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Modelling Volatility Dynamics of Cryptocurrencies Using GARCH Models

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***Abstract** :* The objective of this research paper is to examine the volatility of cryptocurrency returns. Specifically, we consider the following cryptocurrencies: Bitcoin, Ethereum, Tether, Solana, Ripple, Cardano, Dogecoin, Litecoin, Binance Coin, and Stellar. Special attention will be given to Bitcoin and Ethereum due to their significance in terms of key market capitalization and financial performance of cryptocurrencies. The results obtained through time-series econometrics covering the period from 04/2013 to 06/2022 show that return volatility is not constant (heteroskedastic). Using GARCH modeling, we found that volatility shocks are permanent and significant on conditional variance. The sum of ARCH and GARCH terms is close to unity, implying that shocks on the conditional variable have a permanent impact. However, the results of the analyses of TARCH and EGARCH models have revealed a leverage effect for three cryptocurrencies (Bitcoin, Dogecoin, and Tether): negative economic shocks have a greater impact on future return volatility (amplification effect). For other crypto-assets, the models do not mention significant impacts on volatility.

Keywords: Cryptocurrencies, Volatility, GARCH models
JEL Classification: C22, G12

I. INTRODUCTION

Cryptocurrencies have become hot topics, capturing media attention and sparking debates, even controversies, worldwide. Since the creation of Bitcoin in 2009, the first and most famous of crypto-assets, the cryptocurrency landscape has experienced explosive growth. Barely a decade later, a single bitcoin is worth well over $30,000. There are over 19,000 cryptocurrencies in circulation, and their combined value approaches $1.8 trillion, after surpassing $3 trillion just a few months ago. However, this rise in prominence, driven by the decentralized nature and increasing trading volume and market capitalization with the prospect of achieving high short-term returns (Kristoufek 2013), is accompanied by significant volatility, making cryptocurrencies an important area of study for investors and regulators.
Volatility, which measures the magnitude of price fluctuations of a financial asset over a given period, is particularly pronounced in the cryptocurrency market. This high volatility poses significant risk management challenges for investors and financial institutions. Understanding the volatility of an asset is crucial for investors, as it directly influences investment strategies and potential outcomes, especially in the dynamic and constantly evolving cryptocurrency market. For central banks, market volatility estimates often serve as a gauge of financial market vulnerability and the economy as a whole.
Numerous theoretical and empirical research studies have focused on analyzing cryptocurrency volatility. Previous studies have revealed that the cryptocurrency market is extremely risky, with frequent jumps creating highly volatile outliers (Akhtaruzzaman et al., 2022; Trucíos, 2019; Briere et al., 2015). Various explanations have been put forward to explain these volatility patterns, ranging from the complexity of determining intrinsic values to uncertainty regarding regulation and susceptibility to speculative pressures (Romanchenko et al., 2018; Alam, 2017; Hayes, 2017; Bartos, 2015). Nevertheless, due to the urgency of empirical evidence, there is still a strong need to understand and manage cryptocurrency volatility risk, to devise new evidence to quantify this risk across a range of cryptocurrencies, and simultaneously to acknowledge gaps in the existing empirical literature. It is undeniable that volatility modeling is essential for risk management assessment. Moreover, it is necessary to assess their volatility relative to traditional instruments due to the highly volatile environment. Many volatility models have been developed since Engle's (1982) seminal paper, which introduced a class of Autoregressive Conditional Heteroskedasticity (ARCH) models. Bollerslev (1986), following Engle's earlier work, generalized the ARCH (Autoregressive Conditional Heteroskedasticity) model and introduced the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model. Since then, GARCH models have evolved to become the most widely used models for estimating volatility. Therefore, it is essential to find the most appropriate technique for estimating cryptocurrency volatility. The question becomes increasingly critical as the cryptocurrency market grows, with a growing number of currencies, platforms, and buyers. Our research aims to evaluate the effectiveness of GARCH models in modeling cryptocurrency volatility forecasting. This article focuses on modeling the volatility dynamics of the ten largest cryptocurrencies in terms of market capitalization, covering the period from 04/2013 to 06/2022. Specifically, we consider the following cryptocurrencies: Bitcoin, Ethereum, Tether, Solana, Ripple, Cardano, Dogecoin, Litecoin, Binance Coin, and Stellar. Special attention will be given to Bitcoin and Ethereum, due to their significance as key assets in the financial performance of

cryptocurrencies and their high appeal among investors. Through this research paper, we seek to provide empirical tools for risk management in the cryptocurrency domain while contributing to the growing literature on this subject. Understanding GARCH models and their specific application to this context should provide valuable insights for market participants, researchers, and regulators grappling with the rapidly evolving and ever-changing financial sector. The remainder of the article is structured as follows: we will present an initial step of this research, a literature review on cryptocurrency volatility, is developed. In the second step, we present the research methodology and the results obtained.

