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MATHEMATICAL ANALYSIS OF HYDROMAGNETIC FLOW AND HEAT TRANSFER OVER A SHRINKING SHEET FOR LARGE SUCTION.

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Abstract: This work considers the steady two dimensional magnetohydrodynamic boundary layer flow and heat transfer over a shrinking sheet in the presence of an external magnetic field and internal heat generation or absorption. The governing equations are transformed using similarity transformation. Perturbation method is used to linearise the resulting ordinary differential equations at large suction. The resulting equations are solved analytically. The effects of the involved parameters on the velocity and temperature profiles are illustrated graphically and in tabular forms.

Keywords: Boundary layer, MHD, shrinking sheet, heat transfer, heat source/sink, mass suction.

1.INTRODUCTION

The problem of stretching or shrinking sheet flow arises in numerous manufacturing processes. Many industrial processes involve the cooling of materials or filaments by drawing them through an ambient liquid which acts as the cooling system. While they are being drawn through the surrounding liquid they are either stretched or shrank. The stretching or shrinking of these materials induces the boundary layer flow of the cooling liquid. The quality of the final product depends to a large extent on the rate of cooling at the sheet. Therefore, the steady boundary layer flow of an incompressible viscous fluid over a stretching or shrinking sheet is of great importance both from theoretical and practical point of view because of its application in many manufacturing industries and technological processes such as glass blowing, casting and spinning of fibre, wire and paper production, metallurgical and polymer processing. In his pioneering work, Sakiadis (1961) introduced the study of boundary layer flow over a continuous solid surface moving with constant velocity. Crane (1970) found an exact similarity solution for steady laminar boundary layer of a Newtonian fluid over a linearly stretching sheet. The heat and mass transfer of viscous fluid over an isothermal stretching sheet with suction and blowing was studied by Gupta and Gupta (1977). Banks (1983) obtained both numerical and analytical solutions for boundary layer flow over a stretching wall. Wang (1984) examined three dimensional flows due to a stretching sheet. Grubka and Bobba (1985) studied heat and mass transfer characteristics on a stretching sheet. Noor (1992) discussed the characteristics of heat transfer on a stretching sheet. Convective heat transfer on a stretching sheet has been presented by Vajravelu and Nayfeh (1993).

The study of flow and heat transfer in an electrically conducting fluid permeated by a transverse magnetic field is of special interest and has many practical applications in manufacturing processes in industry. The magnetohydrodynamic problem was first investigated by Pavlov (1974) who studied the MHD flow over a stretching wall in an electrically conducting fluid, with uniform magnetic field. Chakrabati and Gupta (1979) studied hydromagnetic flow and heat transfer over a stretching sheet. Vajrevelu and Nayfeh (1992) investigated the hydromagnetic flow of a dusty fluid over a stretching sheet.

Due to its technological appeal, the flow of incompressible fluid due to a shrinking sheet has attracted a lot of attention. Wang (1990), Miclavcic and wang (2006), Hayet et al (2007), have all made useful contributions to shrinking sheet flow. Fang (2008) reported a numerical solution for boundary layer flow over a shrinking sheet with power law velocity. Fang and Zang (2009) obtained interesting analytical solution for MHD flow over a porous shrinking sheet subject to suction. Wang (2008), Bhattacharyya et al (2011) have all investigated stagnation-point flow towards a shrinking sheet under different flow conditions.

Bataller (2007), Chen (2009) have also made important contributions to the effects of heat source/sink on the boundary layer flow over a stretching sheet. Bhattacharyya (2011) discussed the effects of heat source/sink on MHD flow and heat transfer over a shrinking sheet with mass suction. This was extended by Babu et al by considering the effects of radiation on magnetohydrodynamic heat and mass transfer over a shrinking sheet.

The aim of this present paper is to analyse the steady MHD boundary layer flow due to a porous shrinking sheet with mass suction and internal heat generation or absorption. Similarity equations have been obtained and solved by the perturbation method for large suction. Results are shown graphically and in tabular form.

