. \tag{2.31}$$

$$\frac{d}{dt}\hat{\eta}_c = -(\gamma_c\hat{\eta}_b + \frac{\Omega}{2}\hat{\sigma}^{\dagger}\hat{\sigma}_c) - r_a\ \eta_c\ . \tag{2.32}$$

Now summing over N three level atoms, we can write Eqs. (2.24), (2.25), (2.26), (2.28), (2.31) and Eq. (2.32) in the form of

$$\frac{d}{dt}m_a = -\gamma_c\,\hat{m}_a + \frac{\Omega}{2}\hat{m}_b^{\dagger}, \tag{2.33}$$

$$\frac{d}{dt}\hat{m}_b = -\frac{\gamma_c}{2}\hat{m}_b - \frac{\Omega}{2}\hat{m}_a^{\dagger}, \tag{2.34}$$

$$\frac{d}{dt}\hat{m}_c = -\frac{\gamma_c}{2}(\hat{m}_c + \frac{\Omega}{2}\hat{N}_a), \tag{2.35}$$

$$\frac{d}{dt}\hat{N}_a = -\gamma_c\,\hat{N}_a + \frac{\Omega}{2}(\hat{m}_c + \hat{m}_c^{\dagger}) + r_a\,\hat{N}_c\ , \tag{2.36}$$

$$\frac{d}{dt}\hat{N}_b = -\gamma_c\,\hat{N}_b + \gamma_c\,\hat{N}_a\ , \tag{2.37}$$

$$\sum_{i}^{N}|b\rangle\langle a| = N\,|b\rangle\langle a|, \tag{2.39}$$

$$\sum_{i}^{N}|c\rangle\langle b| = N\,|c\rangle\langle b|, \tag{2.40}$$

$$\sum_{i}^{N} |c\rangle\langle a| = N\,|c\rangle\langle a|, \qquad (2.41)$$

$$\sum_{i}^{N} |a\rangle\langle a| = N\,|a\rangle\langle a|, \qquad (2.42)$$

$$\sum_{i}^{N} |b\rangle\langle b| = N\,|b\rangle\langle b|, \qquad (2.43)$$

$$\sum_{i}^{N} |c\rangle\langle c| = N\,|c\rangle\langle c|, \qquad (2.44)$$

Here the operators $\hat N_a$, $\hat N_b$ and $\hat N_c$ representing the number of atoms in the top, middle and bottom levels . Employing the parameter a, we can rewrite Eq. (2.35) as

$$\frac{d}{dt}\hat m_c = -\frac{\gamma_c}{2}\hat m_c + a\frac{\Omega}{2}\hat N_c - \frac{\Omega}{2}\hat N_a \qquad (2.45)$$

We see that we can recover Eq. (2.35) upon setting $a = 1$. We also consider some other value of a later on. By taking $r_a = 0$, we note that the steady state solutions of Eqs. (2.36) and (2.37) are given by

$$\hat N_a = \frac{\Omega}{2\gamma_c}(\hat m_c + \hat m_c^\dagger) \qquad (2.46)$$

$$\hat N_b = \hat N_a . \qquad (2.47)$$

$$= \frac{\Omega}{\gamma_c} \quad - a\frac{\Omega}{\gamma_c}$$

Hence with the aid of Eqs. (2.46) and (2.48), we get

$$\hat N_c = \left(\frac{\gamma_c^2 + a\Omega^2}{\Omega^2}\right)\hat N_a . \qquad (2.49)$$

In addition employing the completeness relation

$$\hat\eta_a + \hat\eta_b + \hat\eta_c = \hat I . \qquad (2.50)$$

we easily arrive at

$$\hat N_a + \hat N_b + \hat N_c = N. \qquad (2.51)$$

With the aid of Eqs. (2.47), (2.49) and Eq. (2.51) we readily find

$$\hat N_a = \frac{\Omega^2 N}{\gamma_c^2 + (2+a)\Omega^2}, \qquad (2.52)$$

Thus in view of this result Eq. (2.49) takes the form

$$\hat N_c = \frac{(\gamma_c^2 + a\Omega^2)}{\gamma_c^2 + (2+a)\Omega^2}N, \qquad (2.53)$$

Finally, combination of Eqs. (2.48), (2.52) and (2.53), we get

$$= \frac{\gamma_c \Omega N}{\gamma_c + (2 + a)\Omega^2} m_c. \quad (2.54)$$

In the presence of N three level atoms, we can rewrite Eq. (2.10) and (2.11) as

$$\frac{d}{dt}\hat{a} = -\frac{\kappa}{2}\hat{a} + \lambda \hat{m}_a. \quad (2.55)$$

$$\frac{d}{dt}\hat{b} = -\frac{\kappa}{2}\hat{b} + \lambda \hat{m}_b. \quad (2.56)$$

in which λ is a constant whose value remains to be fixed . Applying the steady state solution of Eq. (2.10), we get

$$[\hat{a}, \hat{a}^{\dagger}] = \frac{\gamma_c}{\kappa}(\hat{\eta}_b - \hat{\eta}_a), \quad (2.57)$$

and on summing over all atoms, we have

$$[\hat{a}, \hat{a}^{\dagger}] = \frac{\gamma_c}{\kappa}(\hat{N}_b - \hat{N}_a), \quad (2.58)$$

which

$$[\hat{a}, \hat{a}^{\dagger}] = \sum_{i=1}^{N} [\hat{a}_i, \hat{a}_i^{\dagger}]. \quad (2.59)$$

Stands for the commutator of $\hat{a}$ and $\hat{a}^{\dagger}$ when the cavity mode is interacting with all the

.

N three level atoms . On the other hand, using the steady state solution of Eq. (2.55), one can easily verify that

$$[\hat{a}, \hat{a}^{\dagger}] = \frac{2\lambda^2}{\kappa}(\hat{N}_b - \hat{N}_a), \quad (2.60)$$

Thus on account of Eqs. (2.58) and Eq. (2.60), we say that

$$\lambda = \pm\sqrt{\frac{g}{N}} \quad (2.61)$$

And in view of this result, Eqs. (2.55) and (2.56) can be written as

$$\frac{d}{dt}\hat{a} = -\frac{\kappa}{2}\hat{a} + \frac{g}{\sqrt{N}} \quad (2.62)$$

$$\frac{d}{dt}\hat{b} = -\frac{\kappa}{2}\hat{b} + \frac{g}{\sqrt{N}} \quad (2.63)$$

Lastly taking in to account in steady state, the cavity mode operators can be connected with atomic operator as

$$\sqrt{\frac{2g}{N}}\,\hat{m}_a. = \qquad (2.64)$$

$$\sqrt{\frac{2g}{\kappa N}}\,\hat{m}_b. = \qquad (2.65)$$

Then up on adding Eqs. (2.62) and Eq. (2.63)

$$\hat{c} = -\frac{\kappa}{2}\frac{d}{dt} \quad -\hat{c} + \frac{g}{N} \qquad (2.66)$$

Where

$$m = m_a + \hat{m}_b. \qquad (2.67)$$

In addition one can easily verify that for N identical three level atoms

$$m^{\dagger}m = N(\hat{N}_a + \hat{N}_b). \qquad (2.68)$$

$$mm^{\dagger} = N(N_b + N_c). \qquad (2.69)$$

$$m^2 = N m_c. \qquad (2.70)$$

3.The Q Function for cavity mode light

The Q function for the cavity mode light is expressible in terms of anti normally ordered characteristic function as[15].

$$Q(\alpha', \alpha, t) = d^2 z \varphi_a(z', z, t) exp[z'\alpha - z\alpha'], \qquad (3.1)$$

where

$$\varphi_a(z', z, t) = Tr \;\; \rho_{(0)} e^{-z\,\hat{a}(t)} e^{z'\hat{a}^{\dagger}(t)} . \qquad (3.2)$$

In our case using of Eq. (2.66), we have

$$\hat{c}(t) = \hat{a}(t) + \hat{b}(t). \qquad (3.3)$$

Then Eq. (3.2) becomes

$$\varphi_a(z', z, t) = Tr \;\; \rho_{(0)} e^{-z\,\hat{c}(t)} e^{z'\hat{c}^{\dagger}(t)} , \qquad (3.4)$$

and one can say that

$$[\hat{c}, \hat{c}^{\dagger}] = \hat{c}\hat{c}^{\dagger} - \hat{c}^{\dagger}\hat{c} = \eta.$$

where

$$\hat{\eta} = \frac{\gamma_c}{\kappa}[\, N_c - N_a \,], \qquad (3.6)$$

Using of Baker-Housdorff identity

Eq. (3.4) can be written as

$$e^{\frac{1}{2}z z'} \quad e^{-z'\hat{c}^{\dagger}\hat{c}} \qquad (3.8)$$

$$\varphi_a(z', z, t) = e^{\eta z' z}$$

Now assuming $\hat{c}(t)$ is a Gaussian variable with zero mean. we can write as

$$\varphi_a(z', z, t) = e^{\eta z' z} exp \frac{1}{2} \frac{1}{2} [z\hat{c}^\dagger - z'\hat{c}]^2 ,$$

and this can be written as

$$\varphi_a(z', z, t) = e^{\eta z' Z} exp \frac{1}{2} \frac{1}{2}[z^2 \hat{c}^{\dagger 2} + z'^2 \hat{c}^2 - z'z \hat{c}^\dagger\hat{c} - z'z \hat{c}\hat{c}^\dagger] ,$$