II. LITERATURE REVIEW

Several recent studies have focused on examining cryptocurrency volatility and the determinants of their volatility through time series modeling. For instance, academic works available primarily concentrate on GARCH modeling of Bitcoin. Bitcoin, the first and most popular cryptocurrency (see, Dyhrberg (2016); Katsiampa (2017); Lahmiri and al. (2018); Liu and al. (2017); Trucíos (2019)). Christian and al. (2018) sought to identify the determinants of Bitcoin's long-term volatility. Their findings reveal a significantly negative impact of the empirical volatility of the S&P 500 index on Bitcoin's long-term volatility. Additionally, the authors show that the risk premium of the S&P 500 volatility has a significantly positive impact on Bitcoin's long-term volatility.

Along similar lines, Baek and al. (2015) analyzed Bitcoin volatility relative to the S&P 500, as well as factors impacting its returns. The results show that the Bitcoin market is extremely volatile and speculative, suggesting a high level of risk, and that external economic factors have no significant influence on cryptocurrency returns. In fact, this result somewhat contradicts standard economic theory, which considers that at financial market equilibrium, the fundamental (real) value of the asset equals its price (stock price). Volatility seems to be influenced by buyers and sellers in the market, highlighting the significant role of supply and demand dynamics in determining Bitcoin volatility.

Jeffrey Chu, Stephen Chan, Saralees Nadarajah, and Joerg Osterrieder (2017) sought to evaluate the effectiveness of GARCH models in modeling cryptocurrency volatility. The authors tested an entire class of different GARCH models on seven distinct cryptocurrencies. The results suggest that GARCH models with a normal distribution law are well-suited for modeling cryptocurrency price volatility. Asymmetric models, such as IGARCH (Integrated GARCH) and GJRGARCH (Generalized Joint Response GARCH), perform particularly well in this context as they account for asymmetry in price movements, which can be an important characteristic of financial markets.

Katsiampa, P. (2017), on the other hand, analyzed Bitcoin volatility using various GARCH-type models. Her results indicate that the AR (1)-CGARCH (1, 1) model is most suitable for estimating Bitcoin return volatility.

Katsiampa P. (2017) also conducted a comparative analysis of different GARCH models to estimate Bitcoin volatility in a speculative context and gain a better understanding of the persistence of volatility in cryptocurrencies. Volatility persistence refers to the tendency of a period of high or low volatility to persist over time. The authors suggest that it is crucial to use a model that considers both long memory and short memory processes for calculating conditional variance. Long memory implies that past volatility shocks continue to influence current volatility, while short memory focuses on more recent variations. Katsiampa P. (2017) results indicate that there is no significant cause-and-effect relationship between macroeconomic indicators and the value of Bitcoin. This lack of correlation with traditional macroeconomic factors is considered an interesting characteristic (Brière et al. (2015)), which could offer opportunities to investors.

Guesmi, and al. (2016) examined the dynamics and interconnections between the Bitcoin market and other financial markets. The idea was to explore how volatility in Bitcoin interacts with or influences the volatility of other financial indicators, and vice versa. The results suggest that the dynamic conditional correlation (DCC)-GARCH model is most appropriate for modeling the joint dynamics of various financial variables and Bitcoin. Specifically, their research reveals that unexpected changes in the returns of different financial indicators have a significant impact on the conditional volatility of Bitcoin returns. Thus, a shock to financial variables leads to an increase in Bitcoin return volatility. The study's results also indicate that a short position in the Bitcoin market can serve as a hedge to mitigate risk exposure for various financial assets. Indeed, hedging strategies involving gold, oil, stocks from emerging markets, and Bitcoin significantly reduce portfolio risk compared to a portfolio composed solely of gold, oil, and securities from emerging markets. In other words, introducing Bitcoin into a hedging strategy can contribute to more effective portfolio risk management, offering potential benefits to investors.

In a similar vein, Dyhrberg, A.H. (2016) examined the similarities of Bitcoin with gold and the US dollar. Based on GARCH (1,1) and EGARCH models, the author found that Bitcoin exhibited similarities with both gold and the dollar, indicating hedging capabilities and advantages as a medium of exchange.