2.FORMULATION OF THE PROBLEM

Consider the steady, two-dimensional, magnetohydrodynamic boundary layer flow of an electrically conducting fluid, induced by a permeable shrinking sheet with internal heat generation or absorption under the influence of a transversely applied magnetic field. The x axis is chosen to be along the sheet in the direction of flow while the y axis is normal to the sheet and the flow is confined in the region y> 0 as shown in figure1. Neglecting the induced magnetic field and along with boundary layer approximations, the governing equations are given by:

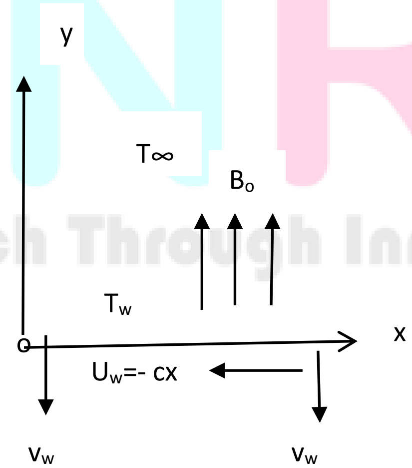

Figure1: Physical model of the problem

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \qquad (2)$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_o}{\rho C_p}(T-T_\infty) \qquad (3)$$

where u and v are the velocity components along the x and y axes, respectively; ρ is the density; υ is the kinematic viscosity; σ is the electrical conductivity of the fluid; B_o is the applied magnetic field strength; T is the temperature of the fluid; T_∞ is the temperature of the free stream; κ is the thermal conductivity of the fluid; and C_p is the specific heat at constant pressure. The corresponding wall conditions for the velocity and temperature fields are:

$$u=U_w=-cx,\quad v=-v_w,\quad T=T_w \quad \text{at } y=0;$$
$$u\to 0,\quad T\to T_\infty \quad \text{as } y\to\infty \qquad (4)$$

Where $c>0$ is shrinking constant and T_w is the temperature of the sheet; v_w is the normal velocity for suction or blowing at the sheet such that $v_w<0$ indicate fluid suction and $v_w>0$ indicate fluid blowing; T_∞ is the free stream temperature.

3.SIMILARITY TRANSFORMATION

The similarity transformations variables include the following

$$\eta=y\sqrt{\frac{c}{v}},\ \psi(x,y)=\sqrt{cvx}f(\eta),\quad u=\frac{\partial\psi}{\partial x},\quad v=-\frac{\partial\psi}{\partial y}$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \qquad (5)$$

Transforming equations (2) - (4), with the non-dimensional variables in equation (6) yields the following ordinary differential equations

$$f'''+ff''-f'^2-M^2f'=0 \qquad (6)$$

$$\theta''+Prf\theta'+Pr\lambda\theta=0 \qquad (7)$$

With boundary conditions

$$f(0)=f_o, f'(0)=-1,\quad \theta(0)=1,$$
$$f'(\infty)=0,\quad \theta(\infty)=0 \qquad (8)$$

Where prime denotes derivatives with respect to similarity variable η and the continuity equation (1) is identically satisfied by the transformation.

Here $M^2=\frac{\sigma B_o^2}{\rho c}$ is the Hartmann number, $Pr=\frac{\mu C_p}{\kappa}$ is the Prandtl number, $\lambda=\frac{Q_o}{\rho C_p c}$ is the heat source ($\lambda>0$) or sink ($\lambda<0$) parameter and $f_o=\frac{v_w}{\sqrt{cv}}$ such that $f_o>0$ indicates suction.

4.ANALYTIC SOLUTION

We define

$$\xi = \eta f, \ f(\eta) = f_o F(\xi), \theta(\eta) = \Theta(\xi), \ \ \varepsilon = \frac{1}{f_o^2} \qquad (9)$$

(See, for example Singh and Dishkit (1988))

Using equation (9) and its derivatives in equations (6), (7) and (8) yields

$$F''' + FF'' - F'^2 - M^2 \varepsilon F' = 0 \qquad (10)$$

$$\Theta'' + PrF\Theta' + Pr\lambda\Theta = 0 \qquad (11)$$

With boundary conditions

$$F(0) = 1, F' = -\varepsilon, \qquad F'(\infty) = 0 \qquad \Theta(0) = 1$$

$$\Theta(\infty) = 0 \qquad (12)$$

Where prime denote differentiation with respect to the ξ.

For large suction, f_o assume large positive values so that $\varepsilon \ll 1$ and as such serves as the perturbation parameter. Therefore, F and Θ can be expanded in terms of the small perturbation parameter as

$$F = 1 + \varepsilon F_1 + \varepsilon^2 F_2 + \varepsilon^3 F_3 + \cdots \qquad (13)$$

$$\Theta = \Theta_o + \varepsilon\Theta_1 + \varepsilon^2\Theta_2 + \cdots \qquad (14)$$

