



## N-TH PRIME AND NEXT PRIME

<sup>1</sup> V. Anantha Padmanabhan

<sup>1</sup>Retired Government servant

9 N.G.O Colony, Pollachi-6, Tamilnadu, Coimbatore.

*Abstract* : Calculation of Nth Prime – Next Prime, A Novel method – Suggested.

*IndexTerms* – N th Prime – Next Prime

### INTRODUCTION:

Finding a New Formula for Calculating the value of Nth Prime and Next Prime are age long questions. In modern times many mathematicians are trying to solve these questions, by attacking the problem by using computerised techniques.

In this paper i suggest a novel method to calculate N th Prime and Next Prime by attacking the problem purely mathematically. I am using one property of the  $\alpha$  sequence and  $\alpha$  series, which i have already suggested in my two papers.

### Short Notes on $\alpha$ Sequence:

(For Ready Reference Sake)

Considered all the odd numbers which are divisible by 5 and 7 separately. The smaller numbers 1,2,3 are omitted. Also all the even numbers and all the numbers divisible by 3 are also omitted.

Thus we get two arithmetic progressions namely

$$\alpha_1 = 5 \ 11 \ 17 \ 23 \dots$$

$$\alpha_2 = 7 \ 13 \ 19 \ 25 \dots$$

In both the ceases the common difference is 6 and the first number being 5 and 7.

### METHODOLOGY:

Now the question is what is the N th Prime. It is obvious that the N th Prime is  $6N + 5$  or  $+7$ . Since the common difference of the set two AP s are 6 and first number being 5 and 7, the above derivation is derived.

Again it may be noticed that both the series consists of prime and composites, only odd numbers greater than 5 even than the derivation above said,  $6N + 5$  or  $+7$  may be a prime or composite.

Now let us considered an example.

What is the 108 Prime?

**Formula:**

$$1^{\text{st}} \text{ Chance: } 6 \times 108 + 5$$

$$2^{\text{nd}} \text{ Chance: } 6 \times 108 + 7$$

**1<sup>st</sup> Chance:**

$$6 \times 108 = 648 + 5 = 653$$

**2<sup>nd</sup> Chance:**

$$6 \times 108 = 648 + 7 = 655$$

Off which the number produced by second chance is 655 which is divisible by 5.

So the number 653 may be a Prime or Composite and if it is a Prime that itself is 108th Prime.

So let us conduct the primality test to the number 653 to now whether it is a prime or a composite.

It is enough to divide 653 up to  $\sqrt{653}$  whether the number 653 is a prime or not.

This property is given in Fermat's Theorem So  $\sqrt{653} = 25.55386...$  Now the  $\alpha$  numbers up to 25 are 5,7,11,13,17,19 and 23. We need not considered the other numbers and If we reduced 653 to a  $\alpha$  number

$$\text{that is } 653 - 1 / 6 = 652 / 6 = 108.66$$

$$653+1 / 6 = 109 \text{ is an integer. So } 653 \text{ is an } \alpha_1 \text{ Number.}$$

Now conduct the primality test

$$1) 653 / 5 = 130.6$$

$$2) 653 / 7 = 93.28$$

$$3) 653 / 11 = 59.36$$

$$4) 653 / 13 = 50.23$$

$$5) 653 / 17 = 38.41$$

$$6) 653 / 19 = 34.36$$

$$7) 653 / 23 = 28.39$$

The number 653 is not divisible by the required  $\alpha$  numbers so it is a prime and itself is the 108 th Prime.

Now another question may arise if  $6N + 5$  or  $+7$  is a composite what we have to do ? How to arrive the Nth prime at that condition.

**SOLUTION:**

Considered the same number 653. Add +2 to it. Since 655 is a number divisible by 5, It is not a prime.

$653 + 4 = 657$  and it is divisible by 3 and so it is not also a prime.  
So add another +2 to number  $657 = 659$

We cannot judge 659 easily and its primality cannot be arrived without further tests.  
 $\sqrt{659} = 25.67$

The same  $\alpha$  number divisors are enough to conduct this tests. They are 5,7,11,13,17,19,23

- 1)  $659 / 5 = 131.8$
- 2)  $659 / 7 = 94.14$
- 3)  $659 / 11 = 59.90$
- 4)  $659 / 13 = 50.69$
- 5)  $659 / 17 = 38.76$
- 6)  $659 / 19 = 34.68$
- 7)  $659 / 23 = 28.65$

Number 659 is not divisible by the set divisors hence 659 is a Prime and it is the 109 th Prime.

Practically when we have a need to now the next prime value we have to add +2 and +4 alternatively, See that newly arrived number that is  $N' + 2$  or  $+4$  is not a even or divisible by 3 and 5. Then we may conduct the primality test to the arrived number. If the arrived number is composite another +2 or +4 and follow the same methodology till we arrived a next prime number.

**CONCLUSION:**

A novel method is suggested to calculate the Nth Prime value and Next Prime value. The result will be accurate.

**REFERENCES**

- [1] <https://www.research.net/publications2112990>
- [2] Fermats Factorization method Para 5.2 Page 110 Book Elementary Number Theory by David M Burton
- [3] V.Anantha Padmanabhan Had already explained what is  $\alpha$  sequence in IJNRD2310036