Many researchers have expressed keen interest in comparing predictive models beyond traditional ARCH (Autoregressive Conditional Heteroskedasticity) and GARCH (Generalized Autoregressive Conditional Heteroskedasticity) categories in estimating financial market volatility. Pilbeam and Langeland (2014) conducted a study focused on foreign exchange markets, comparing the conventional GARCH (1,1) model to two asymmetric GARCH models, namely EGARCH and GJR-GARCH. Their results indicated that the implied volatility model outperformed both symmetric and asymmetric GARCH models, a conclusion supported by other studies (Poon and Granger, 2003, 2005). In this context, they particularly emphasized that during periods of high volatility, the accuracy of all models decreased.

These research findings underscore the importance of evaluating various predictive models in estimating financial market volatility. Moreover, several factors can influence the predictive ability of these models, such as asset type, sample size, data frequency, and time interval, among others. However, no approach has thus far demonstrated supreme superiority in anticipating volatility across all asset classes (Aalborg et al., 2019). Thus, the predictive capacity of a model remains relative and dependent on the specific research context.

III. RESEARCH METHODOLOGY

Our research aims to evaluate the effectiveness of time series modeling using GARCH models in forecasting cryptocurrency volatility. In the following sections, we present the ARIMA and GARCH models. ARIMA, in fact, is essentially a combination of two processes, namely Autoregressive (AR) and Moving Average (MA). It is used when the determinants of the variable to be forecasted are unknown or when data on these occasional variables are not readily available. In autoregressive models (AR), the dependent variable $\boldsymbol{Y_t}$, depends on its own past values. An AR model can be specified as follows:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \cdots + \beta_p Y_{t-p} + e_t$$

Where,
Y_t, represents the response variable at time t;
β_0, is the constant mean of the process;
$Y_{t-1}, Y_{t-2}, \ldots., Y_{t-p}$, are the lagged variables at time $t-1, t-2, \ldots\ t-p$, rrespectively
$\beta_1, \beta_2, \ldots \beta_p$ are the coefficients to be estimated, and e_test is the error term at date t.
Theoretically, if the Partial Autocorrelation Function (PACF) abruptly stops at a certain point, the model is of the AR(p) type. The number of peaks before the abrupt halt is equal to the order of the AR model. In Moving Average (MA) models, the dependent variable Y_t depends on past error values rather than the variable itself. The MA model can be specified as follows:

$$Y_t = \omega_0 - \omega_1 Y\varepsilon_{t-1} - \omega_2 Y\varepsilon_{t-2} - \cdots - \omega_q Y\varepsilon_{t-q} + e_t$$

ω_0 , the constant mean of the process,
$\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots, \varepsilon_{t-p}$, the lagged variables at times $t-1,\ t-2, \ldots\ t-p$ respectively.
$\omega_1, \omega_2, \ldots, \omega_q$, the coefficients to be estimated,
e_t , the error term at time t.

If the autocorrelation function abruptly stops at a certain point, say after q peaks, then the appropriate model is of the MA(q) type. The number of peaks before the halt is designated by q. If neither function abruptly stops but both decline towards zero in some manner, then the appropriate model is of the ARMA(p,q) type. For a univariate series, the model is specified as follows:

$$Y_t = \alpha_0 + \sum_{i=1}^{p} \beta_i Y_{t-i} + \sum_{j=1}^{q} \omega_j Y\varepsilon_{t-j} + e_t$$

Where e_t , is a white noise process, which is a purely random series of numbers with zero mean, normally and independently distributed, and with no correlation between the independent variable and time.
The ARMA model requires stationary series with a constant mean, constant variance, and constant autocorrelation. The ARMA model is extended to the Autoregressive Integrated Moving Average (ARIMA) model when the series is differenced to eliminate non-stationarity. A series is stationary when, fundamentally, two consecutive values of the series depend only on the time interval that separates them and not on time itself. When differencing is used to make a time series stationary, it is common to designate the resulting model as ARIMA(p,d,q).

3.1. GARCH (1,1) Model
The basic GARCH model was developed by Bollerslev (1986). The GARCH model consists of a mean equation and a variance equation. The mean equation is as follows:

$$R_t = \pi + \varepsilon_t$$

Where, R_t epresents the stock returns at time t. π is the mean return of the stocks, and ε_t, is the return residual. The variance equation in the GARCH model is written as follows:

$$\sigma_t^2 = \alpha + \omega_1 \varepsilon_{t-1}^2 + \lambda_1 \sigma_{t-1}^2$$

$$\alpha > 0\ , \omega_1 \geq 0, \lambda_1 \geq 0$$

Where, σ_t^2, is the conditional variance, and the weighted term that depends on the volatility of the previous period $(\omega_1 \varepsilon_{t-1}^2)$ and also the conditional variance of the previous period ($\lambda_1 \sigma_{t-1}^2$). Stationarity is achieved when the coefficients of the volatility of the previous period and the conditional variance are equal to one.