Substituting Equations (13)-(14) and its higher derivatives in Equations (10), (11) and (12), yields the following set of differential equations and boundary conditions:

$$\Theta_o'' + Pr\Theta_o' = 0 \qquad (15)$$

$$\Theta_o(0) = 1, \qquad \Theta_o(\infty) \to 0 \qquad (16)$$

$$F_1''' + F_1'' = 0 \qquad (17)$$

$$\Theta_1'' + Pr\Theta_1' + PrF_1\Theta_0' + Pr\lambda\Theta_o = 0 \qquad (18)$$

$$F_1(0) = 0 \ F_1'(0) = -1 \quad \Theta_1(0) = 0, \quad F_1'(\infty) \to 0 \quad \theta(\infty) \to 0 \qquad (19)$$

$$F_2''' + F_2'' + FF_1'' - F_1'^2 - M^2 F_1' = 0 \qquad (20)$$

$$\Theta_2'' + Pr\Theta_2' + PrF_1\Theta_1' + PrF_2\Theta_o' + Pr\lambda\Theta_1 = 0 \qquad (21)$$

` $$F_2(0) = F_2'(0) = \Theta_2(0) = 0, F_2'(\infty) \to 0, \Theta_2(\infty) \to 0 \qquad (22)$$

$$F_3'' + F_3'' + F_1F_2'' + F_2F_1'' - 2F_1'F_2' - M^2 F_2' = 0 \qquad (23)$$

$$F_3(0) = F_3'(0) = 0, F_3'(\infty) \to 0 \qquad (24)$$

Solving the above equations subject to their corresponding boundary conditions, we obtain the following solutions:

$$\Theta_o = e^{-Pr\xi} \qquad (25)$$

$$F_1 = e^{-\xi} - 1 \qquad (26)$$

$$\Theta_1 = -B_1 e^{-Pr\xi} + B_2\xi e^{-Pr\xi} + B_1 e^{-(1+\mathrm{Pr})\xi} \qquad (27)$$

$$F_2 = -A_1 + A_1 e^{-\xi} + A_1\xi e^{-\xi} \qquad (28)$$

$$\Theta_2 = -\frac{B_5}{1+Pr} e^{-Pr\xi} - \frac{(2+\mathrm{Pr})B_6}{(1+Pr)^2} e^{-Pr\xi} - \frac{B_7}{(4+2\mathrm{Pr})} e^{-Pr\xi}$$

$$-\frac{B_3}{Pr}\xi e^{-Pr\xi}-\frac{B_4}{2Pr}\xi^2 e^{-Pr\xi}-\frac{B_4}{(Pr)^2}\xi e^{-Pr\xi}+\frac{B_5}{(1+\mathrm{Pr})}e^{-(1+Pr)\xi}$$

$$+\frac{B_6}{(1+\mathrm{Pr})}\xi e^{-(1+Pr)\xi}+\frac{B_6(2+Pr)}{(1+Pr)^2}e^{-(1+Pr)\xi}+\frac{B_7}{(4+2Pr)}e^{-(2+Pr)\xi} \qquad (29)$$

$$F_3=-2A_1^2+2A_1^2e^{-\xi}+\frac{A_1^2}{2}\xi^2e^{-\xi}+2A_1^2\xi e^{-\xi} \qquad (30)$$

Where

$$A_1=1-M^2, \qquad B_1=\frac{(Pr)^2}{(1+Pr)}, \qquad B_2=Pr+\lambda$$
$$B_3=Pr\lambda B_1+(Pr)^2B_1+PrB_2-(Pr)^2A_1$$
$$B_4=-Pr\lambda B_2-B_2(Pr)^2$$
$$B_5=-Pr\lambda B_1-(Pr)^3-(Pr)^2B_1-PrB_2+(Pr)^2$$
$$B_6=B_2(Pr)^2+A_1(\mathrm{Pr})^2$$
$$B_7=(Pr)^3$$

The velocity and temperature fields can be evaluated from the expressions:

$$f'(\eta)=F_1'+\varepsilon F_2'+\varepsilon^2F_3' \qquad (31)$$
$$\Theta(\eta)=\Theta_o+\varepsilon\Theta_1+\varepsilon^2\Theta_2 \qquad (32)$$

5.SKIN FRICTION AND NUSSELT NUMBER

The local skin friction coefficient and the dimensionless coefficient of heat transfer, known as the Nusselt number, are two parameters of physical significance.

The local skin friction coefficient is given by

$$C_f=\frac{\tau_w}{\frac{1}{2}\rho U_w^2}$$

Where τ_w ,which is the wall shear stress, is given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}=cx\sqrt{\frac{c}{x}}f''(0)$$

Thus, the skin friction coefficient becomes

$$C_f=2R_e^{-1/2}f''(0)$$

Where $R_e=\frac{cx^2}{\nu}$ is the Reynolds number.