This can be put in the form

$$\varphi_a(z', z, t) = exp - \frac{1}{2} z'z(\hat{c}^\dagger\hat{c} + \hat{c}\hat{c}^\dagger) + \frac{1}{2}(z^2 \hat{c}^{\dagger 2} + z'^2 \hat{c}^2) , \quad (3.11)$$

From Eq. (3.3) and its conjugate we have

$$\hat{c}^\dagger\hat{c} = n = \frac{\gamma_c}{\kappa} - N_a + N_b , \quad (3.12)$$

and

$$d = \hat{c}\hat{c}^\dagger = \frac{\gamma_c}{k} N_b + N_c , \quad (3.13)$$

and also one can write as

$$\hat{c}\hat{c}^\dagger = \eta + \hat{c}^\dagger\hat{c} = n + \eta, \quad (3.14)$$

$$\hat{c}^2 = \hat{c}^{\dagger 2} = \frac{\gamma_c}{\kappa} - \hat{m}_c = \frac{\gamma_c}{\kappa} \frac{x}{1 + (2 + a)x^2} N. \quad (3.15)$$

where

$$\hat{c}^2 = \hat{c}^{\dagger 2} = b = \frac{\gamma_c}{\kappa} \frac{x}{1 + (2 + a)x^2} N, \quad (3.16)$$

with

$$x = \frac{\Omega}{\gamma_c}. \quad (3.17)$$

Applying Eq. (3.12), (3.14) and Eq. (3.16) into Eq. (3.11), we see that

$$\varphi_a(z', z, t) = exp[-z'z \hat{c}\hat{c}^\dagger + \frac{b}{2}(z^2 + z'^2)], \quad (3.18)$$

This can be written as in the form

$$\varphi_a(z', z, t) = exp[-dz'z + (z^2 + z'^2)]\frac{b}{2} \quad (3.19)$$

where

$$d = n + \eta. \quad (3.20)$$

substituting Eq. (3.19) into (3.1), we have

$$Q(\alpha', \alpha, t) \frac{\eta}{\pi} exp[-dz'z \frac{d^2 z}{\pi} - \alpha'z + \alpha z' + (z^z + z'^2)]\frac{b}{2} \quad (3.21)$$

so that carrying out the integration with the help of the relation

$$exp[-dz'z + bz + cz' + Az^2 + Bz'^2]\frac{d^2z}{\pi} = \sqrt{\frac{\eta}{d^2 - 4AB}} exp \frac{dbc + Ac^2 + Bb^2}{d^2 - 4AB}, \quad d > 0. \tag{3.22}$$

we get

$$Q(\alpha', \alpha, t) = \frac{\eta}{\pi(d^2 - b^2)^{\frac{1}{2}}} exp - \frac{d\alpha'\alpha}{d^2 - b^2} + \frac{b}{2(d^2 - b^2)} (\alpha^2 + \alpha'^2), \tag{3.23}$$

$$Q(\alpha', \alpha, t) = \frac{\eta(u^2 - v^2)^{\frac{1}{2}}}{\pi} exp - u\alpha'\alpha + (\alpha^2 + \alpha'^2)\frac{v}{2}. \tag{3.24}$$

One can check that the Q function is normalized to η which is

$$d^2\alpha Q(\alpha', \alpha, t) = \eta. \tag{3.25}$$

where the value of u and v in Eq. (3.24) are given by

$$\mu = \frac{d}{d^2 - b^2}. \tag{3.26}$$

and

$$v = \frac{b}{d^2 - b^2}. \tag{3.27}$$

Eq. (3.24) represents the Q function for cavity mode light produced by coherent light

3.1 Photon Statistics

The statistical properties of light beam can be described in terms of the mean photon number, variance of photon number, phot0n number distribution and power spectrum. Here employing the Q function we wish to calculate the mean photon number, variance of photon number and photon number distribution for the cavity light generated by three level laser. Finally, we determine the power spectrum of cavity mode light produced by driven coherent light.

3.1.1 Mean Photon Number

The expectation value of an operator function $A(\hat{a}^\dagger, \hat{a})$ is expressible in terms of density operator as

$$\hat{A} = Tr(\rho\hat{A}). \tag{3.28}$$

Expanding the density operator in normal order and using the density operator for coherent state Eq. (3.28) can be written as

$$\hat{A} = \frac{d^2\alpha}{\eta} Q(\alpha', \alpha + \eta\frac{\partial}{\partial\alpha'}, t)\hat{A}_n(\alpha', \alpha), \tag{3.29}$$

Where $\hat{A}_n(\alpha', \alpha)$ is the c-number function corresponding to the operator function $\hat{A} = \hat{A}(\hat{c}, \hat{c}^\dagger)$ with the normal order. On the basis of Eq. (3.29), the mean photon number for a cavity mode light is expressed by

$$\bar{n} = \int \frac{d^2\alpha}{\eta} Q\left(\alpha', \alpha + \eta\frac{\partial}{\partial\alpha'}, t\right)\alpha'\alpha. \quad (3.30)$$

When we express $Q(\alpha', \alpha + \eta\frac{\partial}{\partial\alpha'}, t)$ in Eq. (3.30) in terms of $Q(\alpha', \alpha, t)$, we have

$$Q\left(\alpha', \alpha + \eta\frac{\partial}{\partial\alpha'}, t\right) = Q(\alpha', \alpha, t)\exp\left[-\eta(u\alpha' - v\alpha)\left(\frac{\partial}{\partial\alpha'} + \frac{v}{2}\eta^2\frac{\partial^2}{\partial\alpha'^2}\right)\right]. \quad (3.31)$$

On account of Eq. (3.31), we see that

$$\bar{n} = \int \frac{d^2\alpha}{\eta} Q(\alpha', \alpha, t)\exp\left[-\eta(u\alpha' - v\alpha)\left(\frac{\partial}{\partial\alpha'} + \frac{v}{2}\eta^2\frac{\partial^2}{\partial\alpha'^2}\right)\right]\alpha'\alpha, \quad (3.32)$$

Expanding the exponential function in power series, we have

$$\exp\left[-\eta(u\alpha' - v\alpha)\left(\frac{\partial}{\partial\alpha'} + \frac{v}{2}\eta^2\frac{\partial^2}{\partial\alpha^2}\right)\right] = 1 - \eta(u\alpha' - v\alpha)\left(\frac{\partial}{\partial\alpha'} + \frac{v}{2}\eta^2\frac{\partial^2}{\partial\alpha'^2}\right) - \frac{1}{2!}\left[\eta(u\alpha' - v\alpha)\left(\frac{\partial}{\partial\alpha'} + \frac{v}{2}\eta^2\frac{\partial^2}{\partial\alpha'^2}\right)\right]^2 + - - - \quad (3.33)$$

So that substituting Eq. (3.33) in to Eq. (3.32), we write as

$$\bar{n} = \int \frac{d^2\alpha}{\eta} Q(\alpha', \alpha, t)\left[\alpha'\alpha - \alpha\eta(u\alpha' - v\alpha)\right], \quad (3.34)$$

And also this can be written as in the form of

$$\bar{n} = \int \frac{d^2\alpha}{\eta} Q(\alpha', \alpha, t)\left[(1 - \eta u)\alpha'\alpha + \eta v\alpha^2\right], \quad (3.35)$$

Eq. (3.35) can be written as

$$\bar{n} = (1 - \eta u)I_1 + \eta v I_2, \quad (3.36)$$

where

$$I_1 = \int \frac{d^2\alpha}{\eta} Q(\alpha', \alpha, t)\alpha'\alpha, \quad (3.37)$$

$$I_2 = \int \frac{d^2\alpha}{\eta} Q(\alpha', \alpha, t)\alpha^2, \quad (3.38)$$

On account Eq. (3.24), we see that

$$I_1 = [u^2 - v^2]^{1/2}\int \frac{d^2\alpha}{\pi}\exp\left[-u\alpha'\alpha + \frac{v}{2}(\alpha^2 + \alpha'^2)\right]\alpha'\alpha. \quad (3.39)$$