3.2. EGARCH Model
EGARCH , which stands for Exponential GARCH model, is expressed as $log(\sigma_t^2)$. This model captures market asymmetries, where the leverage effect of returns and volatility is examined.

$$\ln(\sigma_t^2) = \alpha + \omega_1 \ln(\sigma_{t-1}^2) + \delta_1 \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \lambda_1 \left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \frac{2}{\pi} \right]$$

With, $\ln(\sigma_t^2)$ representing the log-normal of the conditional variance, the leverage effect between returns and volatility is represented by δ_1, assuming that $\lambda > 0$. A positive and significant coefficient of conditional volatility shows a positive association between risk and return. A negative and significant coefficient of the asymmetrical term indicates an increase in volatility due to bad news in the market, while a positive asymmetrical term shows a decrease in volatility due to good news in the market. The reality is that investors in the market are more inclined towards bad news than good news.
3.3. TGARCH Model
Rabemananjara and Zakoian (1993) proposed the TGARCH model, also known as Threshold GARCH, which examines the asymmetric relationship between risk and return using the differential effect on conditional variance.

$$\sigma_t^2 = \alpha + \omega_1 \varepsilon_{t-1}^2 + \delta_1 S_{t-1} \varepsilon_{t-1}^2 + \lambda_1 \sigma_{t-1}^2$$

Where the leverage effect or the asymmetrical relationship between risk and return is given by the coefficient (δ_1). This model examines the differential effect on conditional variance and is recognized when volatility in the past is greater than zero ($\sigma_{t-1}^2 > 0$), indicating a positive effect and the presence of the leverage effect. Conversely, if volatility in the past is less than zero ($\varepsilon_{t-1}^2 < 0$) it shows a negative effect associated with adverse information.

In our study, we drew inspiration from the work of Wang (2021) to investigate cryptocurrency volatility. To measure volatility, we utilized the closing prices of each cryptocurrency. The return (Rt) of each cryptocurrency was calculated using the classic formula for logarithmic rate of return:

$$\boldsymbol{R_t = ln(\frac{P_t}{P_{t-1}})}$$

Where *Pt*, is the closing price at date t and *Pt*−1, is the closing price at date t-1.
We conducted an Augmented Dickey-Fuller (ADF) test on the time series of cryptocurrency prices. The aim of this test is to determine whether these price series are stationary. A stationary series is a time series whose statistical properties (mean, variance, etc.) remain constant over time. Stationarity is important in the analysis of financial time series, as many statistical models assume that the data are stationary. This analysis is crucial for establishing the relevance of using statistical models in analyzing the volatility of these financial assets. The stationarity of price series is a critical prerequisite as it ensures the validity of the statistical models applied in the study of volatility.

IV. DESCRIPTIVE STATISTICS

The volatility diagram highlights the following points: (***See Appendices 1 and 2***)

- The volatility of Bitcoin's daily returns has been marked by significant instability during different periods:
  - ✓ Peak in 2013: Volatility reached its record high in 2013, influenced by the European sovereign debt crisis. This economic context likely generated uncertainties, thus impacting the stability of Bitcoin's daily returns.
  - ✓ Peak in January 2015: Another significant volatility spike is observed in January 2015.
  - ✓ The instability of Bitcoin's daily returns sparked investor interest as a "safe haven," suggesting that Bitcoin was perceived as a secure asset during these periods of economic uncertainty, leading to significant price fluctuations until 2016.
  - ✓ 2017-2018: The price of Bitcoin experienced high volatility, characterized by an initial increase followed by a decrease in 2018.
  - ✓ 2019: Volatility persisted, influenced by factors such as the Covid-19 pandemic and economic-political tensions between China and the United States. These global events likely had a major impact on investor confidence and contributed to the instability of financial markets in general and cryptocurrencies in particular.
  - ✓ 2020: Bitcoin is considered a safe haven asset amidst economic-political shocks. This perception attracted investors seeking to diversify their portfolios and protect their wealth in a context of economic uncertainty."
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  - ✓ 2020: Bitcoin is considered a safe haven asset amid economic-political shocks. This perception attracted investors seeking to diversify their portfolios and protect their wealth in an economic uncertainty context.
- Binance Coin and Solana display the highest rates of return, reaching 0.39% and 0.23%, respectively. It is noted that these two cryptocurrencies, although recent, are among the top 10 most capitalized cryptocurrencies in the market.
- Dogecoin presents a very high maximum return rate of 151.62%. However, this positive impact is accompanied by an 8% increase in the standard deviation, suggesting a high level of investment risk associated with this cryptocurrency in this context.
- Analysis of the return rates shows that the average daily value is 0.136% and 0.224% for Bitcoin and Ethereum, respectively. The corresponding standard deviations are 4.11% and 6.41%, indicating relative instability and significant variability in the daily returns of these two cryptocurrencies.
- Skewness and kurtosis coefficients, measuring the shape of the distribution of returns, are analyzed. For Bitcoin, the values are -0.52 and 13.83, while for Ethereum, they are -3.1 and 71.18. These values deviate significantly from the reference values of a normal distribution, indicating an abnormal distribution of returns. The presence of a sharp peak and a thick tail suggests significant concentration around the mean but also a high variability in extreme returns.
- The Jarque-Bera normality test confirms that the return rates of Bitcoin and Ethereum significantly differ from a normal distribution, with values of 17432.4 and 527359.7, respectively. This finding