Thus, we have

$$\frac{1}{2}R_e^{1/2}C_f=f''(0)=f_o-\frac{A_1}{f_o}-\frac{A_1^2}{f_o^3} \qquad (34)$$

The Nusselt number is defined as

$$Nu = \frac{xq_w}{k(T_w - T_\infty)}$$

Where $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = (T_w - T_\infty)\sqrt{\frac{c}{\nu}}\theta'(0)$

We, therefore, obtain the expression for the Nusselt number

$$Re^{-1/2}Nu = -\theta'(0) \qquad (35)$$

Where

$$\theta'(0) = -Prf_o + \frac{1}{f_o}(PrB_1 + B_2 - Pr^2) + \frac{1}{f_o^3}\left(\frac{PrB_5}{1+Pr} + \frac{\Pr(2+\Pr)B_6}{1+Pr} + \frac{PrB_7}{4+2Pr} - \frac{B_3}{Pr} - \frac{B_4}{Pr^2} - B_5 + \frac{B_6}{1+Pr} - \frac{B_6(2+\Pr)}{1+Pr} - \frac{B_7}{2}\right) \qquad (36)$$

6.RESULTS AND DISCUSSIONS

The results show the effects of the governing parameters, namely, the Magnetic parameter M, the mass suction parameter, f_o , the Prandtl number, Pr, and the heat source/sink parameter λ.

The effect of the various parameters on the velocity profile are depicted in figures 2 and 3, while the effects of the parameters on the temperature profile are depicted in figure 4 to 7. The impact of Prandtl number, the magnetic and suction parameters with the heat source/sink parameter on the rate of heat transfer are shown in figures 8 to 10.

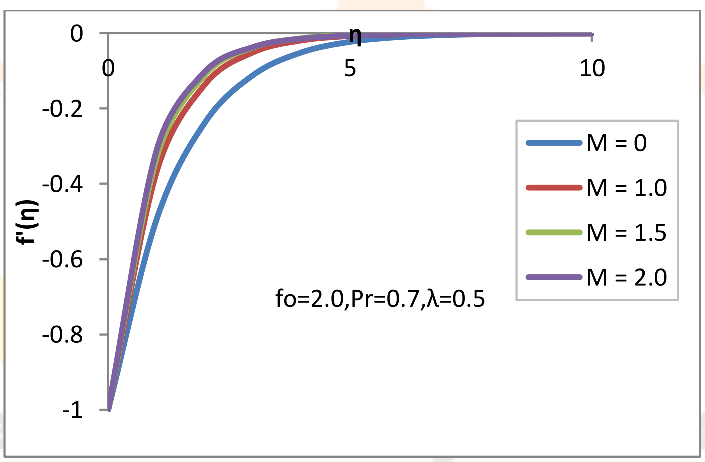

Figure 2: Velocity profile for different values of M.

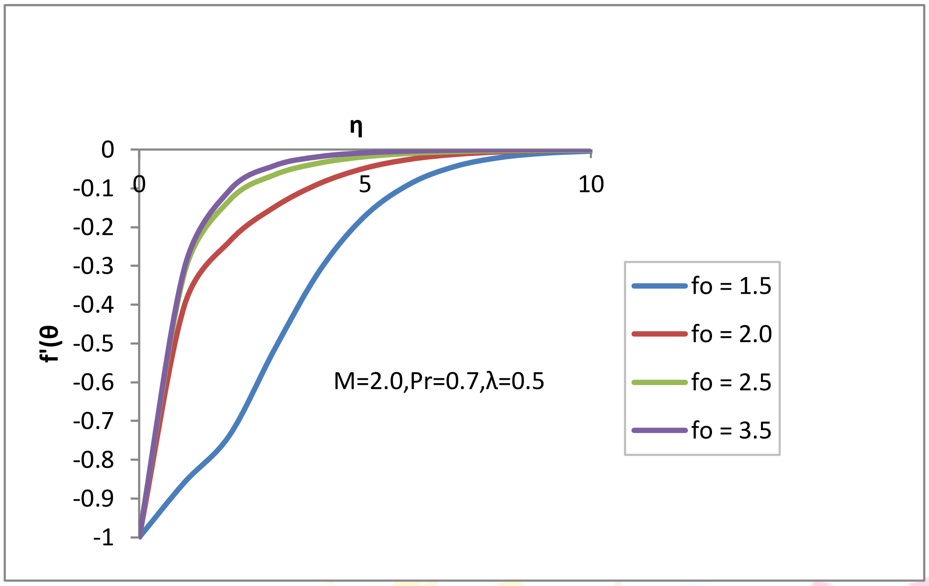

Figure 3: Velocity profile for different values of fo.