This equation can be rewritten as

$$I_1 = -(u^2 - v^2)^{1/2} \text{—} \quad - ua' \alpha + \quad (\alpha^2 + \alpha'^2) - , \tag{3.40}$$

Further more, carrying out the integration,we find

$$I_1 = -(u^2 - v^2)^{1/2} \_\_[(u^2 - v^2)^{1/2}_{du}^{d}]. \tag{3.41}$$

Performing the differentiation, we get

$$I_1 = -[u^2 - v^2]^{1/2}(\quad \frac{-1}{2})[u^2 - v^2]^{-3/2} 2u, \tag{3.42}$$

Following similar procedure, we easily find I_2 using of Eq. (3.24)

$$I_2 = [u^2 - v^2]^{1/2} \quad - ua' \frac{d^2\alpha}{\pi} exp \quad (\gamma\alpha^2 + \alpha' \frac{\gamma}{2}) \quad \alpha^2|_{\gamma=1}, \tag{3.44}$$

This can be written as

$$I_2 = [u^2 - v^2]^{1/2} \frac{2}{v}\frac{d}{d\gamma} exp \quad - ua' \frac{d^2\alpha}{\pi} \alpha + \quad (\gamma\alpha^2 + \alpha' \frac{\gamma}{2}) \quad |_{\gamma=1}, \tag{3.45}$$

Integrating over α Eq. (3.45) becomes

$$I_2 = [u^2 - v^2\gamma]^{-1/2}|_{\gamma=1}, \quad \frac{2(u^2 - v^2)^{1/2}}{v} \frac{d}{d\gamma} \tag{3.46}$$

This gives as

$$I_2 = \frac{2[u^2 - v^2]^{1/2}}{v} \quad (-\frac{1}{2})(u^2 - v^2)^{-3/2}(-v^2), \tag{3.47}$$

Finally we can write I_2 in the form of

$$I_2 = \quad \frac{v}{u^2 - v^2}. \tag{3.48}$$

Substituting Eq. (3.43) and Eq. (3.48) into Eq. (3.36), we obtain

$$\bar{n}(1 - \eta u) + \eta \frac{u}{u^2 - v^2}, \quad \frac{v^2}{u^2 - v^2} \tag{3.49}$$

This can be written in the form of

$$n = \frac{u}{u^2 - v^2} \quad \bar{} \quad \text{———} \quad - \eta. \tag{3.50}$$

Taking in to account Eq. (3.26), the mean photon number, is expressed as

$$n = d - \eta. \quad \bar{} \tag{3.51}$$

More over using Eqs. (3.12) and (3.20),we have

$$\tilde{N}_a n + \frac{\gamma_c}{\kappa} \hat{N}_b \quad , \tag{3.52}$$

Substituting of Eq. (2.47) and Eq. (2.52) into Eq. (3.52), we finally obtain

$$\frac{\gamma_c}{\kappa} \quad \bar{n} = \frac{2x^2}{1 + (2 + a)x^2} \text{———} \quad N \tag{3.53}$$

Eq. (3.53) represents the mean photon number for the cavity mode light produced by the coherent light.

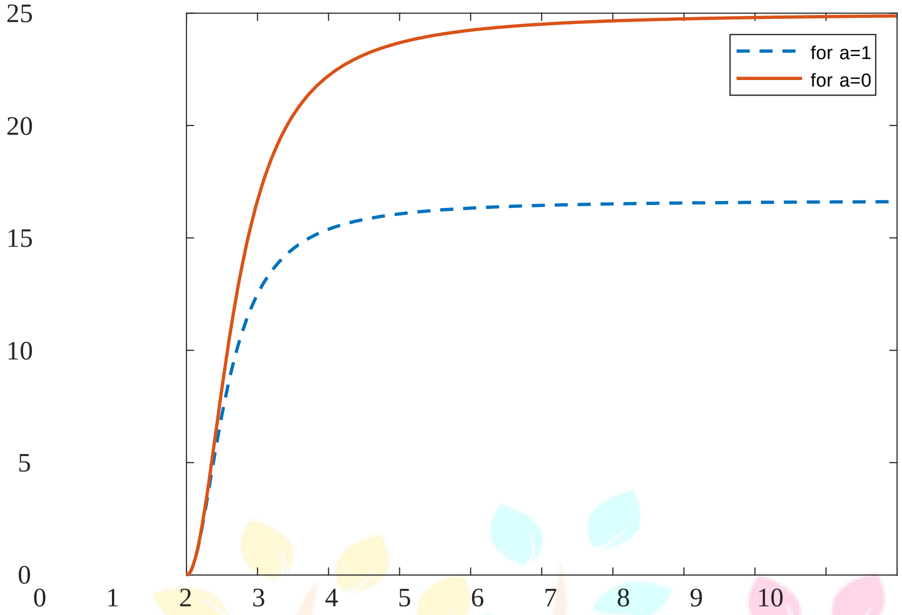

Fig. 3.1: plots of mean photon number(Eq. (3.53)) versus x for $\gamma_c = 0.4, \kappa = 0.8$ and $N = 50$.

The plot in Fig. (3.1) indicates that the mean photon number for $a = 0$ is greater than for $a = 1$. And the mean photon number for the cavity mode light is increase as the parameter x increase for small value of a.

3.1.2 The Variance of Photon Number

The variance of the photon number for cavity mode light can be written as

$$(\Delta n)^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 . \tag{3.54}$$

$$(\Delta n)^2 = \langle \hat{c}^\dagger \hat{c} \hat{c}^\dagger \hat{c} \rangle - \langle \hat{c}^\dagger \hat{c} \rangle^2 , \tag{3.55}$$

and

$$(\Delta n)^2 = \langle \hat{c}^{\dagger 2} \hat{c}^2 \rangle + \langle \hat{c}^\dagger \hat{c} \rangle - \bar{n}^2 . \tag{3.56}$$

From the fact that one can write

$$\langle \hat{c}^{\dagger 2} \hat{c}^2 \rangle = \int d^2\alpha Q(\alpha', \alpha + \eta \frac{\partial}{\partial \alpha'}, t) \alpha'^2 \alpha^2 , \tag{3.57}$$

Where α'^2 and α^2 are c- number variables corresponding to the operator $\hat{c}^{\dagger 2}$ and $\hat{c}^2$.

And also When we write $Q(\alpha', \alpha + \eta \frac{\partial}{\partial \alpha'}, t)$ in terms of $Q(\alpha', \alpha, t)$ we use the following expression.

$$Q(\alpha', \alpha + \eta \frac{\partial}{\partial \alpha'}, t) = Q(\alpha', \alpha, t) exp\left[- \eta[u\alpha' - v\alpha] \frac{\partial}{\partial \alpha'} + \frac{v}{2} \eta^2 \frac{\partial^2}{\partial \alpha'^2} \right] , \tag{3.58}$$

Applying Eq. (3.33) along with Eq. (3.58),we write as

$$\langle \hat{c}^{\dagger 2} \hat{c}^2 \rangle = \int d^2\alpha Q(\alpha', \alpha, t) \left[1 - \eta[u\alpha' - v\alpha] \frac{\partial}{\partial \alpha'} + M \frac{\partial^2}{\partial \alpha'^2} \right.$$

$+$

$$\frac{1}{2!}\left[-\eta(u\alpha' - v\alpha)\frac{\partial}{\partial\alpha'}\right]^2 + M\frac{\partial^2}{\partial\alpha'^2}\alpha'^2\alpha^2, \qquad (3.59)$$

where

$$M = \frac{v}{2}\eta^2. \qquad (3.60)$$

From Eq. (3.59) it then follows that

$$\hat{c}^{\dagger 2}\hat{c}^2 = \int d^2\alpha\, Q(\alpha', \alpha, t)\Big[\alpha'^2\alpha^2 - \eta[u\alpha' - v\alpha]\frac{\partial}{\partial\alpha'}(\alpha'^2\alpha^2) + M\frac{\partial^2}{\partial\alpha'^2}(\alpha'^2\alpha^2)$$

$$+\frac{1}{2!}\left(-\eta[u\alpha' - v\alpha]\frac{\partial}{\partial\alpha'} + M\frac{\partial^2}{\partial\alpha'^2}\right)^2\alpha'^2\alpha^2\Big], \qquad (3.61)$$

And also Eq. (3.61) can be put in the form

$$\hat{c}^{\dagger 2}\hat{c}^2 = \int d^2\alpha\, Q(\alpha', \alpha, t)\Big[\alpha'^2\alpha^2 - \eta[u\alpha' - v\alpha].2\alpha'\alpha^2 + 2M\alpha^2$$

$$+\frac{1}{2!}\left(-\eta[u\alpha' - v\alpha]\frac{\partial}{\partial\alpha'}\right)^2\alpha'^2\alpha^2\Big], \qquad (3.62)$$

