IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

Opus Helios - A treatise on solar incidence due to the movement of the Earth

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Abstract: The Sun is the driver of all energy on the surface of the Earth. Besides all forms of life, ocean currents, winds and weather systems derive their energy from the Sun. At a distance of about 149.6 million km from the Sun, the Earth receives solar radiative energy of nearly 1,361 W/m^2 which remains fairly constant at the location of one astronomical unit away. However, the energy received at the surface of the Earth varies with the angle of incidence on the surface, which in turn depends on the latitude, time of day and also, the position of the earth in its orbit. It is the difference in incident energy that results in thermal and pressure gradients driving the vast magnitude of wind and ocean currents. The incident radiative power per unit area (I) is related to the Solar Constant (I_o) and angle of incidence (i) as $I = I_o \cos(i)$. To understand the variation of incident solar energy on the Earth, it is imperative to understand the variation of this angle (i) with time, as the Earth performs its celestial dance around the Sun. This paper is an attempt to completely describe and determine the angular position of the Sun with reference to any particular location on the Earth at any particular time. Consequently, it is possible to chart out the incident solar energy across the span of a year at various latitudes on the Earth. Using this relation, it is also possible to determine the length of a day at various times of the year at various locations on the Earth and also how the peak angle of the Sun at its Zenith with respect to any location on the Earth varies with the time of the year.

One-Sentence Summary: This paper derives the mathematical relationship that explains the distribution of incident solar energy over the surface of the Earth and its variation through the seasons with a presentation of the findings at various test locations on the Earth.

Keywords: Solar Incidence; Seasonal Variation; Incident Energy; Time of Equivalent Direct Exposure

Main Text:

1.0 Introduction

The rotation of the Earth and its revolution around the Sun causes the incident solar energy on the Earth to vary continuously through the day. However, this daily variation does not repeat on a daily basis due to the tilt of the Earth's axis with respect to its orbit around the Sun. In this paper, the mathematic relationship between the angle of incidence and the host of other parameters like angular velocities of the Earth's rotation and revolution, obliquity and latitude / longitude of a location will be derived and incident solar energy, length of day and peak daytime elevation angle of the Sun at various test locations on the Earth will be graphically compared.

2.0 The Sunray Vector

The main parameters that will be dealt with are:

i. Radius of the Earth - r
ii. Distance of the centre of the Earth from the centre of the Sun - R
iii. Latitude - λ
iv. Longitude - φ
v. Orbital Tilt - δ
vi. Angular velocity of the Earth around its axis - ω
vii. Angular velocity of the Earth around the Sun - Ω

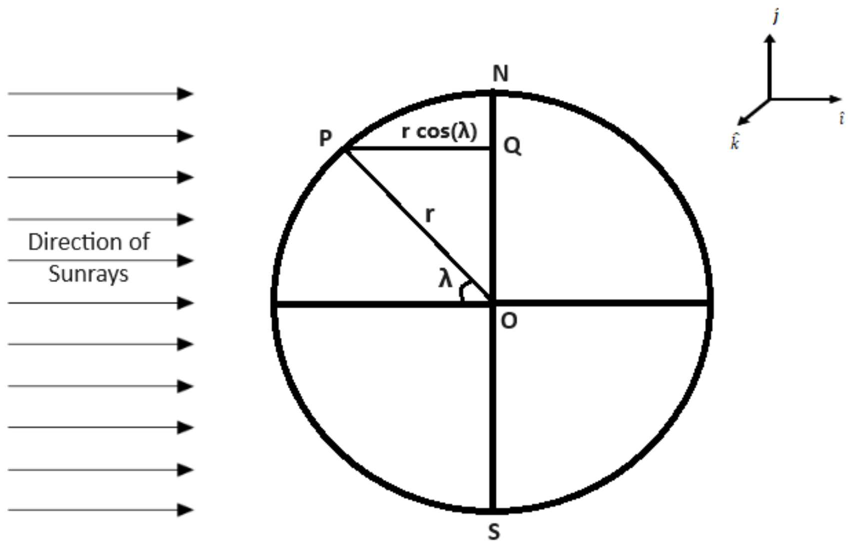


Fig. 1: The Earth with respect to the co-ordinate system chosen

The Right-Hand Co-ordinate system is considered taking the unit vector $\hat{\imath}$ along the direction from the centre of the Sun towards the centre of the Earth intersecting the surface at the equator on the prime meridian during the vernal equinox. This is an idealized starting point for setting up the initial system as once the system is set in motion, the unit vector $\hat{\imath}$ may never exactly intersect the equator at the prime meridian but a few degrees of longitude apart. The unit vector $\hat{\jmath}$ is considered along the axis of rotation of the Earth and points North. The unit vector $\hat{k}$ is mutually perpendicular to the other two unit vectors. Changes due to Milankovitch cycles play out over very long periods of time (tens of thousands of years) and the obliquity (orbital tilt) and direction of the Earth's axis are considered constant for all practical purposes in this paper. Also, the effects of eccentricity of the Earth's orbit, although affect the Solar Constant, do so very minimally and its effect on the angle of incidence of sunlight on the Earth is negligible. So for all practical purposes, the eccentricity is considered nil and the orbit of the Earth is considered circular. Also, the diameter of the Earth subtends an angle of about 17.5 arc seconds at the centre of the Sun and for all practical purposes, the Earth is assumed to receive parallel rays from the Sun.

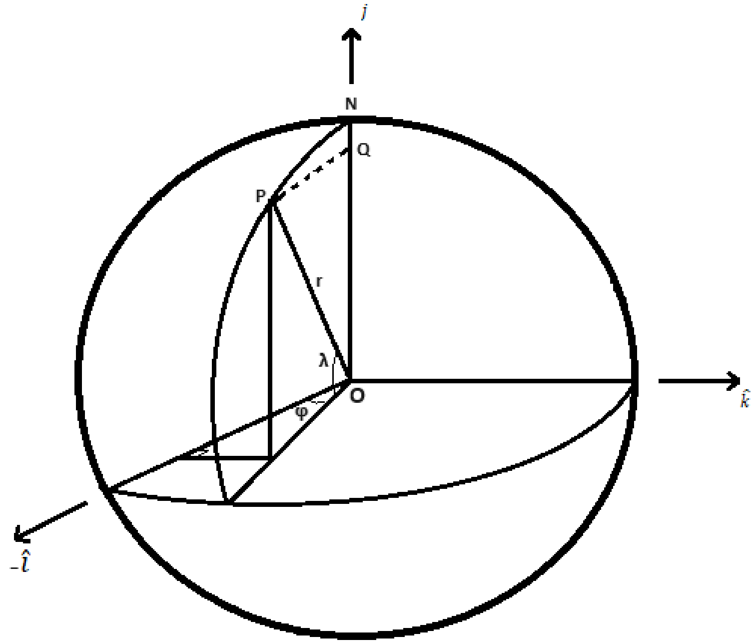


Fig. 2: Test location at point P on the surface of the Earth

Consider a location at point P on the surface of the Earth with a latitude λ and longitude φ.

From Fig.2,

$\overrightarrow{OP} = r\cos(\lambda)\cos(\varphi)(-\hat{\imath}) + r\sin(\lambda)\,\hat{\jmath} + r\cos(\lambda)\sin(\varphi)\,\hat{k}$

As the Earth rotates from West to East in time t, the Earth rotates through an angle $\theta = \omega t$ and the location at point P moves to P' as shown in the following figure.

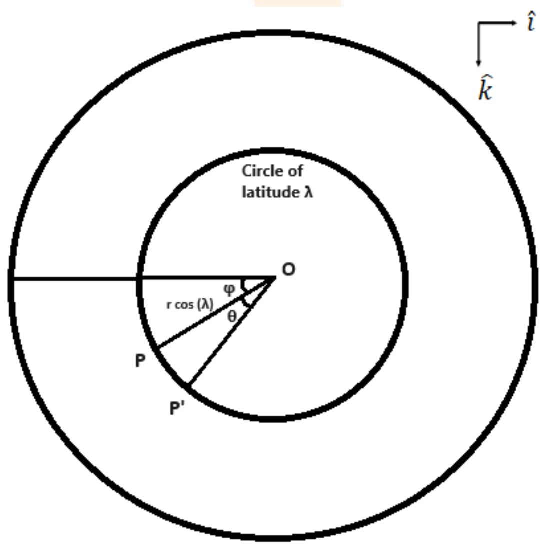


Fig. 3: View of the Earth from above the North Pole. The Earth moves from West to East and point P moves to P' in time t

$\overrightarrow{OP'} = r\cos(\lambda)\cos(\theta+\varphi)(-\hat{\imath}) + r\sin(\lambda)\,\hat{\jmath} + r\cos(\lambda)\sin(\theta+\varphi)\,\hat{k}$ --------- $\boxed{1}$

Now, we shall consider the movement of the Earth around the Sun.