Figure2 shows the effect of magnetic parameter on the velocity profile. It is seen that the velocity increases with an increase in the magnetic parameter. Thus, the thickness of the momentum boundary layer decreases. The magnetic field, during the motion of the electrically conducting fluid, generates a resistive Lorentz force which enhances the flow in the boundary layer region by suppressing the generated vorticity due to the shrinking velocity. Figure3 illustrates the impact of mass suction on the velocity profile. It is seen that the velocity profile increases as the mass suction increases. This reduces the momentum boundary layer thickness. Suction is important to sustain flow in a shrinking sheet as it stabilizes the boundary layer

The effect of the magnetic parameter M, on the temperature profile is shown in figure4. It is noted that the temperature decreases as M increases. This is because the magnetic field accelerates the velocity of the fluid and therefore temperature near the sheet is reduced. Figure5 shows the impact of the wall mass suction, f_o, on the temperature distribution. The Temperature decreases with increasing mass suction, and consequently the thermal boundary layer decreases. This is due to the fact that suction enhances the fluid velocity and therefore reduces the temperature near the sheet. Figure6 illustrates the variation of temperature field for various values of Pr. It is observed that with increasing values of Pr the temperature decreases rapidly. This is due to the fact that large Prandtl number implies low thermal conductivity. The effect of heat source/sink parameter, λ, on the temperature profile is depicted in figure7. It is seen that as the heat source increases or sink decreases, the temperature increases. Figure8 presents the variation of Nusselt number θ'(0) with Prandtl number (Pr) and heat source/sink parameter, λ. It is seen that heat transfer rate increases with increase in Pr but decreases with increase in heat source parameter.

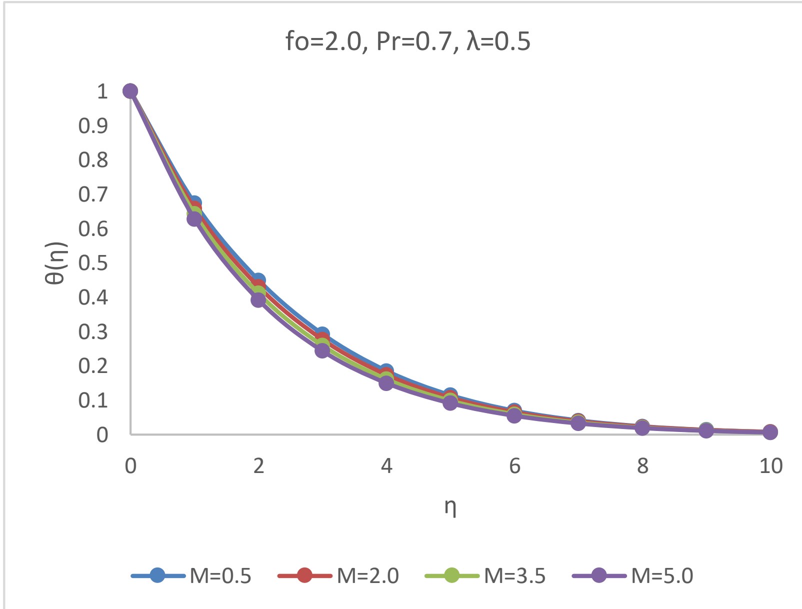

Figure 4: Temperature profile for different values of M.

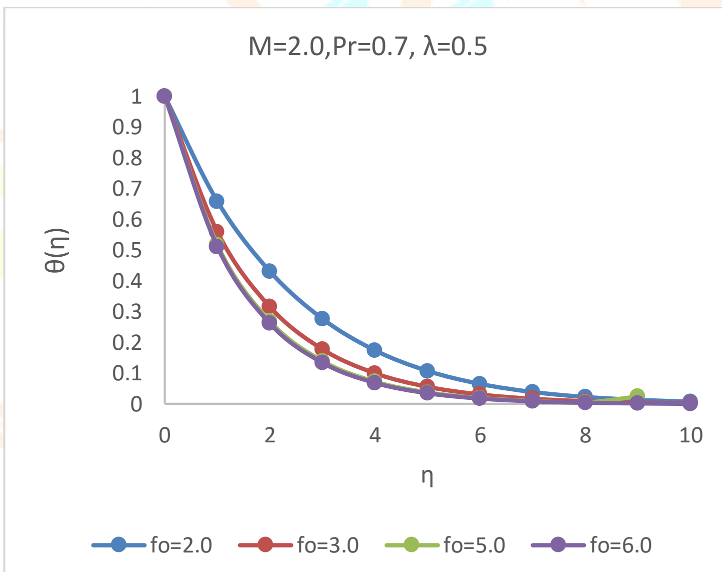

Figure 5: Temperature profile for different values of fo.