From Eq. (3.62) one can say that

$$\hat{c}^{\dagger 2}\hat{c}^2 = \int d^2\alpha\, Q(\alpha', \alpha, t)\Big[\alpha'^2\alpha^2 - 2\eta u\alpha'^2\alpha^2 + 2\eta v\alpha'\alpha^3 + 2M\alpha^2$$

$$+\frac{1}{2}\eta^2[u\alpha' - v\alpha]^2\frac{\partial^2}{\partial\alpha'^2}(\alpha'^2\alpha^2)\Big], \qquad (3.63)$$

Then we can put Eq. (3.63) as

$$\hat{c}^{\dagger 2}\hat{c}^2 = \int d^2\alpha\, Q(\alpha', \alpha, t)\Big[\alpha'^2\alpha^2 - 2\eta u\alpha'^2\alpha^2 + 2\eta v\alpha'\alpha^3 + 2M\alpha^2$$

$$+ u^2\eta^2\alpha'^2\alpha^2 - 2uv\eta^2\alpha'\alpha^3 + \eta v^2\alpha^4\Big], \qquad (3.64)$$

And also one can write Eq. (3.64) as

$$\hat{c}^{\dagger 2}\hat{c}^2 = \int d^2\alpha\, Q(\alpha', \alpha, t)\Big[(u\eta - 1)^2\alpha'^2\alpha^2 + 2\eta v(1 - u\eta)\alpha'\alpha^3 + v\eta^2\alpha^2 + v^2\eta^2\alpha^4\Big], \qquad (3.65)$$

This can be rewritten as

$$\hat{c}^{\dagger 2}\hat{c}^2 = (u\eta - 1)^2 I_1 + 2v\eta(1 - u\eta)I_2 + v\eta^2 I_3 + v^2\eta^2 I_4, \qquad (3.66)$$

in which

$$I_1 = \int\frac{d^2\alpha}{\eta}Q(\alpha', \alpha, t)\alpha'^2\alpha^2, \qquad (3.67)$$

$$I_2 = \int\frac{d^2\alpha}{\eta}Q(\alpha', \alpha, t)\alpha'\alpha^3, \qquad (3.68)$$

$$I_3 = \int\frac{d^2\alpha}{\eta}Q(\alpha', \alpha, t)\alpha^2, \qquad (3.69)$$

$$I_4 = \int\frac{d^2\alpha}{\eta}Q(\alpha', \alpha, t)\alpha^4, \qquad (3.70)$$

Using of Eq. (3.24), we have

$$I_1 = (u^2 - v^2)^{1/2} \frac{d^2\alpha}{\pi} exp[-u\alpha'\alpha + \frac{v}{2}(\alpha^2 + \alpha'^2)]\alpha'^2\alpha^2, \quad (3.71)$$

and this expression can be put in the form

$$I_1 = (u^2 - v^2)^{1/2} \frac{d^2}{du^2} \frac{d^2\alpha}{\pi} exp[-u\alpha'\alpha + \frac{v}{2}(\alpha'^2 + \alpha^2)], \quad (3.72)$$

Carrying of the integration with the help of Eq. (3.22), we get

$$I_1 = (u^2 - v^2)^{1/2} \frac{d^2}{du^2}[u^2 - v^2]^{-1/2}, \quad (3.73)$$

Then it follows that

$$I_1 = (u^2 - v^2)^{1/2} \frac{d}{du}(-u(u^2 - v^2)^{-3/2}), \quad (3.74)$$

Applying differentiation, we find

$$I_1 = (u^2 - v^2)^{1/2} \left[\frac{-1}{(u^2 - v^2)^{3/2}} \quad \frac{3u^2}{(u^2 - v^2)^{5/2}}\right], \quad (3.75)$$

One can rewrite this as

$$I_1 = \frac{(2u^2 + v^2)}{(u^2 - v^2)^2}. \quad (3.76)$$

Following similar procedure, we also find that

$$I_2 = \frac{3uv}{(u^2 - v^2)^2}, \quad (3.77)$$

$$I_3 = \frac{v}{u^2 - v^2}, \quad (3.78)$$

and

$$I_4 = \frac{3v^2}{(u^2 - v^2)^2}, \quad (3.79)$$

Substituting Eq. (3.76), (3.77), (3.78) and Eq. (3.79) into Eq. (3.66), we get

$$\hat{c}^{\dagger 2}\hat{c}^2 = 2\frac{(2u^2 + v^2)}{(u^2 - v^2)^2} + \frac{u}{u^2 - v^2} - 4\frac{}{}, \quad (3.80)$$

This can be written as

$$\hat{c}^{\dagger 2}\hat{c}^2 = 2(d - \eta)^2 + b^2, \quad (3.81)$$

Taking in to account Eq.(3.20), Eq. (3.81), takes the form

$$\hat{c}^{\dagger 2}\hat{c}^2 = 2\bar{n}^2 + b^2, \quad (3.82)$$

Employing Eq. (3.16), (3.53) and Eq. (3.82) into Eq. (3.56), we get

$$(\Delta n)^2 = \frac{3x^2\gamma_c + N(1 + a)x^4}{[1 + \kappa(2 + a)x^2]^2}. \quad (3.83)$$

Eq. (3.83) represents the variance of the photon number for cavity mode light. And the variance of photon number in the cavity is maximum when the number of a tomes are maximum.

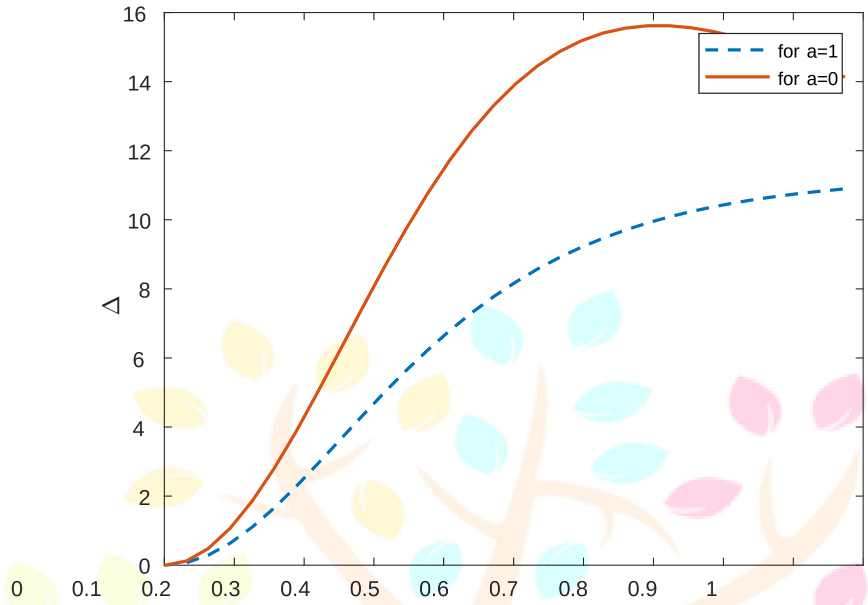

Fig. 3.2: plots variance of photon number(Eq. (3.83)) versus x for $\gamma_c = 0.4, \kappa = 0.8$ and $N = 10$.

The plot in Fig. (3.2) indicates that like mean photon number the variance of photon number for cavity mode is greater at $a = 0$ than that of $a = 1$. And also from the graph we observe that the variance of photon number for the cavity mode light is greater when the parameter x is greater as the value of a is smaller.

3.1.3 The Photon Number Distribution

We next seek to obtain, employing Q function the photon number distribution for the cavity mode produced by the system under consideration. The photon number distribution for cavity mode light is expressible in terms of the Q function as

$$P(n,t) = \frac{\pi}{n!}\frac{\partial \alpha^n}{\partial \alpha'^n \partial \alpha^n} Q(\alpha',\alpha,t)\, e^{\alpha'\alpha}\Big|_{\alpha'=\alpha=0}, \qquad (3.84)$$

Now using Eq. (3.24), the photon number distribution for the cavity mode under consideration can be written in the form

$$P(n,t) = \frac{1}{n!}[u^2 - v^2]^{1/2} \frac{\partial \alpha^n}{\partial \alpha'^n \partial \alpha^n} \exp\Big[(1-u)\alpha'\alpha + \frac{v}{2}(\alpha'^2 + \alpha^2)\Big]_{\alpha'=\alpha=0}, \qquad (3.85)$$

Then expanding the exponential functions in power series, we have:

$$e^{(1-u)\alpha'\alpha} = \sum_k \frac{(1-u)^k(\alpha'\alpha)^k}{k!}, \qquad (3.86)$$

$$e^{\frac{u\alpha^2}{2}} = \sum_l \frac{v^l\alpha^{2l}}{2^l l!}$$

$$ {}^{2} \quad \frac{}{l}, \qquad (3.87) $$

and

$$ e^{\frac{v\alpha'^2}{2}} = \sum_m \frac{v^m \alpha'^{2m}}{2^m m!}, \qquad (3.88) $$