indicates that statistical methods based on a normal distribution are not suitable for analyzing the returns of these two cryptocurrencies.

V. ADF TEST

The data of analyzed cryptocurrency return rates show fluctuations around their mean, without presenting a clear upward or downward trend. In particular, the average return rate of Bitcoin is 0.136%. This indicates that, on average, Bitcoin return rates do not show consistent progression in any particular direction over the study period.

To verify the stationarity of the return rates, we conducted an Augmented Dickey-Fuller (ADF) unit root test based on 4 lags, with a constant but without considering the trend. The objective here is to determine whether the series of return rates is stationary, meaning it exhibits a trend or not.

Table 1. Unit Root Test results (ADF)
(Cryptocurrencies listed by market capitalization, 2023)

H0 : R has a Unit Root			
ADF Test with constant			
Number of lags: 4 (fixe)			
Cryptocurrencies		**T-statistic**	**Prob. ***
BTC	**Bitcoin**	-25.17879	0.0000
ETH	**Ethereum**	-24.10165	0.0000
USDT	**Tether**	-41.83516	0.0000
SOL	**Solana**	-13.68408	0.0000
XRP	**Ripple**	-23.97242	0.0000
ADA	**Cardano**	-17.95251	0.0000
DOGE	**Dogecoin**	-25.45405	0.0000
LTC	**Litecoin**	-25.72454	0.0000
BCH	**Binance coin**	-16.77815	0.0000
XLM	**Stellar**	-23.88048	0.0000
Critical Value		**-3.432449 (1%)**	
		-2.862353 (5%)	
		-2.567247 (10%)	

The results of the unit root test via ADF indicate that the cryptocurrency return series studied are stationary, confirming the stability of the statistical properties of these series over the study period.

VI.THE ARCH TEST EFFECT

The equation for the return rate (R) of cryptocurrencies is formulated as follows:

$$R_t = \alpha_0 + \sum_{i=1}^{n} \alpha_i R_{t-1} + \varepsilon_t$$

Where, *Rt*, is the return rate at a given time,
 $\alpha 0$ is the constant term, αi are the coefficients associated with past lags (*Rt*−1),
 ϵt,. is the error term.

Statistical tools such as Akaike Information Criteria (AIC) and the F-statistic are used here to select the most appropriate lag order. In the context of the ARIMA model, the optimal lag order is the one that minimizes the Akaike Information Criterion (AIC) and maximizes the F-statistic. The AIC is a model selection criterion that balances model fit and complexity, while the F-statistic evaluates the overall significance of the model.

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VII.ARCH-LM HETEROSKEDASTICITY TEST

Here, the ARCH-LM test aims to determine if the residuals of a regression exhibit conditional autocorrelation, which means that the variance of the residuals depends on their past values. This situation can indicate conditional heteroskedasticity in the data.

Conditional autocorrelation occurs when the correlation between residuals at different time points is significant, indicating that the volatility of the residuals varies based on past observations.

The ARCH-LM test is performed while respecting the determined number of lags for each cryptocurrency (in this case, the lag order is 4 for Bitcoin, for example).