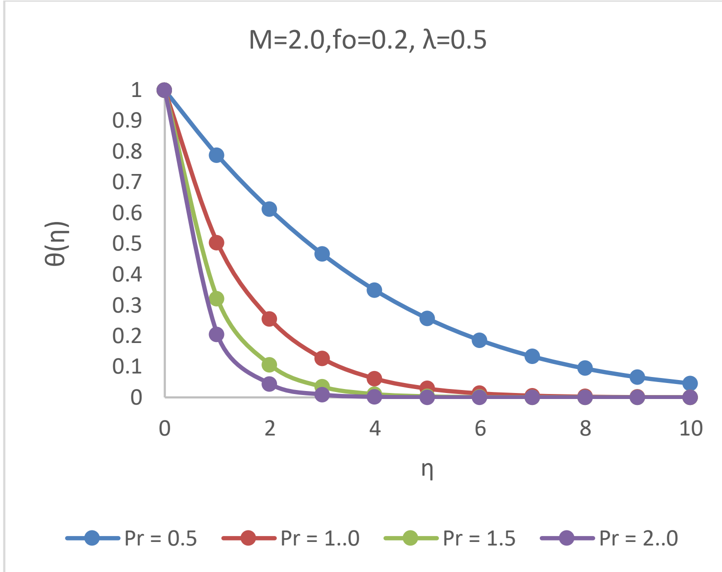

Figure 6: Temperature profile for different values of Pr.

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f_o	Babu *et al* (2013)	Present Study
2	2.414300	2.375000
3	3.302750	3.296296
4	4.236099	4.234375

Table1: Values for Skin friction $f''(0)$ for different values of the mass suction parameter, f_o when M^2=2.

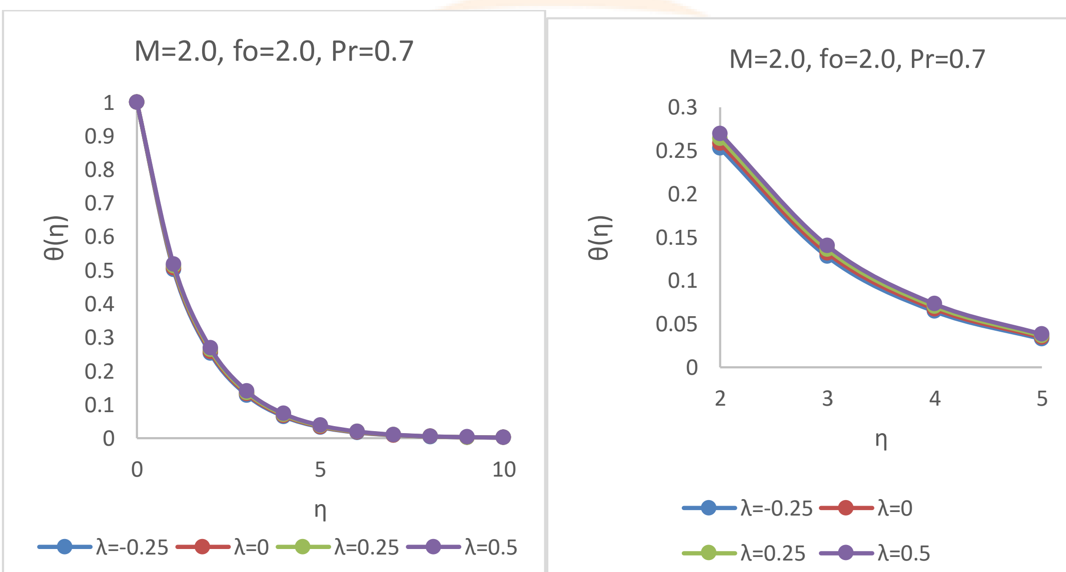


Figure 7: Temperature profile for different values of λ.