Then Eq. (3.85) becomes

$$ P(n,t) = \frac{1}{n!}[u^2 - v^2]^{1/2} \sum_{klm} \frac{(1-u)^k v^{l+m}}{2^{l+m} k! l! m!} \frac{\partial \alpha^n}{\partial \alpha'^n} [\alpha'^{k+2m} \alpha^{k+2l}]_{\alpha^*} \frac{}{\partial \alpha^n}, \qquad (3.89) $$

Applying the properties of knocker delta and up on differentiating with the help of the identity

$$ \frac{\partial^m}{\partial x^m} x^n = \frac{n!}{(n-m)!} x^{n-m}, \qquad (3.90) $$

We arrive at

$$ P(n,t) = \frac{1}{n!}[u^2 - v^2]^{1/2} \sum_{klm} \frac{(1-u)^k v^{l+m}(k+2l)!(k+2m)!}{2^{l+m} k! m!(k+2l-n)!(k+2m-n)!} \delta k + 2l, n \delta k + 2m, n, \qquad (3.91) $$

Finally on setting of the result that $m = l$ and $k = n - 2$, the photon number distribution takes the form

$$ P(n,t) = [u^2 - v^2]^{1/2} \sum_{l=0}^{[n]} \frac{n!(1-n)^{n-2l} v^{2l}}{2^{2l} l!^2 (n-2l)!}, \qquad (3.92) $$

where $[n] = n/2$ for even n and $[n] = [n-1]/2$ for odd. Eq. (3.92) represents that the probability to observe n number of photons in the cavity decrease as n increase. There is a finite probability to observe odd number of photons in the cavity. This is due to the cavity damping. Moreover, the probability of observing even number of photons in general greater than the probability of observing odd number of photons

3.1.4 Power Spectrum

We next seek to obtain the power spectrum of the cavity mode light when both light modes a and b have the same central frequency w_0. The power spectrum of the cavity mode light is expressible by [14].

$$ P(w) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle \hat{c}^\dagger(t)\hat{c}(t+\tau)\rangle_{ss} e^{i(w-w_0)\tau} d\tau, \qquad (3.93) $$

where "ss" stands for a steady state. And upon integrating both sides of Eq. (3.93) over w, we get

$$ \int_{-\infty}^{\infty} P(w)dw = \int_{-\infty}^{\infty} \langle \hat{c}^\dagger(t)\hat{c}(t+\tau)\rangle_{ss} e^{-iw_0\tau} d\tau X \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iw\tau} dw. \qquad (3.94) $$

so that using the fact that

$$ \delta(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iw\tau} dw, \qquad (3.95) $$

We find

$$\int_{-\infty}^{\infty} P(w)dw = \int_{-\infty}^{\infty} \langle \hat{c}^{\dagger}(t)\hat{c}(t+\tau)\rangle_{ss} e^{-i(w-w_0)\tau}\delta(\tau)d\tau. \quad (3.96)$$

In view of the relation

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(x)|_{x=0}, \quad (3.97)$$

It then follows

$$\int_{-\infty}^{\infty} P(w)dw = \bar{n}, \quad (3.98)$$

in which $\bar{n}$ is the steady state mean photon number. On the basis of this result, we assert that $P(w)dw$ is the steady state mean photon number in the frequency interval between w and $w + dw$[13,14]. We hence realize that the local mean photon number in the interval between $w' = -\lambda$ and $w' = \lambda$ is expressible by

$$\bar{n}_{\pm\lambda} = \int_{-\lambda}^{\lambda} P(w')dw'. \quad (3.99)$$

in which $w' = w - w_0$.

It proves to be convenient to rewrite Eq. (3.93) as

$$P(w) = \frac{1}{2\pi}\int_{-\infty}^{0} \langle \hat{c}^{\dagger}(t)\hat{c}(t+\tau)\rangle_{ss} e^{i(w-w_0)\tau} d\tau + \frac{1}{2\pi}\int_{0}^{\infty} \langle \hat{c}^{\dagger}(t)\hat{c}(t+\tau)\rangle_{ss} e^{i(w-w_0)\tau} d\tau, \quad (3.100)$$

so that replacing t by $-\tau$ in the first integral, we find

$$P(w) = \frac{1}{2\pi}\int_{0}^{\infty} \langle \hat{c}^{\dagger}(t)\hat{c}(t-\tau)\rangle_{ss} e^{-i(w-w_0)\tau} d\tau + \frac{1}{2\pi}\int_{0}^{\infty} \langle \hat{c}^{\dagger}(t)\hat{c}(t+\tau)\rangle_{ss} e^{i(w-w_0)\tau} d\tau, \quad (3.101)$$

We observe that one integral is the complex conjugate of the other. Hence the power spectrum can be written as

$$P(w) = \frac{1}{\pi} Re \int_{0}^{\infty} \langle \hat{c}^{\dagger}(t)\hat{c}(t+\tau)\rangle_{ss} e^{i(w-w_0)\tau} d\tau, \quad (3.102)$$

in which "Re" denotes the real part. We now proceed to determine the two time correlation function that appears in Eq. (3.93). To this end, we realize that the expectation value of the solution of Eq. (2.66) can be written as

$$\hat{c}(t+\tau) = \hat{c}(t)\, e^{-\kappa\tau/2} + \frac{g}{\sqrt{N}}\int_0^{\tau} e^{-\kappa\tau'/2}\hat{m}(t+\tau')\, d\tau', \quad (3.103)$$

Applying the large - time approximation schemes on Eq. (2.34), we calculate the power spectrum of cavity mode.

$$\hat{m}_b(t) = -\frac{\Omega}{\gamma_c}\hat{m}_a^{\dagger}(t), \quad (3.104)$$

Now applying the complex conjugate of this result, one can put Eq. (2.33) in the form

$$\frac{d}{dt}\hat{m}_a = -\mu\hat{m}_a, \quad (3.105)$$

in which

$$\mu = \gamma_c + \frac{\Omega^2}{2\gamma_c}. \quad (3.106)$$

With the atoms considered to be initially in the bottom level, the solution of Eq. (3.105) turns out to be

$$\langle \hat{m}_a(t) \rangle = 0. \tag{3.107}$$

Moreover, using the same approximation scheme, we obtained from Eq (2.33) that

$$\hat{m}_a(t) = \frac{\Omega}{2\gamma_c} \hat{m}_b^\dagger(t), \tag{3.108}$$

Then with the aid of the complex conjugate of this relation, we can put Eq. (2.34) in the form

$$\frac{d}{dt}\hat{m}_b = -\frac{1}{2}\mu \hat{m}_b , \tag{3.109}$$

With the atoms considered to be initially in the bottom level, the solution of Eq. (3.109) is found to be

$$\langle \hat{m}_b \rangle = 0. \tag{3.110}$$

Hence assuming the cavity light to be initially in a vacuum state we find the expectation value of the solution of Eq. (2.66) to be

$$\langle \hat{c}(t) \rangle = 0. \tag{3.111}$$

On account of Eqs. (3.105) and (3.109), we get

$$\frac{d}{dt}\hat{m}_b = -\frac{1}{2}\mu \hat{m}_b - \frac{1}{2}\mu \hat{m}_a , \tag{3.112}$$

and in view of Eq. (2.67), we see that

$$\frac{d}{dt}\hat{m} = -\frac{1}{2}\mu \hat{m} , \tag{3.113}$$

On the basis of this equation , one can write

$$\frac{d}{dt}\hat{m} = -\frac{1}{2}\mu \hat{m} + \hat{F}_m(t), \tag{3.114}$$

in which $\hat{F}_m(t)$ is a noise operator with a vanishing mean, the solution of Eq.'(3.114) can be put in the form

$$\hat{m}(t+\tau) = \hat{m}(t)e^{-\mu\tau/2} + e^{-\mu\tau/2}\int_0^\tau e^{\mu\tau'/2}\hat{F}_m(t+\tau')d\tau' , \tag{3.115}$$

Thus combination of Eqs. (3.103) and (3.115) yields

$$\hat{c}(t+\tau) = \hat{c}(t)e^{-\kappa\tau/2} + \frac{2g\hat{m}(t)}{\sqrt{N}(\kappa-\mu)}\left(e^{-\mu\tau/2} - e^{-\kappa\tau/2}\right) + \frac{g}{\sqrt{N}}\int_0^\tau d\tau' e^{(\kappa-\mu)\tau'/2}\int_0^{\tau'} d\tau'' e^{\mu\tau''/2}\hat{F}_m(t+\tau''), \tag{3.116}$$