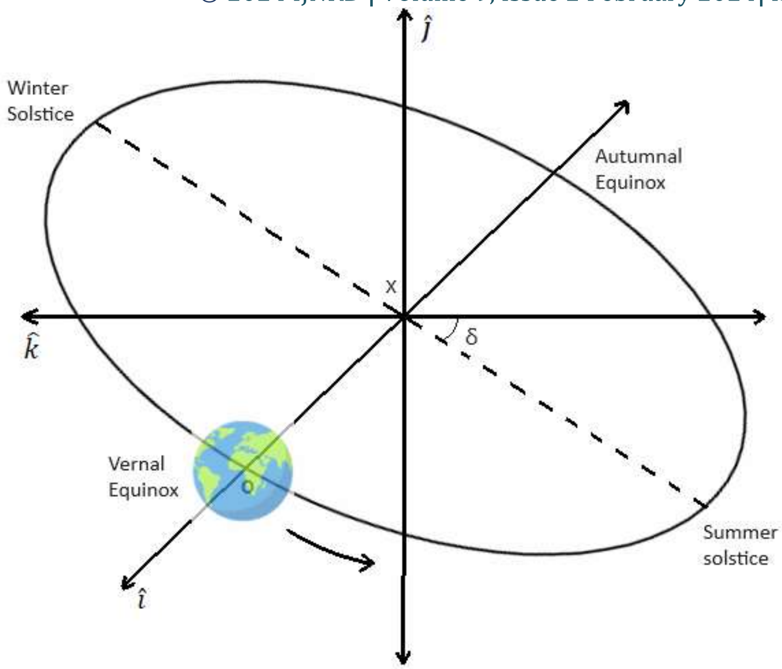


Fig. 4: The Earth in its orbit around the Sun. The solstice and equinox are named with reference to the Northern Hemisphere

The plane of the Earth's orbit around the sun is tilted by an angle δ as shown and the Earth moves counter clock-wise around the Sun when viewed from above the North Pole. As the Earth revolves counter clock-wise in time t, the path of the centre of the Earth subtends an angle $\alpha = \Omega t$ at the centre of the Sun (point X) and the Earth's centre moves from point O to O'. When the orbital plane is viewed normally, the unit vectors considered for the two-dimensional circular motion are $\hat{\imath}$ (pointing to the left) and [- $\hat{\jmath}$ sin(δ) - $\hat{k}$ cos(δ)] (pointing downwards).

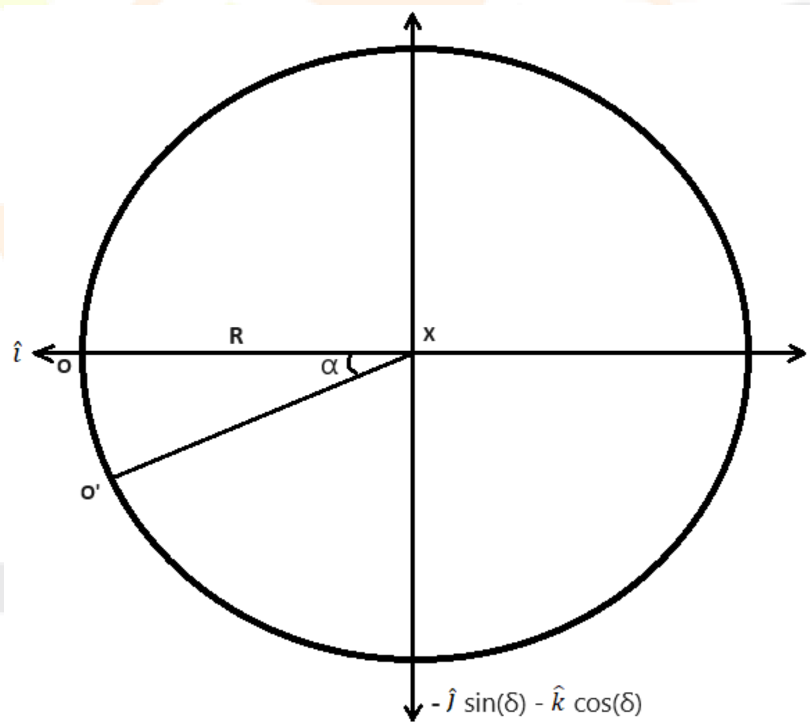


Fig. 5: View of the Earth's orbit normal to the orbital plane. The centre of the Earth moves from point O to O' in time t

$\overrightarrow{XO} = \mathrm{R}\ \hat{\imath}$

$\overrightarrow{XO'} = \mathrm{R}\cos(\alpha)\ \hat{\imath} - \mathrm{R}\sin(\alpha)\ [\sin(\delta)\ \hat{\jmath} + \cos(\delta)\ \hat{k}]$

$\quad = \mathrm{R}\cos(\alpha)\ \hat{\imath} - \mathrm{R}\sin(\delta)\sin(\alpha)\ \hat{\jmath} - \mathrm{R}\cos(\delta)\sin(\alpha)\ \hat{k}$

Due to the movement of the centre of the Earth from O to O', the test Point P which moved from P to P' will now move to P". However, the vector $\overrightarrow{O'P"}$ will be the same as the vector $\overrightarrow{OP'}$

Fig. 6: Test location on the surface of the Earth (P") with respect to the centres of the Earth (O') and the Sun (X)

$\therefore \overrightarrow{XP"} = \overrightarrow{XO'} + \overrightarrow{O'P"}$

$\quad = \overrightarrow{XO'} + \overrightarrow{OP'}$

$\therefore \overrightarrow{XP"} = [\mathrm{R}\cos(\alpha) - \mathrm{r}\cos(\lambda)\cos(\theta+\varphi)]\ \hat{\imath} + [\mathrm{r}\sin(\lambda) - \mathrm{R}\sin(\delta)\sin(\alpha)]\ \hat{\jmath}\ +$

$[\mathrm{r}\cos(\lambda)\sin(\theta+\varphi) - \mathrm{R}\cos(\delta)\sin(\alpha)]\ \hat{k}$ ---------- 2

where $\theta = \omega t$ and $\alpha = \Omega t$

The vector $\overrightarrow{XP"}$ in the above equation represents the sunray vector incident at point P" on the surface of the earth.

The angle of incidence (i) is the angle between the vectors $\overrightarrow{OP'}$ (the normal from the centre of the Earth) and $-\overrightarrow{XP"}$ (the vector in the opposite direction of the sunray vector pointing away from the surface of the Earth).

Fig. 7: Angle of incidence (i)

$\cos(i) = \dfrac{\overrightarrow{OP'} \cdot \overrightarrow{-XP"}}{|\overrightarrow{OP'}| \cdot |\overrightarrow{-XP"}|}$ ------- 3

$\overrightarrow{OP'} \cdot \overrightarrow{-XP"} = [\mathrm{r}\cos(\lambda)\cos(\theta+\varphi)\ (-\hat{\imath}) + \mathrm{r}\sin(\lambda)\ \hat{\jmath} + \mathrm{r}\cos(\lambda)\sin(\theta+\varphi)\ \hat{k}]\ .$

$\quad - \{[\mathrm{R}\cos(\alpha) - \mathrm{r}\cos(\lambda)\cos(\theta+\varphi)]\ \hat{\imath} + [\mathrm{r}\sin(\lambda) - \mathrm{R}\sin(\delta)\sin(\alpha)]\ \hat{\jmath} + [\mathrm{r}\cos(\lambda)\sin(\theta+\varphi) - \mathrm{R}\cos(\delta)\sin(\alpha)]\ \hat{k}\}$

$= [\mathrm{r}\cos(\lambda)\cos(\theta+\varphi)]\ [\mathrm{R}\cos(\alpha) - \mathrm{r}\cos(\lambda)\cos(\theta+\varphi)] - \mathrm{r}\sin(\lambda)\ [\mathrm{r}\sin(\lambda) - \mathrm{R}\sin(\delta)\sin(\alpha)] - \mathrm{r}\cos(\lambda)\sin(\theta+\varphi)\ [\mathrm{r}\cos(\lambda)\sin(\theta+\varphi) - \mathrm{R}\cos(\delta)\sin(\alpha)]$

$= \mathrm{R}\,\mathrm{r}\cos(\lambda)\cos(\alpha)\cos(\theta+\varphi) - \mathrm{r}^2\cos^2(\lambda)\cos^2(\theta+\varphi) + \mathrm{R}\,\mathrm{r}\sin(\lambda)\sin(\delta)\sin(\alpha) - \mathrm{r}^2\sin^2(\lambda) + \mathrm{R}\,\mathrm{r}\cos(\lambda)\cos(\delta)\sin(\alpha)\sin(\theta+\varphi) - \mathrm{r}^2\cos^2(\lambda)\sin^2(\theta+\varphi)$

$= \quad R\, r\, [\cos(\lambda) \cos(\alpha) \cos(\theta+\varphi) + \sin(\lambda) \sin(\delta) \sin(\alpha) + \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta+\varphi)] - r^2$

$$\left|\overrightarrow{OP'}\right|^2 = r^2 \cos^2(\lambda) \cos^2(\theta+\varphi) + r^2 \sin^2(\lambda) + r^2 \cos^2(\lambda) \sin^2(\theta+\varphi) = r^2$$

$$\therefore \left|\overrightarrow{OP'}\right| = r$$

$$\left|-\overrightarrow{XP''}\right|^2 = \left|\overrightarrow{XP''}\right|^2$$

$= \quad R^2 \cos^2(\alpha) - 2 R r \cos(\lambda) \cos(\alpha) \cos(\theta+\varphi) + r^2 \cos^2(\lambda) \cos^2(\theta+\varphi) + r^2 \sin^2(\lambda) - 2 R r \sin(\lambda) \sin(\delta) \sin(\alpha) + R2 \sin^2(\delta) \sin^2(\alpha) + r^2 \cos^2(\lambda) \sin^2(\theta+\varphi) - 2 R r \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta+\varphi) + R^2 \cos^2(\delta) \sin^2(\alpha)$