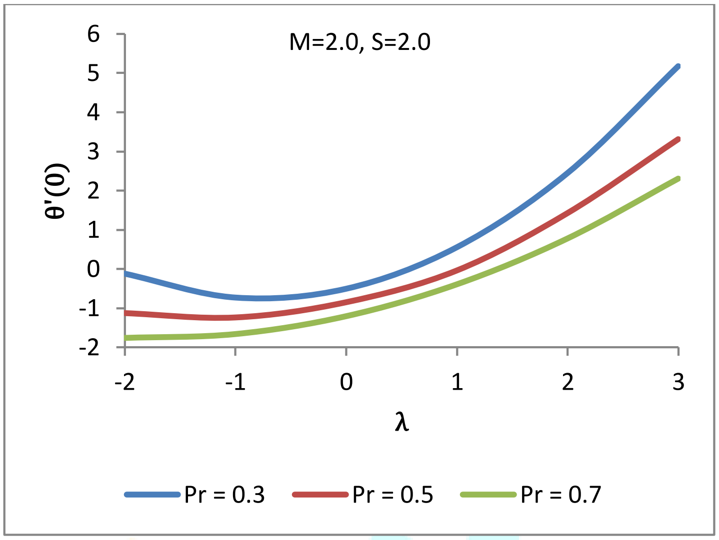


Figure 8: Temperature gradient against λ for different values of Pr.

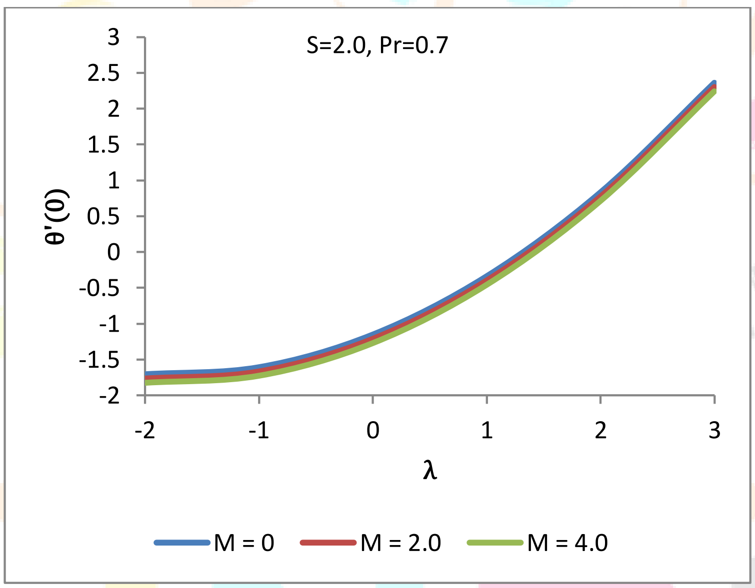
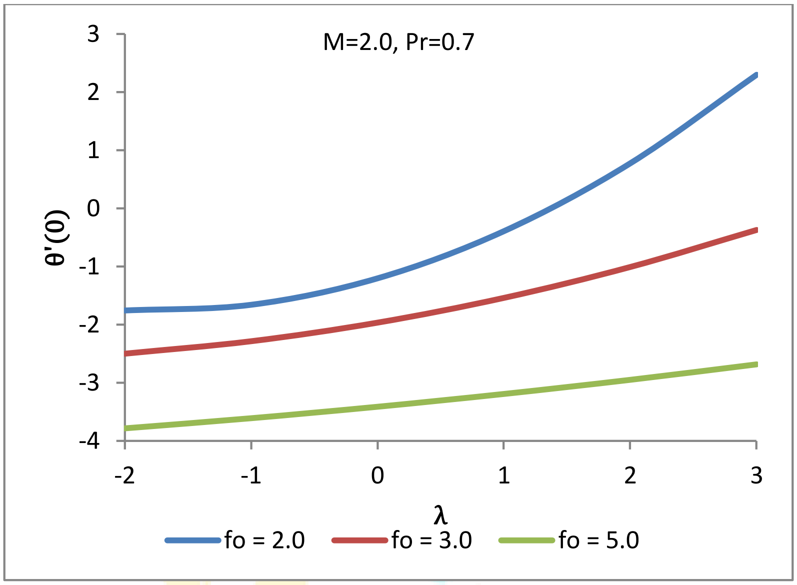


Figure 9: Temperature gradient against λ for different values of M.



. **Figure 10**: Temperature gradient against λ for different values of fo.

Figures9 depicts the impact of the magnetic parameter on the rate of heat transfer. It is observed that the rate of heat of heat transfer increases slightly with increase in magnetic parameter while the rise in the heat source parameter reduces the rate of heat transfer. Furthermore, rise in the value of mass suction, f_o , and heat sink, $\lambda<0$, enhance the rate of heat transfer at the sheet as depicted in figure10.

It is observed from Table1 that the values of $f''(0)$, the skin friction coefficient, enhances with rising values of suction and are in good agreement with the numerical result obtained by Babu *et al* (2013).

7.CONCLUSION

The steady MHD boundary layer flow of a linearly shrinking sheet immersed in a quiescent surrounding fluid is analysed. The study enables us to state the following

(i) Due to the increase in the magnetic parameter and mass suction, the velocity increases and, thus the thickness of the momentum boundary layer decreases.