Multiplying both sides on the left by $\hat{c}^\dagger$ and taking the expectation value of the resulting equation, we have

$$\langle \hat{c}^{\dagger}(t)\hat{c}(t+\tau)\rangle = \langle \hat{c}^{\dagger}(t)\hat{c}(t)\rangle e^{-\kappa\tau/2} + \frac{2g\langle \hat{c}^{\dagger}(t)\hat{m}(t)\rangle}{\sqrt{N}(\kappa-\mu)}\left(e^{-\mu\tau/2} - e^{-\kappa\tau/2}\right) + \frac{g}{\sqrt{N}} e^{-\kappa\tau/2} \int_0^{\tau} d\tau\, e^{(\kappa-\mu)\tau/2} \int_0^{\tau} d\tau\, e^{\mu\tau/2} \langle \hat{c}^{\dagger}(t)F_m(t+\tau)\rangle, \quad (3.117)$$

so that in view of the fact that

$$\langle \hat{c}^{\dagger}(t)F_m(t+\tau)\rangle = 0, \quad (3.118)$$

we readily arrive at

$$\langle \hat{c}^{\dagger}(t)\hat{c}(t+\tau)\rangle = \langle \hat{c}^{\dagger}(t)\hat{c}(t)\rangle e^{-\kappa\tau/2} + \frac{2g\langle \hat{c}^{\dagger}(t)\hat{m}(t)\rangle}{\sqrt{N}(\kappa-\mu)}\left[e^{-\mu\tau/2} - e^{-\kappa\tau/2}\right], \quad (3.119)$$

Applying the large - time approximation to Eq. (2.66) and Eq. (3.119), there emerges

$$\langle \hat{c}^{\dagger}(t)\hat{c}(t+\tau)\rangle = \langle \hat{c}^{\dagger}(t)\hat{c}(t)\rangle \left(\frac{\kappa}{\kappa-\mu} e^{-\mu\tau/2} - \frac{\mu}{\kappa-\mu} e^{-\kappa\tau/2}\right). \quad (3.120)$$

Hence on substituting Eq. (3.120) into Eq. (3.102) and carrying out the integration, we get

$$P(w) = \frac{\kappa}{\kappa-\mu}\frac{\mu/2\pi}{(w-w_0)^2 + (\mu/2)^2} - \frac{\mu}{\kappa-\mu}\frac{\kappa/2\pi}{(w-w_0)^2 + (\kappa/2)^2}. \quad (3.121)$$

Therefore, inserting Eq. (3.121) into Eq. (3.99) and carrying out the integration applying the relation

$$\int_{-\lambda}^{\lambda} \frac{dx}{x^2 + a^2} = \frac{2}{a} tan^{-1}(\lambda/a), \quad (3.122)$$

We arrive at

$$\bar{n}_{\pm\lambda} = \bar{n} z(\lambda). \quad (3.123)$$

where $\bar{n}$ is the mean photon number and with $z(\lambda)$ defined by

$$Z(\lambda) = \frac{2\kappa/\pi}{\kappa-\mu} tan^{-1}\left(\frac{2\lambda}{\mu}\right) - \frac{2\mu/\pi}{\kappa-\mu} tan^{-1}\left(\frac{2\lambda}{\kappa}\right). \quad (3.124)$$

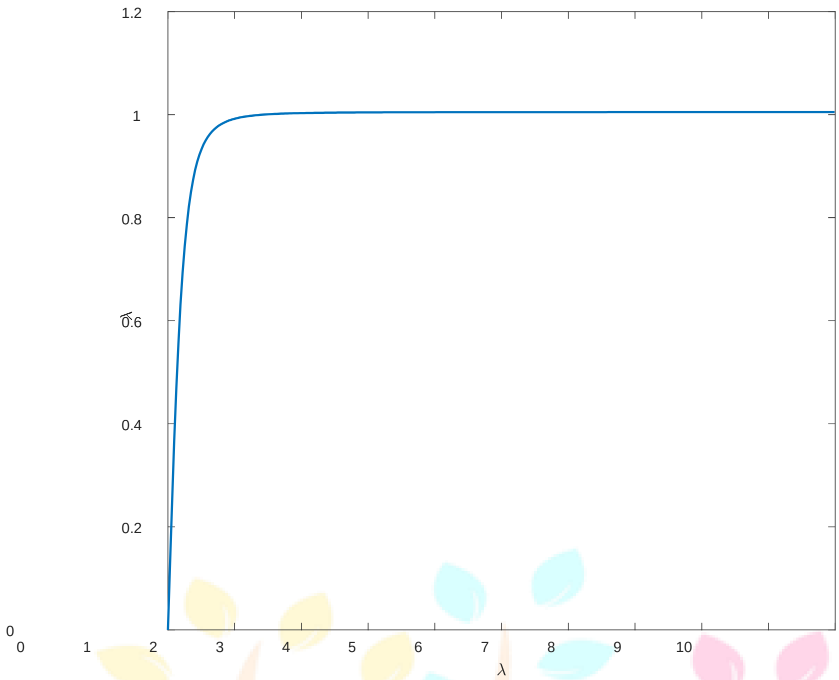

Fig. 3.3: plots of power spectrum(Eq. (3.124)) versus λ for $\kappa = 0.8$ and $\mu = 50$.

From the plot in Fig. (3.3) we easily find $z(0.5) = 0.66$, $z(1) = 0.86$ and $z(2) = 0.96$. The combination of these results with Eq. (3.123) yields $\bar{n}_{\pm 0.5} = 0.66\bar{n}$, $\bar{n}_{\pm 1} = 0.86\bar{n}$ and $\bar{n}_{\pm 2} = 0.96\bar{n}$. From this we can observe that a large part of the total mean photon number is confined in a relatively small frequency interval

4.Quadrature Squeezing

In this chapter we wish to calculate the quadrature squeezing of the cavity mode, out put mode and squeezing spectrum in a given frequency interval.

4.1 Global Quadrature Squeezing

The squeezing properties of the cavity light are described using the Hermitian operators defined by[13].

$$\hat{c}_+ = \hat{c}^\dagger + \hat{c}. \tag{4.1}$$

and

$$\hat{c}_- = i[\hat{c}^\dagger - \hat{c}]. \tag{4.2}$$

The operators $\hat{c}_+$ and $\hat{c}_-$ represent physical quantities called the plus and minus quadratures. Employing Eq. (3.3) it can be readily established that

$$[\hat{c}_+, \hat{c}_-] = 2i\frac{\gamma_c}{\kappa}[\hat{N}_c - \hat{N}_a], \tag{4.3}$$

On the basis of Eq. (4.3), the uncertainty relations for $\hat{c}_+$ and $\hat{c}_-$ can be written as

$$\Delta\hat{c}_+\Delta\hat{c}_- \geq \frac{\gamma_c}{\kappa}\hat{N}_c\hat{N}_a . \tag{4.4}$$

The variances of the quadrature operator are expressible as

$$\Delta c^2 = \pm [\hat{c}^\dagger \pm \hat{c}]^2 \mp [\hat{c}^\dagger \pm \hat{c}]^2. \quad (4.5)$$

This can be written as

$$\Delta c^2 = \pm \hat{c}^{\dagger 2} \pm \hat{c}^\dagger\hat{c} \pm \hat{c}\hat{c}^\dagger \pm \hat{c}^2 \mp \hat{c}^{\dagger 2} \mp 2\hat{c}\hat{c}^\dagger \mp \hat{c}^2. \quad (4.6)$$

and it is possible to write Eq. (4.6) as

$$\Delta c^2 = \eta + 2\hat{c}^\dagger\hat{c} \pm \hat{c}^2 \pm \hat{c}^{\dagger 2} - 2\hat{c}\hat{c}^\dagger \mp \hat{c}^2 \mp \hat{c}^{\dagger 2}, \quad (4.7)$$

We next proceed to obtain the expectation values involved in Eq, (4.7). To this end, the expectation value for an operator expressible by

$$\hat{c} = \int d^2\alpha Q(\alpha', \alpha + \eta \frac{\partial}{\partial \alpha'}, t)\alpha, \quad (4.8)$$

employing Eq. (3.31) and Eq. (3.33) into Eq. (4.8), we see that

$$\hat{c} = \int d^2\alpha Q(\alpha', \alpha, t)\left(1 - \eta[u\alpha' - v\alpha]\frac{\partial}{\partial \alpha'} + \frac{v}{2}\eta^2\frac{\partial \alpha^2}{\partial \alpha'^2}\right)\alpha, \quad (4.9)$$

Applying Eq. (3.24) and carrying out the integration we obtain

$$\hat{c} = 0. \quad (4.10)$$

Following the same procedure, we obtain as

$$\hat{c}^\dagger = 0. \quad (4.11)$$

and

$$\hat{c}^2 = \int \frac{d^2\alpha}{\eta} Q(\alpha', \alpha, t)\alpha^2, \quad (4.12)$$