$= \quad R^2 + r^2 - 2 R r [\cos(\lambda) \cos(\alpha) \cos(\theta+\varphi) + \sin(\lambda) \sin(\delta) \sin(\alpha) + \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta+\varphi)]$

$$\therefore \left|-\overrightarrow{XP''}\right| = \sqrt{R^2 + r^2 - 2\,R\,r\,[\cos(\lambda) \cos(\alpha) \cos(\theta+\varphi) + \sin(\lambda) \sin(\delta) \sin(\alpha) + \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta+\varphi)]}$$

$$\therefore \cos(i) = \frac{R\,r\,[\cos(\lambda) \cos(\alpha) \cos(\theta+\varphi) + \sin(\lambda) \sin(\delta) \sin(\alpha) + \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta+\varphi)] - r^2}{r\sqrt{R^2+r^2 - 2\,R\,r\,[\cos(\lambda) \cos(\alpha) \cos(\theta+\varphi) + \sin(\lambda) \sin(\delta) \sin(\alpha) + \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta+\varphi)]}}$$

$$= \frac{R\,[\cos(\lambda) \cos(\alpha) \cos(\theta+\varphi) + \sin(\lambda) \sin(\delta) \sin(\alpha) + \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta+\varphi)] - r}{\sqrt{R^2+r^2 - 2\,R\,r\,[\cos(\lambda) \cos(\alpha) \cos(\theta+\varphi) + \sin(\lambda) \sin(\delta) \sin(\alpha) + \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta+\varphi)]}}$$

where $\theta = \omega t$ and $\alpha = \Omega t$ and for given values of ω, Ω, R, r, λ, φ and δ, the angle of incidence is a function of time

Taking $\rho = \frac{r}{R}$

$$\cos(i) = \frac{[\cos(\lambda) \cos(\alpha) \cos(\theta+\varphi) + \sin(\lambda) \sin(\delta) \sin(\alpha) + \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta+\varphi)] - \rho}{\sqrt{1 + \rho^2 - 2\,\rho\,[\cos(\lambda) \cos(\alpha) \cos(\theta+\varphi) + \sin(\lambda) \sin(\delta) \sin(\alpha) + \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta+\varphi)]}}$$

Taking $\sigma(t) = \cos(\lambda) \cos(\alpha) \cos(\theta + \varphi) + \sin(\lambda) \sin(\delta) \sin(\alpha) + \cos(\lambda) \cos(\delta) \sin(\alpha) \sin(\theta + \varphi)$

$$\cos(i) = \frac{\sigma(t) - \rho}{\sqrt{1 + \rho^2 - 2\,\rho\,\sigma(t)}} \qquad \text{----------} \boxed{4}$$

Thus, from the above relations, it is seen that for any location on Earth with a given latitude and longitude, the angle of incidence is a function of time.

3.0 Interpretation of the relation

The values of the constants are taken as follows:

1. Angular velocity of rotation of the Earth (ω): The Earth completes one full rotation around its axis in 23 hours 56 minutes 4 seconds. Thus there are 86,164 seconds in one rotation of the Earth. Hence the angular velocity is given by

   $\omega = \frac{2\pi}{T} = \frac{2\pi}{86{,}164} = 7.29212 \times 10^{-5}$ rad/s

2. Angular velocity of revolution of the Earth around the Sun (Ω): One sidereal year in which the Earth makes one revolution around the Sun takes 365.256363 days (each day being 24 hours long). Thus the angular velocity $\Omega = 1.99099 \times 10^{-7}$ rad/s.
3. Radius of the Earth (r): r = 6371 km
4. Radius of the Earth's orbit (R): R = 149,600,000 km
5. Orbital tilt (δ): $\delta = 23.43645$ deg or 0.409043 rad

With these values, the charts for incident energy, length of day and peak daytime elevation of the Sun at various locations are plotted for a year.

3.1 Incident Energy

The incident radiative power per unit area (I) is related to the Solar Constant (I_o) and angle of incidence (i) as $I = I_o \cos(i)$. The total incident energy during the day is

$$E = \int I\,dt = I_o \int \cos(i)\,dt. \qquad \text{----------} \boxed{5}$$

The integral will have to be taken within the time limits where cos(i) is positive or $-90^o < i < 90^o$, i.e. as long as the sun is above the horizon ($i = 0^o$ indicates the Sun vertically overhead above the test location). This relation neglects the effect of refraction through the earth's atmosphere which becomes significant when the Sun is near the horizon as the refractive effects cause Solar incidence even when the Sun goes below the horizon. I_o is about 1,361 W/m^2 which means that 1,361 Joule of energy is incident on 1 sq.m. area every second. If the Sun were to stay overhead (at 0^o to the radial vector from the Earth's centre) all day long, then cos(i) would be 1 and every second, each sq.m. area would receive 1,361 Joule and the total energy received per sq.m. in one day would be 1,361 x 86,400 = 117,590,400 Joule

The expression $I_o \int \cos(i)\,dt$ is solved numerically and the relation between i and t as established in the equation 4 above, is set out on a computer spreadsheet and the value of cos(i) is determined for each time step. In this exercise, the time step is taken as one minute. The energy incident during each time step is then quantified in Watt-minutes (or x60 Joule). The summation of all ***positive values*** of cos(i) for 1,440 time steps (one solar day) becomes representative of the total solar energy incident at that location for that particular day (negative values indicate Sun below the horizon and are not considered for the calculation of incident energy representation). In the above extreme example of the Sun persistently overhead, all time steps would have positive values of cos(i) and the value of $\int \cos(i)\,dt$ would be $\sum_{t=1}^{t=1440} \cos(O)$ = 1440 minutes. The total incident energy in a day per sq.m. would then be $I_o \int \cos(i)\,dt$ = 1,361 x 1440 = 1,959,840 Watt-minutes. The term $\int \cos(i)\,dt$ is indicative of the equivalent time of direct normal exposure of the test location to the Sun for a particular day and the term $\int \cos(i)\,dt$ is indicated as ***T_{EDE} (Time of Equivalent Direct Exposure)*** expressed in minutes (as we have taken each time step as a minute). Thus, the incident energy during the day $E = I_o T_{EDE}$.

The exercise to determine T_{EDE} is carried out separately for all the days of the year and the data is plotted for the year. A similar exercise can be carried out at various latitudes and the incident energy graph comparison looks like this:

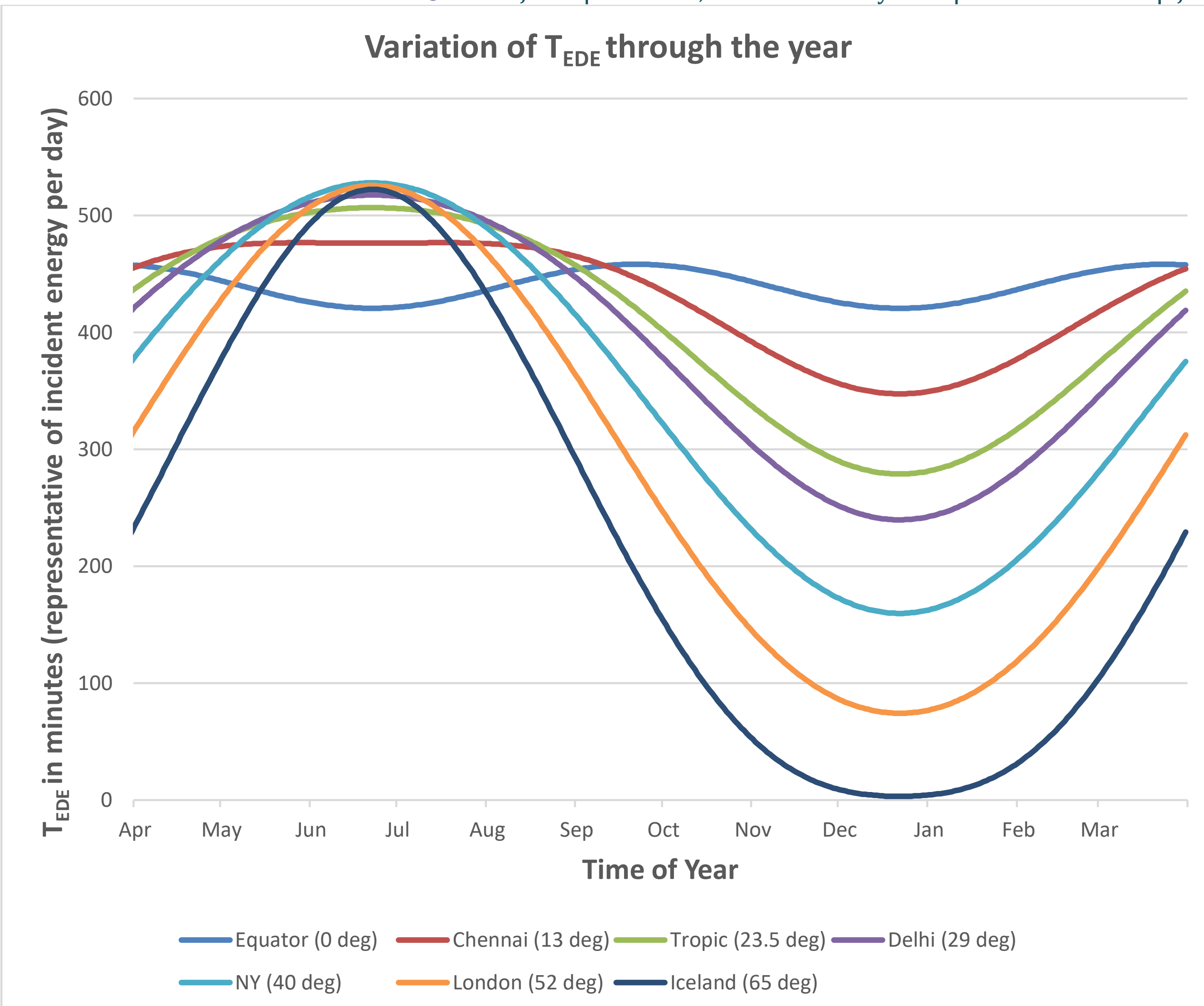


Fig. 8: Variation of T_{EDE} through the year

From the above graph, the following can be inferred:

i. The test locations (indicated in the legend of the graph above) have been taken at the equator and at various latitudes in the Northern Hemisphere. It is seen that in the Northern Hemisphere, T_{EDE} and consequently, E is the maximum during the summer solstice and minimum during the winter solstice. However, at the equator, the maximum values occur during the equinoxes.

ii. It is seen that among the seven test locations considered, the highest value of T_{EDE} occurs at 40 deg latitude peaking to 527.90 minutes on the solstice. This corresponds to an energy incidence of 1,361 x 527.90 = 718,472 Watt-minutes or 43,108,314 Joule in a day. In comparison, the peak T_{EDE} at the equator is 458.25 minutes during the equinox. This corresponds to an energy incidence of 1,361 x 458.25 = 623,678 Watt-minutes or 37,420,695 Joule. This means that the peak daily solar incidence at New York is 15.20% higher than that at the equator.

iii. Near the Arctic Circle, the daily solar incidence drops to near zero at the winter solstice. North or the Arctic Circle, the daily solar incidence remains nil till the Sun emerges from the horizon.

iv. Between the equator and the Tropic of Cancer, at 13^{o}N, the daily solar incidence remains fairly constant from May to July.

v. As we move away from the equator, the variation between maximum and minimum daily solar incidence increases substantially. Also the rate of change of the daily solar incidence increases as we proceed from the equator to the poles as seen from the slope of the curves.