The regression equation is then formulated by including the appropriate lags, as illustrated for Bitcoin:

$$R_t = \alpha_0 + \alpha_1 R_{t-1} + \alpha_2 R_{t-2} + \alpha_3 R_{t-3} + \alpha_4 R_{t-4} + \varepsilon_t$$

- If the ARCH-LM test is statistically significant, it indicates that the residuals of the regression exhibit conditional autocorrelation. In this case, another approach to modeling the data may be necessary.
- If the test is not significant, it suggests that the linear serial regression is adequate, and one can continue to use this approach to model the data.

Table 2. ARCH-LM Test of Cryptocurrency Returns Residuals
(Cryptocurrencies listed by market capitalization, 2023)

ARCH HETEROSKEDASTICITY TEST

BTC	F-statistic	40.48289	Prob. F (4,3521)	0.0000
	Obs.*R-squared	155.0316	Prob. Chi-Square (4)	0.0000
ETH	F-statistic	83.85426	Prob. F (2,2696)	0.0000
	Obs.*R-squared	158.0626	Prob. Chi-Square (2)	0.0000
USDT	F-statistic	1102.787	Prob. F (2,2854)	0.0000
	Obs.*R-squared	1245.426	Prob. Chi-Square (2)	0.0000
SOL	F-statistic	41.07988	Prob. F (1,991)	0.0000
	Obs.*R-squared	39.52438	Prob. Chi-Square (1)	0.0000
XRP	F-statistic	77.62959	Prob. F (4,3423)	0.0000
	Obs.*R-squared	285.1083	Prob. Chi-Square (4)	0.0000
ADA	F-statistic	62.48417	Prob. F (2,1908)	0.0000
	Obs.*R-squared	117.4708	Prob. Chi-Square (2)	0.0000
DOGE	F-statistic	27.81477	Prob. F (2,3294)	0.0000
	Obs.*R-squared	54.75548	Prob. Chi-Square (2)	0.0000
LTC	F-statistic	33.94854	Prob. F (4,3521)	0.0000
	Obs.*R-squared	130.9371	Prob. Chi-Square (4)	0.0000
BCH	F-statistic	58.85493	Prob. F (4,1972)	0.0000
	Obs.*R-squared	210.8456	Prob. Chi-Square (4)	0.0000
XLM	F-statistic	426.0480	Prob. F (1,3066)	0.0000
	Obs.*R-squared	374.3120	Prob. Chi-Square (1)	0.0000

We can highlight the following points:
Significant Heteroskedasticity: The results of the ARCH-LM test indicate that the sequence of cryptocurrency return rates exhibits significant heteroskedasticity. This means that the volatility of these return rates is not constant over time and may be influenced by past variations in volatility.
Strong ARCH Effect:
The observation of a strong ARCH effect suggests that the sequence of regression residuals exhibits conditional autocorrelation, showing that the volatility of the return rates is strongly influenced by past volatility variations.
The associated F-statistic for Bitcoin is 40.482, indicating significant heteroskedasticity for this specific model. The Obs.*R-squared (155.031) measures the heteroskedasticity of the error variance.
A high value of this measure, as observed, indicates a strong presence of heteroskedasticity in the model. Statistically **Significant Model Overall:**
The probability associated with the F-statistic (4.352) indicates that the model is statistically significant overall, reinforcing the validity of the results observed in the context of the ARCH-LM test.
Errors Do Not Follow a Normal Distribution:
The probability associated with the error normality test (Chi-Square(4)) is often performed in conjunction with the ARCH test. This probability indicates that the errors do not follow a normal distribution.

VIII.CRYPTOCURRENCY VOLATILITY STUDY

This section focuses on the study of cryptocurrency return volatility, particularly for Bitcoin and Ethereum, using the GARCH(1,1) model. Bitcoin and Ethereum were chosen due to their importance as key indicators of cryptocurrency financial performance and their popularity among investors.
The estimation results (*See Annex 3*) highlight the following points:
Volatility Clustering Characteristics: The presence of significant ARCH and GARCH terms indicates that return volatility exhibits clustering characteristics. This suggests that shocks may have a lasting impact on conditional variance.