Using of Eq. (3.24), we easily find

$$\hat{c}^2 = \frac{v}{u^2 - v^2} = b, \quad (4.13)$$

similarly

$$\hat{c}^{\dagger 2} = b, \quad (4.14)$$

and

$$\hat{c}^\dagger\hat{c} = \bar{n}, \quad (4.15)$$

Substituting Eq. (4.10), (4.11), (4.13), (4.14), and (4.15) into Eq. (4.7), we get

$$\Delta c^2 = \eta + 2\bar{n} \pm 2b. \quad (4.16)$$

Taking into account Eq. (3.16), (3.6) and Eq. (3.53) into Eq. (4.16), we obtain

.
$$\Delta c_{\pm}^2 = \frac{\gamma_c}{\kappa} \frac{1 + x^2 \pm 2x}{1 + (a+2)x^2} N. \tag{4.17}$$

For the place and minus quadrature variances we have

$$\Delta c_{+}^2 = \frac{\gamma_c}{\kappa} \frac{1 + x^2 + 2x}{1 + (a+2)x^2} N. \tag{4.18}$$

and

$$\Delta c_{-}^2 = \frac{\gamma_c}{\kappa} \frac{1 + x^2 - 2x}{1 + (a+2)x^2} N. \tag{4.19}$$

From Fig. (4.1) we say that the cavity light is in a squeezed state for $x < 2$ and the squeezing occurs in the minus quadrature. And the plus and minus quadrature variances are greater for $a = 0$ than that of $a = 1$. As the number of a tomes increase the plus quadrature is increase while the minus quadrature is decrease. Moreover, the plus and minus quadrature have a common starting point depending on the number N, γ_c and κ.

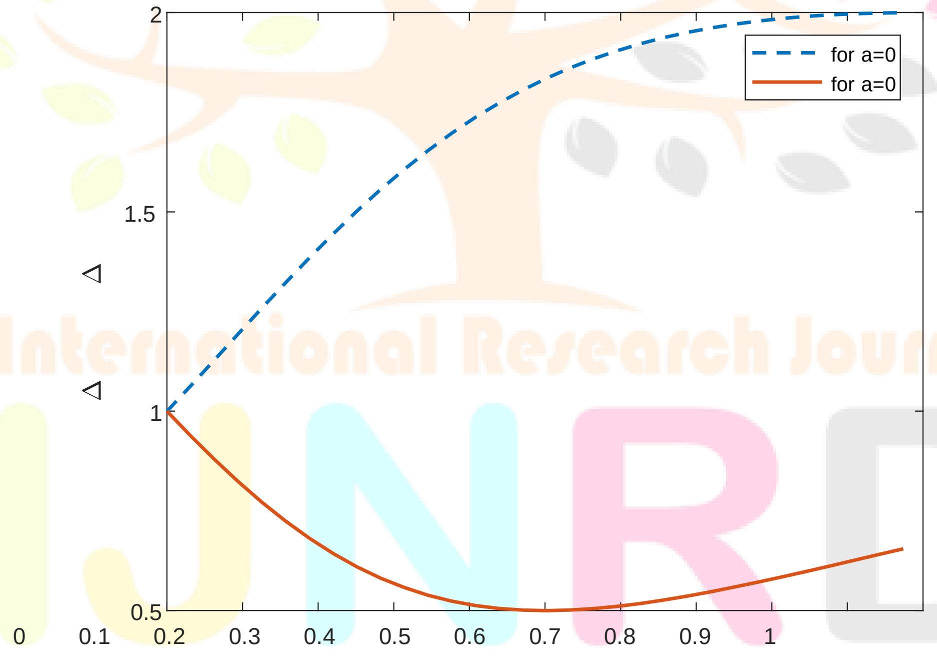

Fig. 4.1: plots of quadrature variance (Eq. (4.18) and (4.19)) versus x for $\gamma_c = 0.4$, $\kappa = 0.8$ and $N = 2$.

Moreover on setting $\Omega = 0$ in Eq. (4.17), we get

$$(\Delta c^2)_v = (\Delta c^2)_{v+} = \frac{\gamma_c}{\kappa} N. \tag{4.20}$$

Eq. (4.20) represents the quadrature variance of a cavity mode light in vacuum state. And also we next determine the quadrature squeezing of the cavity mode light relative to the quadrature variance of the cavity mode of vacuum state. We then define the quadrature squeezing of the cavity mode light relative to the vacuum state by

$$S = \frac{(\Delta c_-^2)_v - (\Delta c_-^2)}{(\Delta c_-^2)_v}. \tag{4.21}$$

Hence with the aid of Eq. (4.19) and Eq. (4.20), one can put Eq. (4.21) as

$$S = \frac{2x - x^2}{1 + (2 + a)x^2}. \tag{4.22}$$

We observe from Eq. (4.22) that, unlike mean photon number, the quadrature squeez-

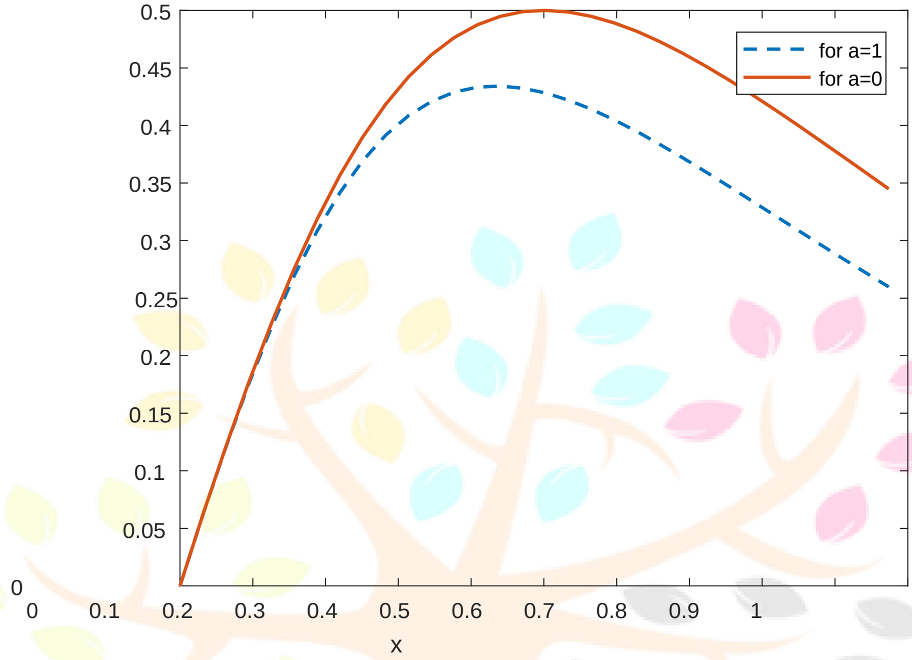

Fig. 4.2: plots of global quadrature squeezing (Eq. (4.22)) versus x

ing doesn't depend on the number of atoms. This impulse that the quadrature squeezing of the cavity mode is independent of the number of photons. The plot in Fig. (4.2) indicates that the maximum quadrature squeezing is 43.3% for $a = 1$ and 50% for $a = 0$ at $x = 0.5$.

On the other hand, we define the quadrature squeezing of the out put light by

$$S^{out} = \frac{(\Delta c_-^{out})_v^2 - (\Delta c_-^{out})^2}{(\Delta c_-^{out})_v^2}. \tag{4.23}$$

where $(\Delta c^{out})_v^2$ is the quadrature variance of the out put light in vacuum state. Since all calculations are carried out by putting the vacuum noise in normal order, the out put and cavity mode variable can be related by [5]

$$\hat{c}^{out} = \sqrt{\kappa}\,\hat{c}, \tag{4.24}$$

It can also ready variable that

$$(\Delta c^{out})^2 = \kappa(\Delta c^2)_v \tag{4.25}$$

and

$$(\Delta c^{out})^2 = \kappa(\Delta c^2), \tag{4.26}$$

(4.26) and Eq. (4.21), we easily get

$$S^{out} = S. \tag{4.27}$$

From Eq. (4.27) we observe that the quadrature squeezing of the out put light is equal to that of the cavity light.