3.2 Length of Day

The length of day (or the time for which the Sun stays above the horizon) can also be determined from the above exercise by counting the number of time steps with positive values of cos(i) out of the 1,440 time steps during a day. The length of day at the various test locations is depicted in the graphs below:

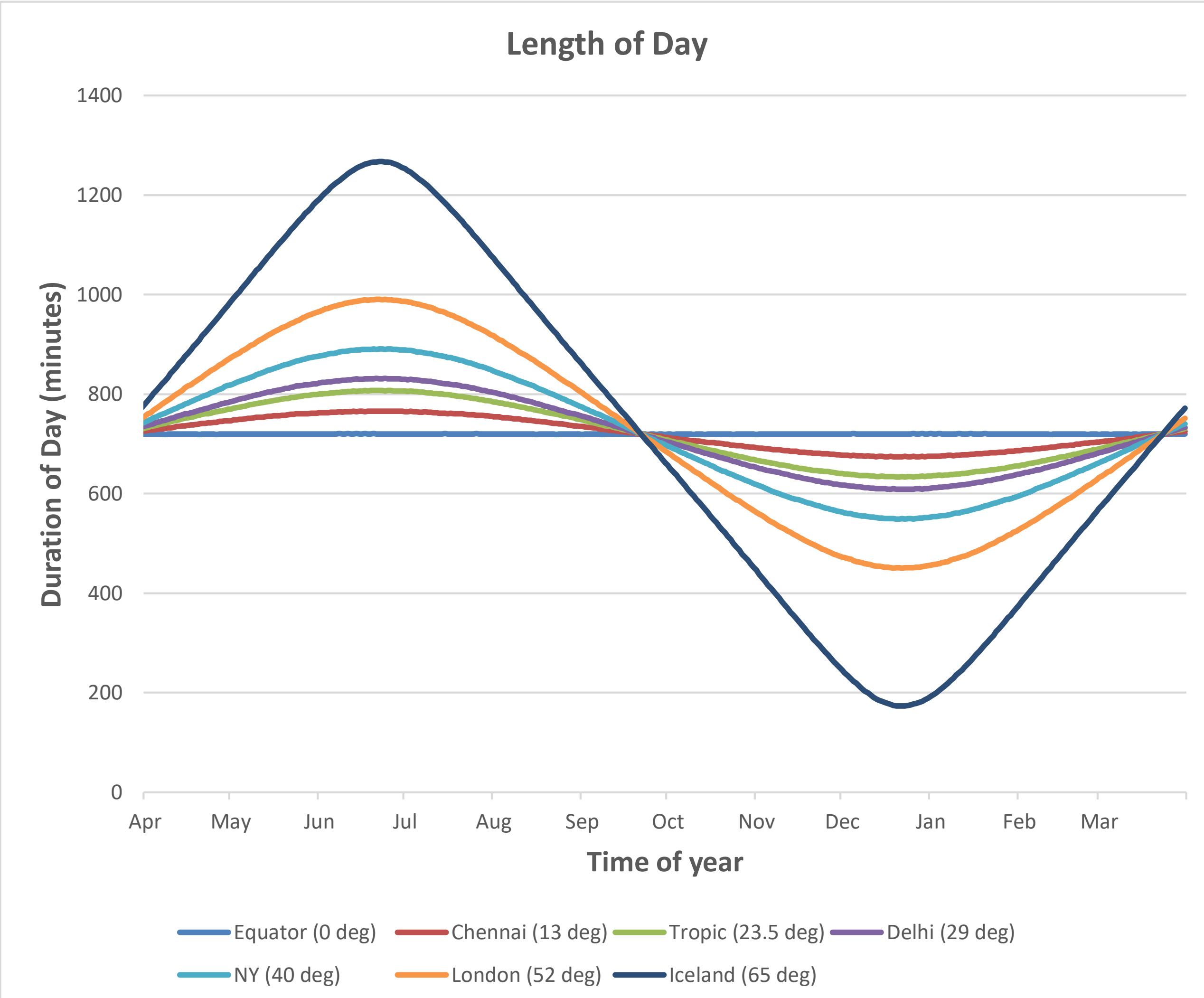


Fig. 9: Variation of the length of day through the year

From the above graph, it can be seen that the seasonal variation in the length of day increases as we move from the equator towards the poles. At the equator, the length of day is the same throughout the year at 720 minutes (12 hours). At 65^{o}N, the length of day is the highest on the summer solstice at 1267 minutes (21 hours 7 minutes) and the least on the winter solstice at 173 minutes (2 hours 53 minutes). All locations have 12 hours of day and 12 hours on night on the vernal and autumnal equinox.

3.3 Peak Daytime Elevation angle of the Sun

The peak elevation of the angle of the Sun during the day at a given location on the Earth can also be determined from this exercise. The elevation angle would be 90^o less the angle of incidence (i). So the peak elevation angle of the Sun would occur when the angle of incidence is minimum, which corresponds to the maximum positive value of cos(i) out of the 1440 time steps. For example, a peak elevation angle of 90^o would correspond to an angle of incidence of 0^o and cos(i) = 1. The peak elevation angle in the day can plotted for all the days of the year and the graph for the various test locations is as below:

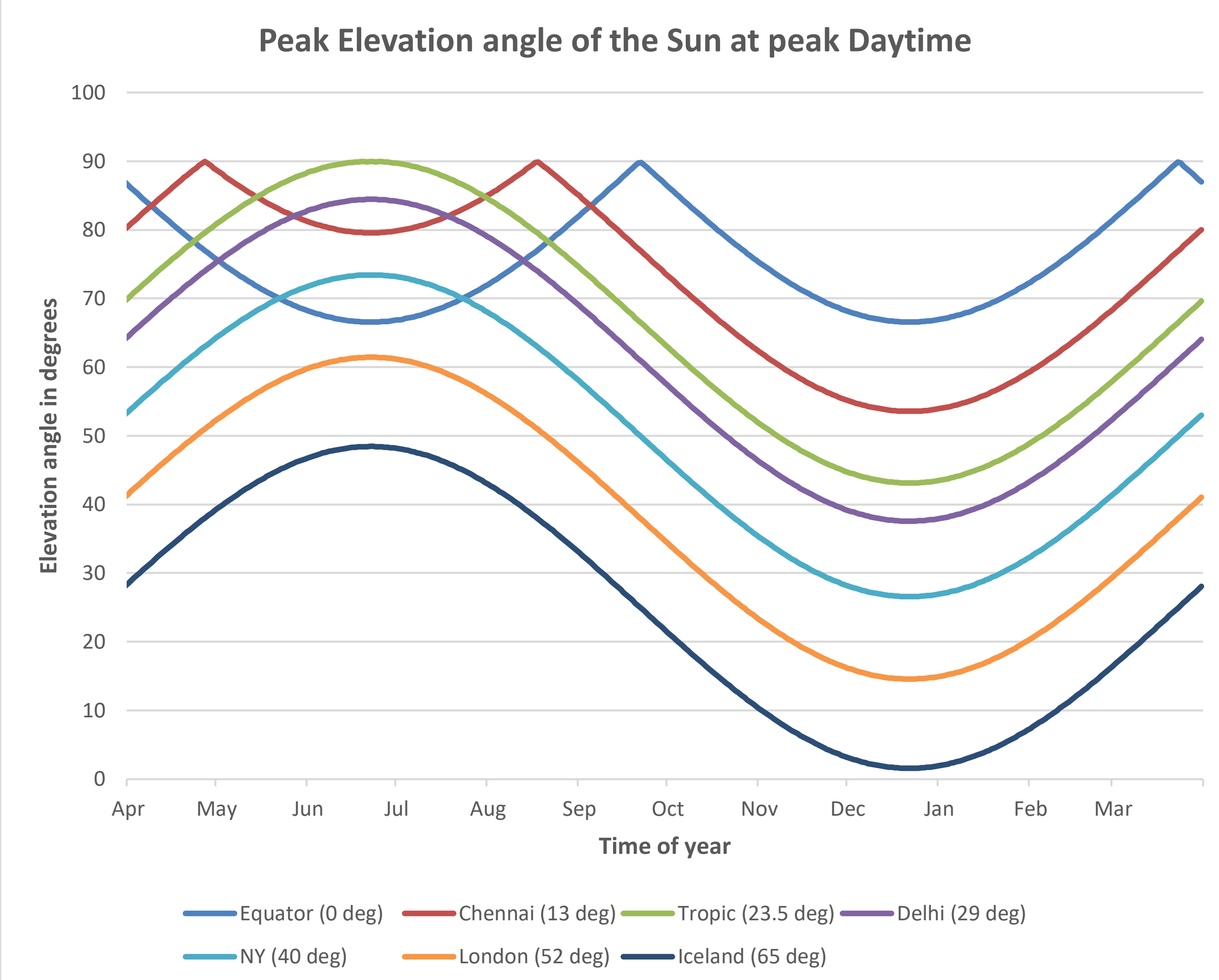


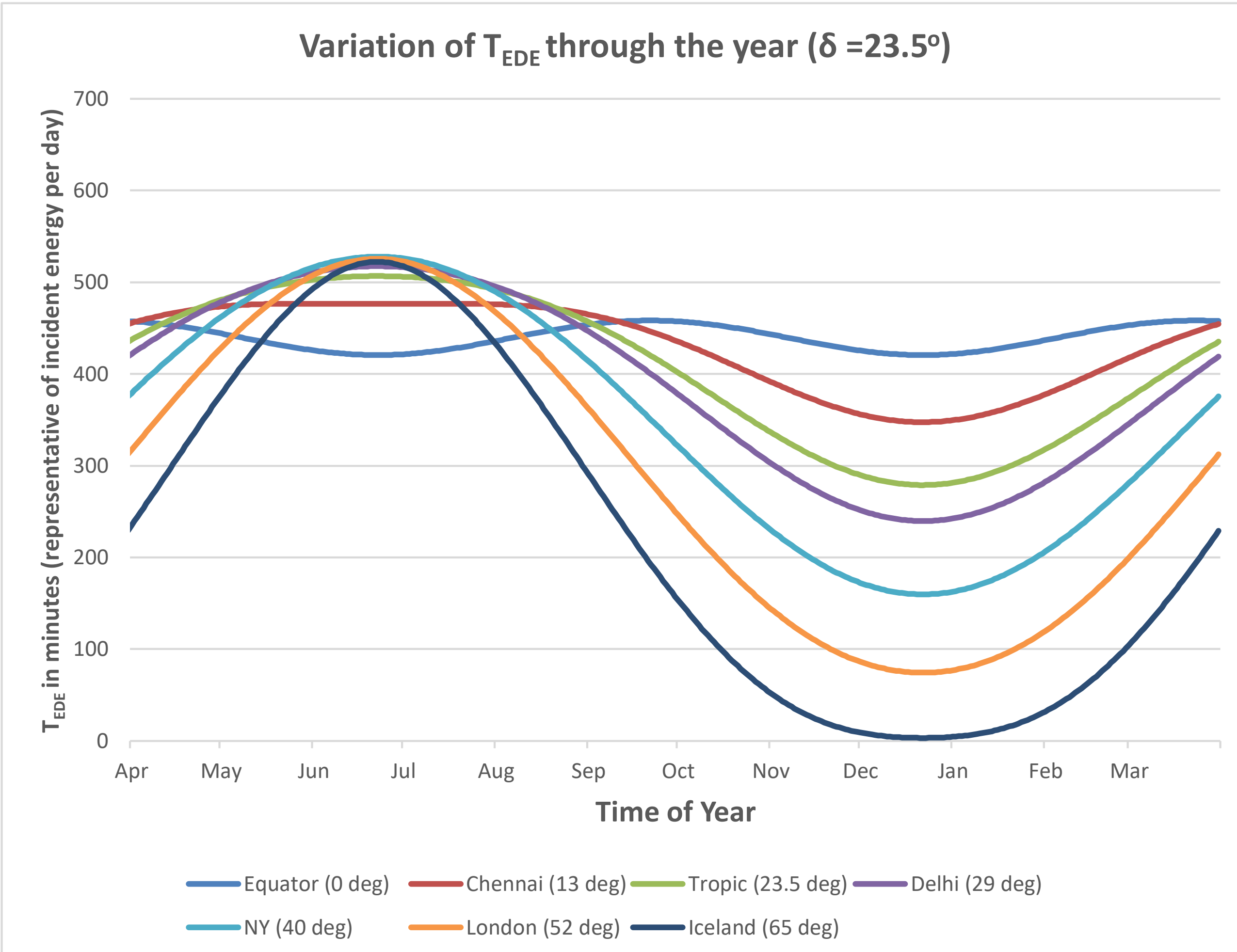
Fig. 10: Variation of the peak elevation angle of the Sun through the year

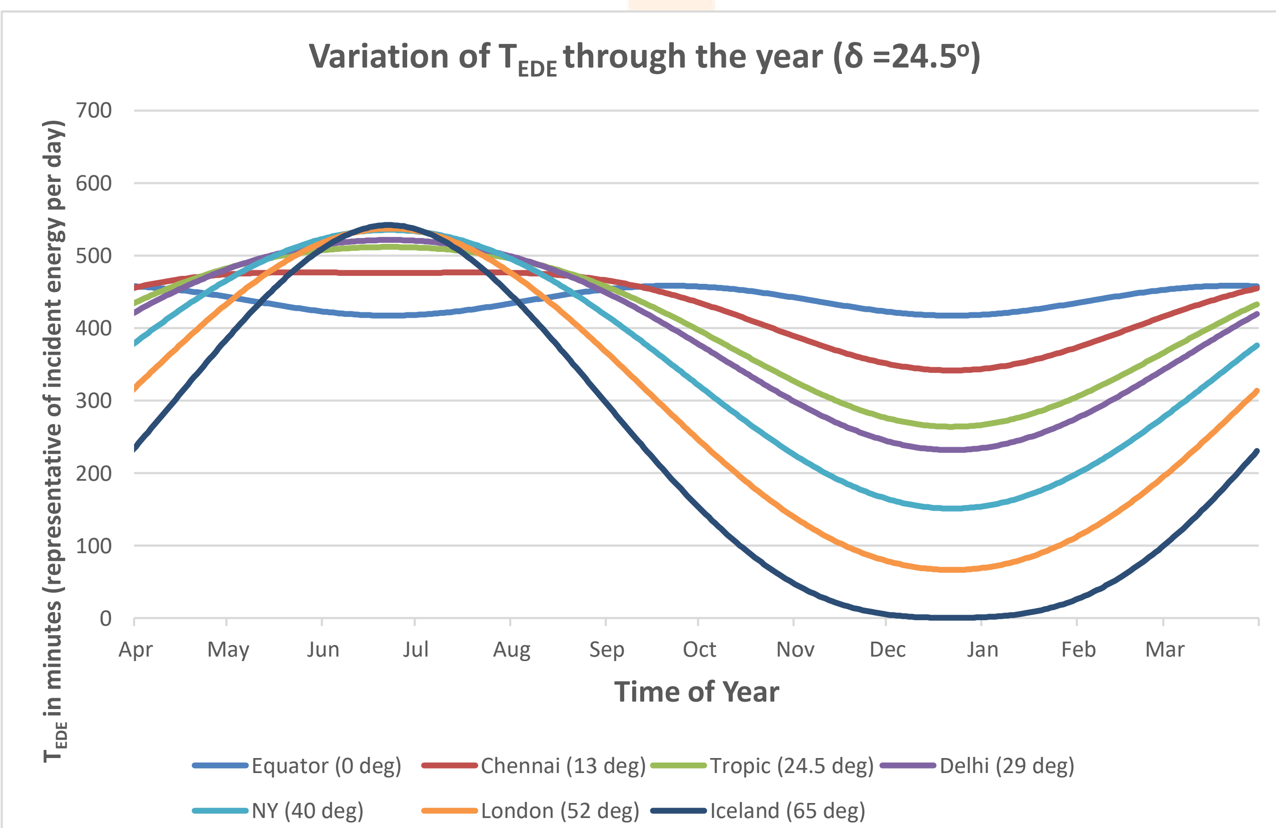
From the above graph it is seen that between the equator and the tropics the Sun peaks at an elevation angle of 90^o twice a year. At the equator, the peak angle of 90^o occurs during the vernal and autumnal equinox. As we move towards the tropic of Cancer, the two occurrences of the peak angle of 90^o draw nearer to the summer solstice from the equinox (as can be seen for the graph corresponding to 13^{o}N. At the tropic of Cancer, the peak angle of 90^o occurs once on the summer solstice. As we proceed away from the tropic to the pole, the peak angle never reaches 90^o any time of the year. At 65oN, during the winter solstice, the Sun just glances above the horizon at its peak. North of the Arctic Circle, the Sun does not rise above the horizon around the winter solstice.