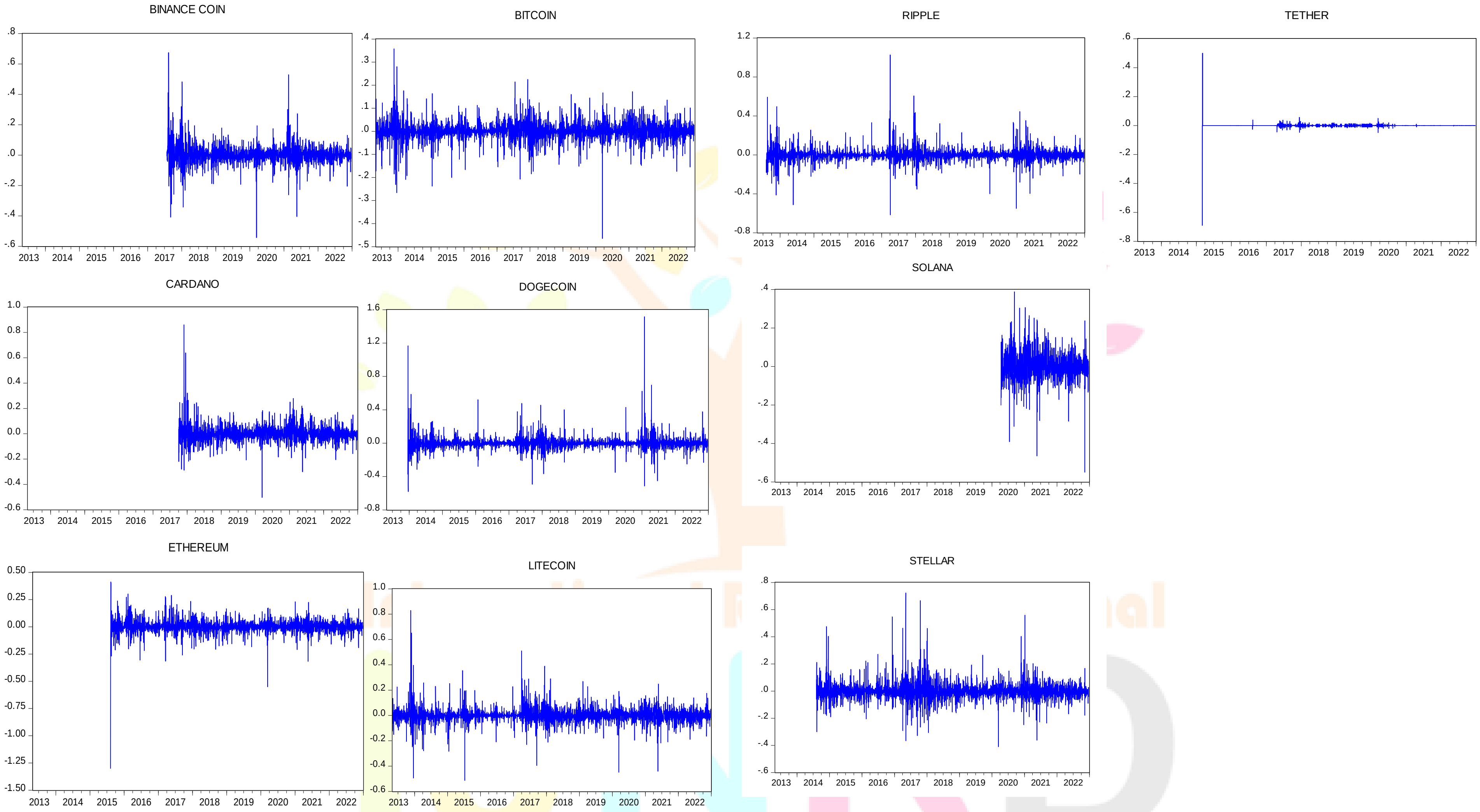
Model Convergence: Model convergence is achieved after a limited number of iterations (24 for Bitcoin and 22 for Ethereum). This suggests that the model is effective in predicting conditional variance of returns.
Sum of ARCH and GARCH Terms: The sum of ARCH and GARCH terms is close to unity for both cryptocurrencies studied. This observation indicates that the impact of shocks on conditional variance is not transient but permanent.
Mean Reverting Process for Ethereum: For Ethereum, the sum of ARCH and GARCH terms is less than 1 (0.852). However, conditional variance shows that the influence of past fluctuations is limited and gradually attenuates to zero. This suggests a mean reverting process, where volatility gradually returns to more stable levels.