(ii) The temperature decreases with increase in magnetic parameter, Prandtl number and mass suction.

(iii) Increase in heat source/sink parameter increases/ decreases heat at the sheet.

(iv) Increase in Prandtl number, magnetic parameter, mass suction and heat source enhances the rate of heat transfer at the sheet.

(v) The result obtained is in agreement with previously reported cases in the literature.

8.ACKNOWLEDGMENT

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REFERENCES

[1] Sakiadis, B.C. (1961). Boundary layer behavior on continuous solid surfaces: the boundary layer on a flat surface. *AIChE Journal*, 7: 221-225

[2] Crane, L.J. (1970). Flow past a stretching plane. *Journal of Applied Mathematics and Physics*, 21: 645-647.

[3] Gupta, P.S. and Gupta, A.S. (1977). Heat and mass transfer on a stretching sheet with suction or blowing. *Canadian Journal of Chemical Engineering*, 55: 744-746.

[4] Banks, W.H.H. (1983). Similarity solution of boundary layer for stretching wall. J. Mech. *Theo. et Appl.*, 2: 375.

[5] Wang, C.Y. (1984). Three dimensional flow due to a stretching flat surface. *Physics of fluid*, 27: 1915-1917.

[6] Grubka, L.J. and Bobba, K.M. (1985). Heat transfer characteristic of a continuously stretching surface with variable temperature. *Transactions of ASME Journal of heat and Mass transfer*, *107: 248-250*.

[7] Vajravelu, K. and Nayfeh, J. (1993). Convective heat transfer at a stretching sheet. *Accta Mechanica*, 96: 47-54.

[8] Pavlov, K.B. (1974). Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of plane surface. *Magnitnaya Gidrodinamica*, 14: 146-147.

[9] Chakrabati, A. and Gupta, A.S. (1979). Hydromagnetic flow and heat transfer over a tretching sheet. *Quarterly of Mechanics and Applied Mechanics*, 37: 73-78.

[10] Wang, C.Y. (1990). Liquid film on unsteady stretching sheet. *Quarterly of Applied Mathematics*, 489(4): 601-610.

[11] Hayat, T., Abbas, Z. and Sajid, M. (2007). On the analytic solution o magnetohydrodynamic flow of second grade fluid over a shrinking sheet. *Journal of Applied Mechanics*, 74: 601-610.

[12] Battaler, R.C. (2007). Effects of heat source/sink radiation and work done by deformation on the flow and heat transfer of a viscoelastic fluid over a stretching sheet. *Computers and Mechanics with applications*, 53(3): 305-316.

[13] Fang, T. and Zhang, J. (2009). Closed-form exact solution of MHD viscous flow over a stretching sheet. *Communication in Non-linear Science and Numerical Simulation*, 14: 2853-2857.

[14] Chen, C.H. (2009). Magnetohydrodynamic mixed convection of a power-law fluid past a stretching sheet in the presence of thermal radiation and internal heat generation or absorption. *International Journal of Non-linear Mechanics*, 44(6): 596-603.

[15] Krishnendu Bhattacharyya (2011). Effects of hear source/sink on MHD flow and heat transfer over a shrinking sheet with mass suction. *Chemical Engineering Bulletin*, 15: 12-17

[16] Babu, P.R., Rao, J.A. and Sheri, S. (2013). Radiation effect on MHD heat and mass transfer flow over a shrinking sheet with mass suction. *Journal of Applied Fluid Mechan*ics, 7(4): 641-650.

[17] Singh, A.K. and Dikshit, C.K. (1988). Hydromagnetic flow past a continuously moving semi- infinite plate for large suction. *Astrophysics and Space Science*, 148: 249-256.

[18] Miklavcic, M. and Wang, C.Y. (2006). Viscous flow due to a shrinking sheet. *Quarterly of Applied Mathematics 64(2): 283-290.*

[19] Noor, N.F., Kechil, S.A. and Hashim, I. (2009). Simple non-perturbative solution of MHD viscous flow due to shrinking sheet. *Communication in Non-linear Science and Numerical Simulation.*

[20] Wang, C.Y. (2008). Stagnation point flow towards a shrinking sheet. International Journal of non-Linear Mechanics. 43: 377-382.

[21] Bhattcharyya, K., Mukhopadhyay, S. and Layek, G. (2011). Slip effects on boundary layer stagnation point flow and heat transfer towards a shrinking sheet. Internationl Journal of Heat and Mass Transfer. 54: 308.