4.0 Effect of changing the Orbital Tilt Parameter

It is well known that the orbital tilt or obliquity or the tilt of the Earth's axis to the plane of its orbit around the Sun causes seasonal variations. Presently, the orbital tilt of the Earth is about 23.5^o. But this tilt varies slowly and over a cycle averaging about 40,000 years, the orbital tilt varies between 22.1^o and 24.5^o. As the orbital tilt changes, the seasons become more severe and the graphical comparison of T_{EDE}, day length and peak elevation angle of the sun for δ values of 23.5^o, 24.5^o and a hypothetical situation of 29^o is depicted below:

4.1 T_{EDE} (representative of the incident Energy)





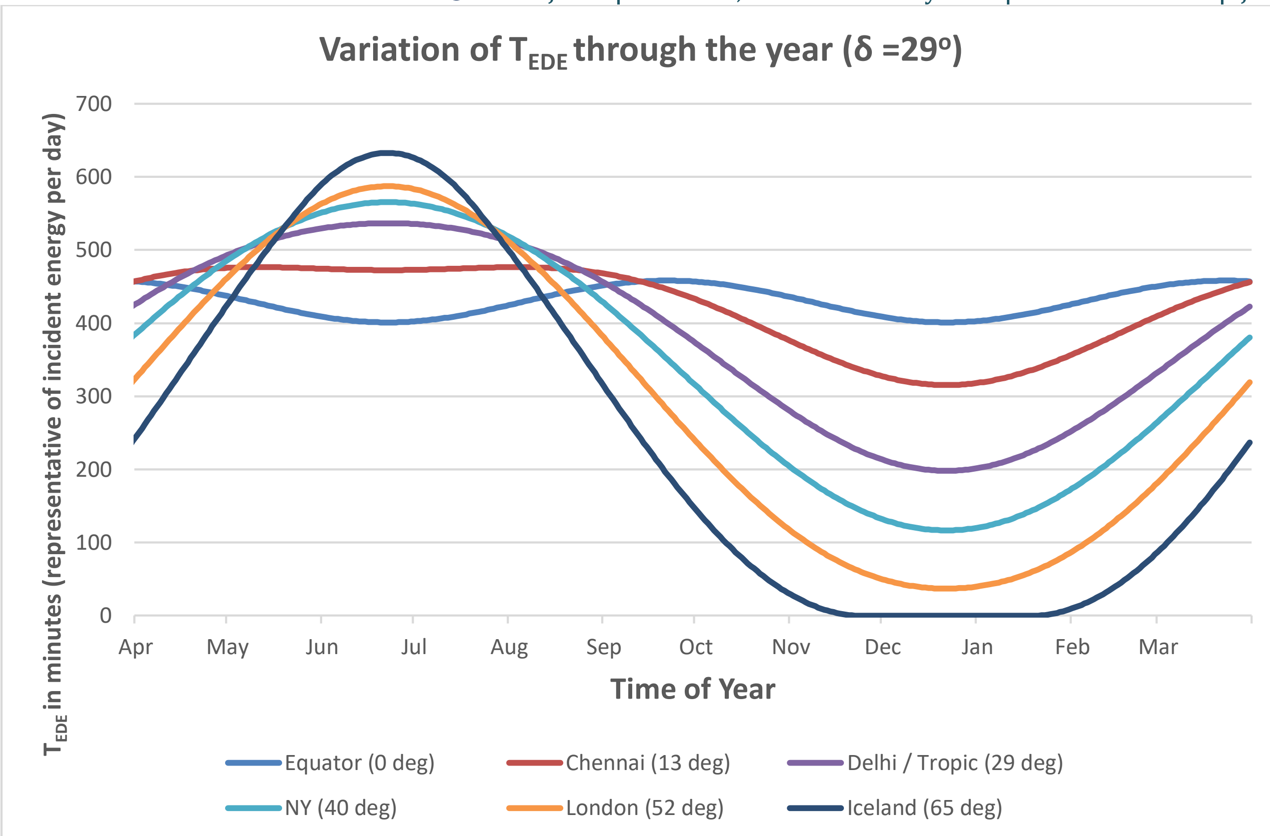


Fig. 11: Comparison of daily variation of T_{EDE} for different values of δ

δ =23.5º	**δ =24.5º**	**δ =29º**	**Inference**
The peak T_{EDE} at the equator is 458.25 minutes during the equinox. This corresponds to an energy incidence of 1,361 x 458.25 = 623,678 Watt-minutes or 37.420 MJ in a day.	The peak T_{EDE} at the equator is 458.24 minutes during the equinox. This corresponds to an energy incidence of 1,361 x 458.24 = 623,664 Watt-minutes or 37.420 MJ in a day.	The peak T_{EDE} at the equator is 458.19 minutes during the equinox. This corresponds to an energy incidence of 1,361 x 458.19 = 623,597 Watt-minutes or 37.416 MJ in a day.	The peak T_{EDE} at the equator does not vary much
Among the test locations, the highest value of T_{EDE} occurs at 40 deg latitude peaking to 527.90 minutes on the solstice. Corresponding energy incidence of 718,472 Watt-minutes or 43.108 MJ in a day (15.20% higher than that at the equator).	Among the test locations, the highest value of T_{EDE} occurs at the northernmost latitude (65ºN in this case) peaking to 541.87 minutes on the solstice. Corresponding energy incidence of 737,485 Watt-minutes or 44.249 MJ in a day (18.25% higher than that at the equator).	Among the test locations, the highest value of T_{EDE} occurs at the northernmost latitude (65ºN in this case) peaking to 632.86 minutes on the solstice. Corresponding energy incidence of 861,322 Watt-minutes or 51.679 MJ in a day (38.12% higher than that at the equator). The T_{EDE} stays above 600 minutes for 38 days from 4th June to 11th July. Further, this location stays in continuous darkness for 59 days from 23rd Nov to 20th Jan as the T_{EDE} drops to zero and the Sun stays below the horizon.	The peak T_{EDE} varies drastically for locations closer to the Poles.

Table 1: Inferences from Comparison of daily variation of T_{EDE} for different values of δ

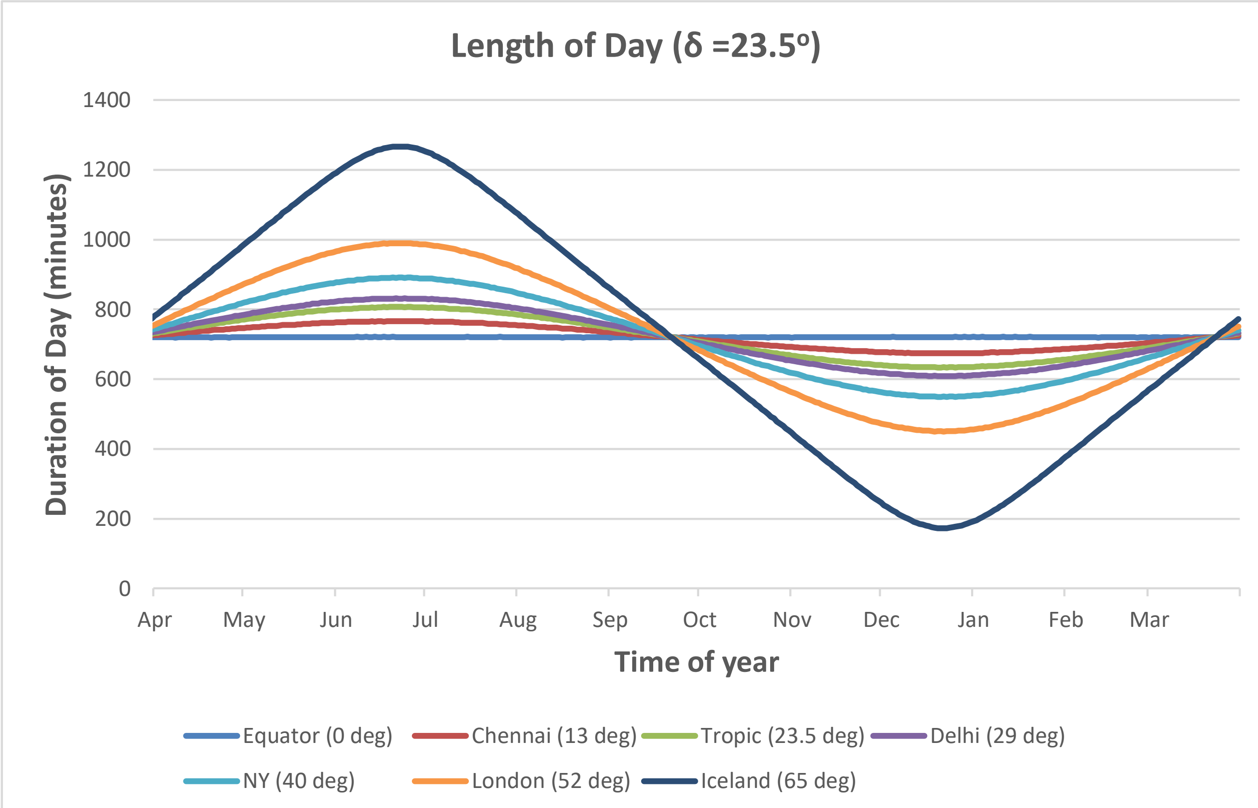
A comparison of the total energy (in terms of T_{EDE}) received in Summer and Winter at the various test locations is represented in the table below. The quantity for Summer is taken as the summation of T_{EDE} of each day from the vernal equinox to the autumnal equinox. For Winter the quantities are taken from the autumnal equinox to the vernal equinox.