4.2 Local Quadrature Squeezing

In this section we seek to determine the quadrature squeezing of the cavity and out put modes in a given frequency interval. To this end, we seek to determine the squeezing spectrum which is expressible by

$$S_{\pm(w)} = \frac{1}{\pi} Re \int_0^\infty \langle \hat{c}_\pm(t), \hat{c}_\pm(t+\tau) \rangle_{ss} e^{i(w-w_0)\tau} d\tau. \tag{4.28}$$

Upon integrating both sides over w, we get

$$\int_{-\infty}^{\infty} S_{\pm(w)} dw = (\Delta c_\pm^2)_{ss}, \tag{4.29}$$

in which

$$(\Delta c_\pm^2) = \langle \hat{c}_\pm(t), \hat{c}_\pm(t) \rangle_{ss}, \tag{4.30}$$

is the quadrature variance for the light mode at steady state. On the basis of the result given by Eq. (4.29), we assert that $S_\pm(w)dw$ is the steady state quadrature variance of the light mode in the interval between w and $w + dw$. In view of Eq. (3.111), we note that

$$\langle \hat{c}_\pm(t), \hat{c}_\pm(t+\tau) \rangle_{ss} = \langle \hat{c}_\pm(t)\hat{c}_\pm(t+\tau) \rangle_{ss}, \tag{4.31}$$

We now proceed to determine the two time correlation that appears in Eq. (4.31) for the cavity light. To this end, we realize that the solution of Eq. (2.66) can be written as

$$\frac{g}{\sqrt{N}} e^{-\kappa\tau/2} \int_0^\tau e^{\kappa\tau'/2} \hat{m}(t+\tau')d\tau', \tag{4.32}$$

substituting Eq. (3.115) in to Eq. (4.32), we get

$$\hat{c}(t+\tau) = \hat{c}(t)e^{-\kappa\tau/2} + \frac{2g\hat{m}(t)}{\sqrt{N}(\kappa-\mu)}\left[e^{-\kappa\mu/2} - e^{-\kappa\tau/2}\right]$$
$$+ \frac{g}{\sqrt{N}} e^{-\kappa\tau/2} \int_0^\tau d\tau' e^{(\kappa-\mu)\tau'/2} \int_0^{\tau'} d\tau'' e^{\mu\tau''/2} \hat{F}_m(t+\tau''). \tag{4.33}$$

On account of this equation, we see that

$$\langle \hat{c}(t)\hat{m}(t) \rangle [e^{-\mu\tau/2} - e^{-\tau/2}], \tag{4.34}$$

$$\hat{c}^{\dagger}(t)\hat{c}(t+\tau) = \hat{c}^{\dagger}(t)\hat{c}(t+\tau)\, e^{-\kappa\tau/2} + \frac{2g}{\sqrt{N}(\kappa-\mu)}$$

Applying the large time approximation, we write from Eq. (3.3), that

$$m(t) = \frac{\kappa\sqrt{N}\,\hat{c}(t)}{2g}, \quad (4.35)$$

Substituting Eq. (4.35) into Eq. (4.34), we have

$$\hat{c}^{\dagger}(t)\hat{c}(t+\tau) = \hat{c}^{\dagger}(t)\hat{c}(t) \quad \frac{\kappa}{\kappa-\mu} e^{-\kappa\tau/2} - \frac{\mu}{\kappa-\mu} e^{-\kappa\tau/2}, \quad (4.36)$$

Following the same procedure, one can also readily establish that

$$\hat{c}(t)\hat{c}^{\dagger}(t) = \hat{c}(t)\hat{c}^{\dagger}(t+\tau) \quad \frac{\kappa}{\kappa-\mu} e^{-\kappa\tau/2} - \frac{\mu}{\kappa-\mu} e^{-\kappa\tau/2}, \quad (4.37)$$

And

$$\hat{c}(t)\hat{c}(t+\tau) = \hat{c}^2(t) \quad \frac{\kappa}{\kappa-\mu} e^{-\kappa\tau/2} - \frac{\mu}{\kappa-\mu} e^{-\kappa\tau/2}, \quad (4.38)$$

Therefore, on account of Eqs. (4.36), (4.37) and Eq. (4.38), Eq. (4.31)takes the form

$$\hat{c}_{\pm}(t), \hat{c}_{\pm}(t+\tau)_{ss} = \Delta c_{\pm}^2 \frac{\kappa}{\kappa-\mu} e^{-\kappa\tau/2} - \frac{\mu}{\kappa-\mu} e^{-\kappa\tau/2}, \quad (4.39)$$

Now introducing of Eq. (4.39) into Eq. (4.28) and carrying out the integration, we find the spectrum of the quadrature fluctuation for the cavity light to be

$$S_{-}(w) = \Delta c_{-}^2 \frac{\kappa}{\kappa-\mu} \frac{\mu/2\pi}{(w-w_0)^2 + [\mu/2]^2} - \frac{\mu}{\kappa-\mu} \frac{\kappa/2\pi}{(w-w_0)^2 + [\kappa/2]^2}, \quad (4.40)$$

We realize that the variance of the minus quadrature in the interval between $w = -\lambda$ and $w = \lambda$ is expressible as

$$(\Delta c^2)_{\pm\lambda} = \int_{-\lambda}^{\lambda} S_{-}(w)\,dw. \quad (4.41)$$

in which $w = w - w_0$. Applying Eq. (4.40) and carrying out the integration, we readily get

$$(\Delta c^2)_{\pm\lambda} = z(\lambda)(\Delta c^2_{-}), \quad (4.42)$$

$$Z(\lambda) = \frac{2\kappa/\pi}{\kappa-\mu} \tan^{-1}(2\lambda/\mu) - \frac{2\mu/\pi}{\kappa-\mu} \tan^{-1}(2\lambda/\kappa). \quad (4.43)$$

On account Eq. (4.42), the quadrature variance of a cavity mod light in vacuum state can be written in the same frequency interval as

$$(\Delta c^2)_{v\pm\lambda} = z(\lambda)(\Delta c^2_{-})_v, \quad (4.44)$$

We define the quadrature squeezing for the cavity light in the $\lambda_{\pm}$ frequency interval by

$$S_{\pm\lambda} = \frac{(\Delta c^2)_{v\pm\lambda} - (\Delta c^2)_{\pm\lambda}}{(\Delta c^2)_{v\pm\lambda}} \qquad (4.45)$$

so that on account of Eq. (4.42), (4.44) and (4.21), there follows

$$S_{\pm\lambda} = S. \qquad (4.46)$$

This shows that the quadrature squeezing of the cavity light in a given frequency interval is equal to that of the cavity light in the entire frequency interval. We also notice that as λ increase the local quadrature squeezing approaches the global quadrature squeezing. It is not hard to realize that the mean photon number is very small when the quadrature squeezing is relatively large. Finally, defining the quadrature squeezing of the out put light in the aforementioned frequency interval by

$$S^{out}_{\pm\lambda} = \frac{(\Delta c^{out}_{-})^2_{v\pm\lambda} - (\Delta c^{out}_{-})^2_{\pm\lambda}}{(\Delta c^{out}_{-})^2_{v\pm\lambda}} \qquad (4.47)$$

and taking in to account the fact that

$$(\Delta c^{out}_{-})^2_{v} = z(\lambda)(\Delta c^{out}_{v\pm\lambda})^2, \qquad (4.48)$$

and

$$(\Delta c^{out}_{-})^2_{\pm\lambda} = z(\lambda)(\Delta c^{out}_{-})^2, \qquad (4.49)$$

substituting Eq. (4.48) and Eq. (4.49) in to Eq. (4.47), we arrive at

$$S^{out}_{\pm\lambda} = S^{out}. \qquad (4.50)$$

From Eq. (4.50) we observe that the quadrature squeezing of the out put light in certain frequency interval is the same as that of the out put light in the entire frequency interval.

5.Conclusion

In this thesis, we have studied the squeezing and statistical properties of alight generated by three level laser whose cavity modes are coupled to vacuum reservoir. The three level atoms available in the cavity are pumped from the bottom to the top level by means of electron bombardment

We carry out our analysis by putting the noise operators associated with the vacuum reservoir in normal order. Making use of the quantum langevin equation at steady state solution, we have determined the time

evolution for the expectation values of atomic operators and stimulated emission decay constant.

On the other hand, using of the Q function in anti normal ordered, we have calculated the mean photon number, variance of photon number, photon number distribution, and quadrature variances of cavity mode light. It is found that both the mean and variance of photon number for $a = 0$ are greater than that for $a = 1$.

the meanand variance of photon number for $a = 0$ are greater than that for $a = 1$. Moreover, the mean photon number is very small when the quadrature squeezing is relatively very large. We observe from power spectrum we observed that a large part of the total mean photon number is confined in a relatively very small frequency interval.total,mean photon number is confined in a relatively very small frequency interval . This study shows that the cavity light produced by the system under consideration can be in squeezed state for $x < 2$ and the squeezing occurs in the minus quadrature , it is observe that, the maximum quadrature squeezing is 43.3% for $a = 1$ and 50% for $a = 0$ at $x = 0.5$. And also the quadrature squeezing doesn't depend on the number of atoms. And the quadrature squeezing of the cavity light in a given frequency interval is equal to that of the cavity light. Finally, we can conclude that the quadrature squeezing of the out put light in a certain frequency interval is the same as that of the out put light in the entire frequency.

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