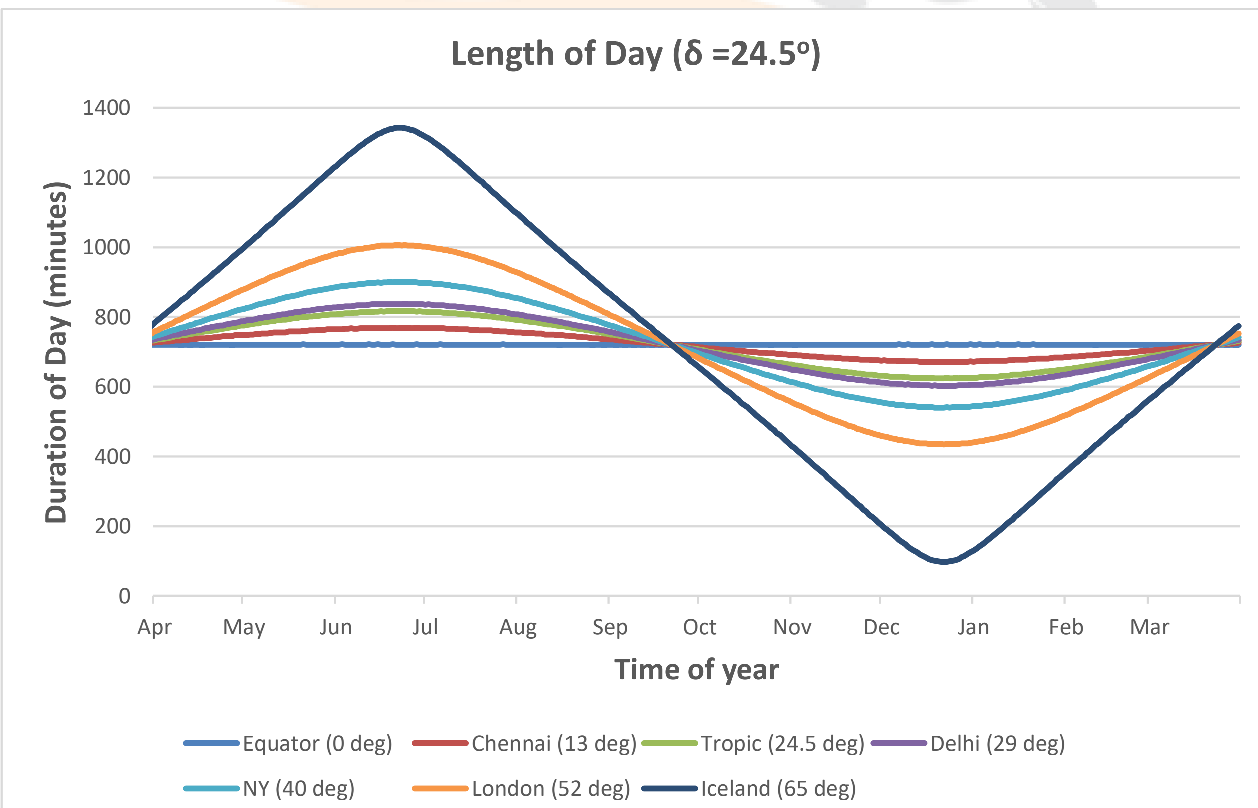
Delta	**T_{EDE} (minutes)**	**Equator (0^{o}N)**	**Chennai (13^{o}N)**	**Tropic (δ^{o}N)**	**Delhi (29^{o}N)**	**NY (40^{o}N)**	**London (52^{o}N)**	**Iceland (65^{o}N)**
δ =23.5^o	T_{EDE} during Summer	80233	85849	87465	87265	84780	79261	71425
	T_{EDE} during Winter	80233	70870	60981	54983	41979	26791	11080
	Total T_{EDE} in the year	160466	156719	148446	142248	126759	106051	82505
	Ratio of Summer to Winter T_{EDE}	1.00	1.21	1.43	1.59	2.02	2.96	6.45
δ =24.5^o	T_{EDE} during Summer	79925	85885	87854	87773	85638	80539	73358
	T_{EDE} during Winter	79925	70268	59063	54114	41012	25831	10440
	Total T_{EDE} in the year	159849	156153	146918	141888	126650	106371	83799
	Ratio of Summer to Winter T_{EDE}	1.00	1.22	1.49	1.62	2.09	3.12	7.03
δ =29^o	T_{EDE} during Summer	78489	85888	89789	89789	89179	85967	82140
	T_{EDE} during Winter	78489	67630	50439	50439	37007	22010	8584
	Total T_{EDE} in the year	156977	153518	140229	140229	126186	107977	90724
	Ratio of Summer to Winter T_{EDE}	1.00	1.27	1.78	1.78	2.41	3.91	9.57

Table 2: Summer, Winter and Total (Annual) T_{EDE} for the various locations at different values of δ. The Energy (in MJ) = $\frac{\text{TEDE x 60 x 1361}}{\text{1,000,000}}$

From the above table, it is seen that the total incident energy through the year is the highest at the equator and reduces towards the poles. As δ increases, the total annual incident energy decreases at the various test locations from the equator to 40^{o}N. However, this value increases with increasing δ at 52^{o}N and 65^{o}N.

4.2 Length of Day





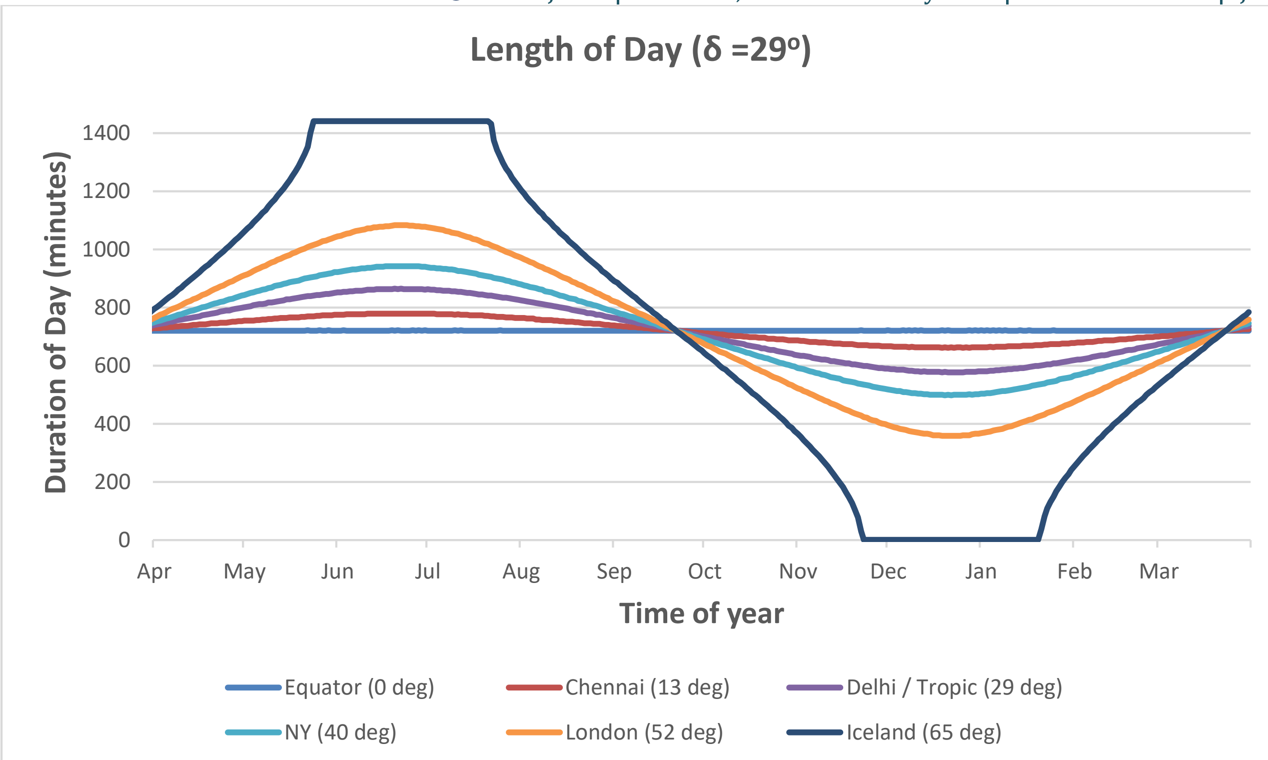
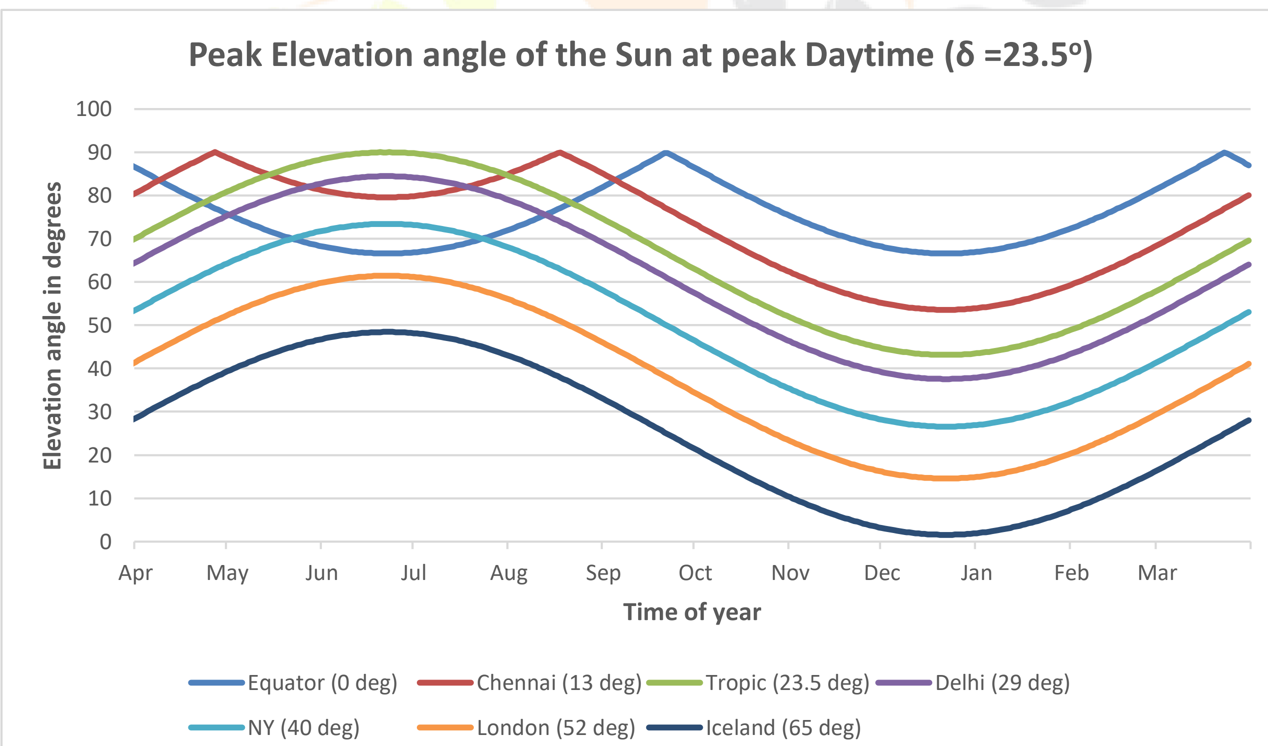
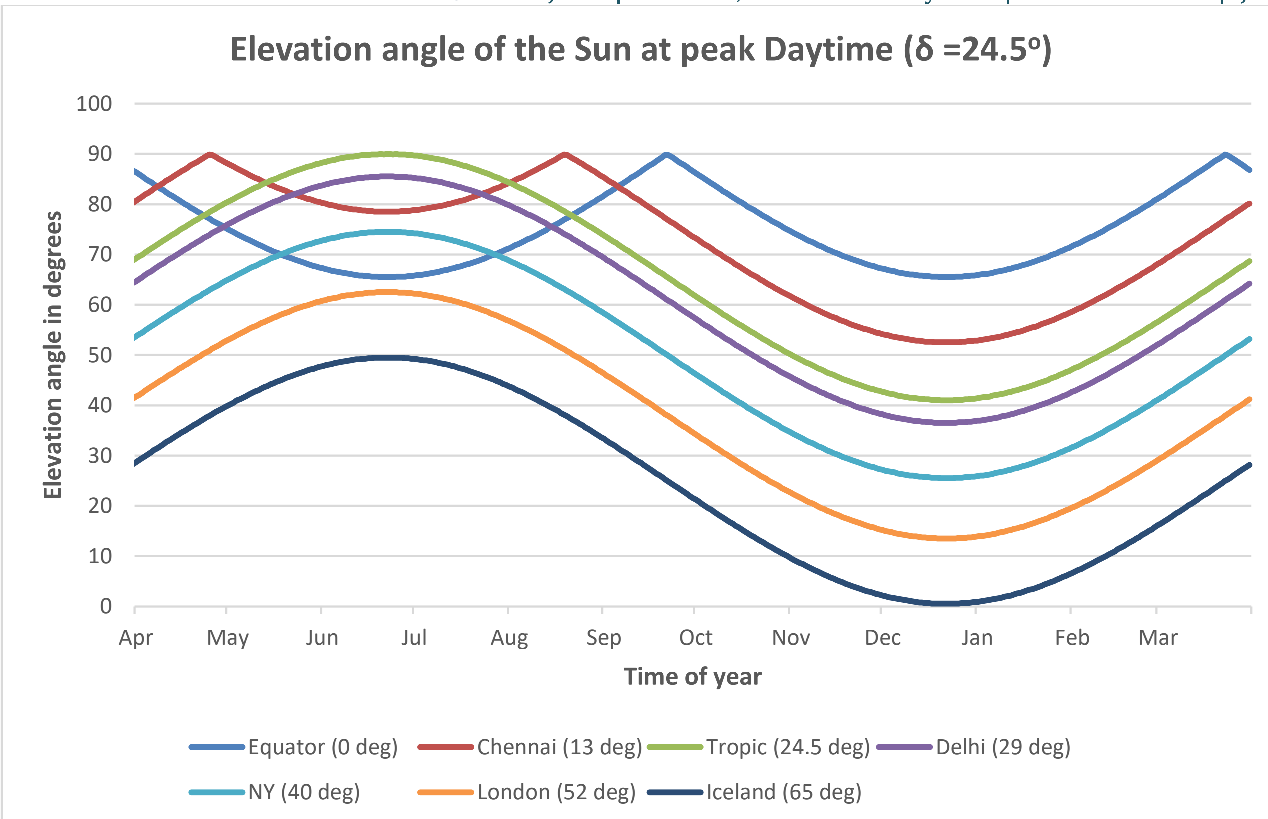


Fig. 12: Comparison of daily variation of length of day for different values of δ

From the above comparative figures, it is seen that as we move from the equator to the poles, the rate of change in the length of day drastically changes. As δ increases, the long summer days become longer still and the shorter winter days become shorter still.

4.3 Peak Daytime Elevation angle of the Sun





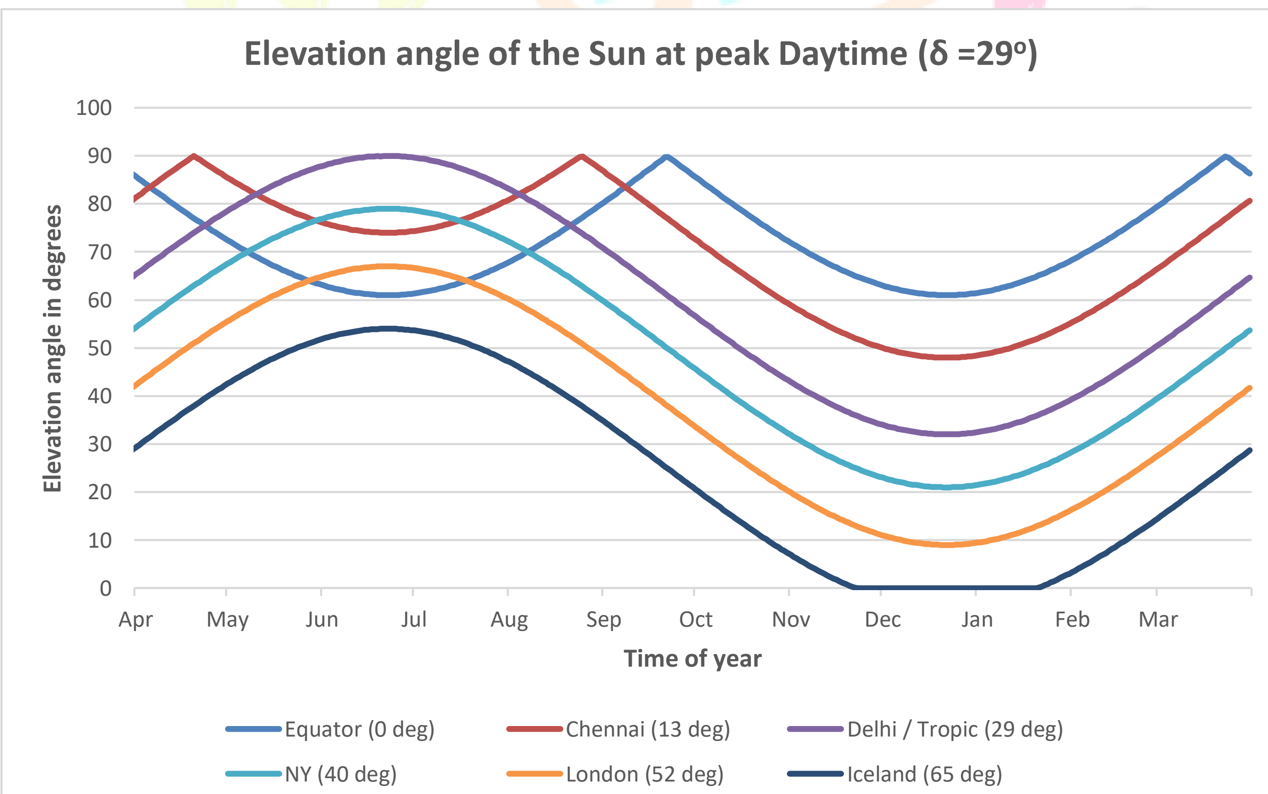


Fig. 13: Comparison of daily variation of peak elevation angle of the Sun for different values of δ

As seen with the Day Length charts, with increasing δ, the peak elevation angle of the Sun over a location increases during summer and decreases during winter.

5.0 CONCLUSION

The spatial and temporal variation in incident Solar Energy on the Earth is almost purely a consequence of the geometric setup of the movement of the Earth around the Sun. Effects like cloud cover, refraction, greenhouse effect and other atmospheric effects will cause slight deviation from the energy results plotted here. However, understanding the kinematics of the motion of the Earth, thus affecting Solar incidence serves as a foundation over which other deviation-causing effects can be imposed to match observations. The study of solar energy incidence can be applied to study weather, temperature variations, wind and ocean currents, etc. The quantity of heat received at any location affects the quantum of evaporation and condensation. This in turn can be applied to precipitation studies.

Varying the values of r, R, ω, Ω, δ and I_o, this study can be applied to other planets of the Solar System to study the heat distribution and weather phenomena in extra-terrestrial systems.

References and Notes

1. NASA Science Reference Systems

Acknowledgments: The contents presented in this paper are the author's individual work inspired by the curiosity and awe towards the diversity in nature brought about by the energy showered upon the Earth by the Sun. The values and charts presented in this paper are made based on the voluminous database created by the author consisting of all associated values for each time step of the calculation of the various parameters presented for the various test locations. There is no external specific source of funding for this article

List of Supplementary Materials:

Fig. 1: The Earth with respect to the co-ordinate system chosen

Fig. 2: Test location at point P on the surface of the Earth

Fig. 3: View of the Earth from above the North Pole. The figure illustrates the movement of a test location as the Earth moves from West to East

Fig. 4: The Earth in its orbit around the Sun. The solstice and equinox are named with reference to the Northern Hemisphere

Fig. 5: View of the Earth's orbit normal to the orbital plane.

Fig. 6: Test location on the surface of the Earth with respect to the centres of the Earth and the Sun

Fig. 7: Angle of incidence (i)

Fig. 8: Variation of T_{EDE} through the year

Fig. 9: Variation of the length of day through the year

Fig. 10: Variation of the peak elevation angle of the Sun through the year

Fig. 11: Comparison of daily variation of T_{EDE} for different values of δ

Fig. 12: Comparison of daily variation of length of day for different values of δ

Fig. 13: Comparison of daily variation of peak elevation angle of the Sun for different values of δ

Table 1: Inferences from Comparison of daily variation of T_{EDE} for different values of δ

Table 2: Summer, Winter and Total (Annual) T_{EDE} for the various locations at different values of δ. The Energy (in MJ) = $\frac{\text{TEDE x 60 x 1361}}{1{,}000{,}000}$