IX.ASYMMETRY TEST OF CRYPTOCURRENCY RETURN

Two statistical models, TARCH and EARCH, are used to determine whether negative shocks have a greater impact on future return volatility than positive shocks. The estimation results (see Annex 4,5) highlight the following points:
Concept of Leverage Effect: The leverage effect suggests that a change in future return volatility may be more influenced by negative events than by positive events, such as inflation, economic crises, or the COVID-19 pandemic, or more generally, other politico-economic events.
TARCH Model Results: (***See Appendix 4***) The results show a significant leverage effect for Bitcoin, Dogecoin, and Tether. This indicates that negative shocks have a greater impact on the future volatility of returns for these three assets. The estimated coefficient of the variable ARCH(1)*(RESID(−1)< 0) is positive and significant, indicating that special price fluctuations have leverage effect on the price of Bitcoin.
Confirmation by EARCH Model: (***See Appendix 5***) The results of the EARCH model confirm those of the TARCH model for Bitcoin and Dogecoin, showing that these cryptocurrencies have a leverage effect on their return rates. The C1 rate measures the percentage impact that shocks will have on the future volatility of the return rate. For Bitcoin, it shows an impact of 26.5%, and for Dogecoin, an impact of 65.07%.
Absence of Leverage Effect for Other Cryptocurrencies: The results of the TARCH and EARCH models do not show significant evidence of a leverage effect for the other tested cryptocurrencies.

X. CONCLUSION

Cryptocurrencies are relatively new financial assets characterized by high volatility, making the ability to predict their future volatility crucial for investors. Our empirical results obtained through time series econometrics from April 2013 to June 2022 show that cryptocurrency return volatility is not constant (heteroskedastic). By using GARCH modeling, we found that volatility shocks in returns are permanent and significant on conditional variance. The sum of ARCH and GARCH terms is close to unity, indicating that shocks on the conditional variable have a permanent impact. However, the results of the TARCH and EGARCH models highlighted a leverage effect for three cryptocurrencies (Bitcoin, Dogecoin, and Tether): negative economic shocks have a greater impact on future return volatility (amplification effect). For other crypto-assets, the models did not indicate significant impacts. Finally, our results underscore the importance of the GARCH model in forecasting cryptocurrency volatility, providing investors and portfolio managers with relevant information for effective portfolio and risk management.



APPENDIX 1. Volatility Diagrams of Daily Rate of Return

APPENDIX 2: Descriptive Statistics of Daily Returns of Cryptocurrencies for the Period 04/2013 - 12/2022

	BTC	ETH	USDT	SOL	XRP	ADA	DOGE	LTC	BCH	XLM
MEAN	0.001362	0.002245	-6.68E-05	0.002361	0.001180	0.001193	0.001654	0.000786	0.003905	0.001098
MEDIAN	0.001558	0.000432	0.000000	-0.000228	-0.001691	0.000000	-0.001813	0.000000	0.000991	-0.001578
MAXIMUM	0.357451	0.412405	0.500545	0.387766	1.027463	0.861235	1.516211	0.828968	0.675064	0.723152
MINIMUM	-0.464730	-1.302139	-0.690671	-0.549522	-0.616380	-0.503705	-0.581047	-0.513925	-0.542809	-0.410040
STD. DEV.	0.041529	0.064122	0.016719	0.079155	0.069613	0.068190	0.080004	0.061386	0.067151	0.069667
SKEWNESS	-0.525826	-3.098461	-15.24924	-0.367103	1.576980	1.826007	3.701441	1.122544	0.931769	1.771042
KURTOSIS	13.82961	71.14718	1299.755	8.444352	30.32891	25.81173	65.05747	26.35277	19.02941	20.35815
JARQUE-BERA	17432.40	527359.7	2.01E+08	1251.214	108351.0	42630.27	537553.9	81045.27	21538.46	40146.85
PROBABILITY	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SUM	4.814584	6.068535	-0.191020	2.348814	4.054344	2.287656	5.462430	2.778262	7.752009	3.372179
SUM SQ. DEV.	6.093272	11.10949	0.799413	6.227866	16.64570	8.909083	21.13484	13.31300	8.946247	14.89548
OBSERVATIONS	3534	2703	2861	995	3436	1917	3303	3534	1985	3070

APPENDIX 3: Estimation Results of the GARCH(1,1) Model

Dependent Variable: R

Method: ML-ARCH (Marquardt) Normal Distribution

Coeff	BTC	ETH	USDT	SOL	XRP	ADA	DOGE	LTC	BCH	XLM
Variance Equation										
C0	7.67E-05	0.000257	1.27E-07	0.000925	0.000381	0.000172	0.000281	0.000127	0.000103	0.000283
	(0.0202)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
C1	0.1343982	0.162549	0.335423	0.217166	0.418946	0.127082	0.545282	0.083824	0.165271	0.206942
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
C2	0.831939	0.768116	0.811691	0.640606	0.592485	0.838880	0.606348	0.884401	0.823815	0.754399
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Log Likelihood	6616.889	4098.953	13335.89	1194.108	5144.706	2709.355	4726.361	5325.340	3021.732	4350.574
AIC	-3.746113	-3.031435	-9.325561	-2.392572	-2.995166	-2.824392	-2.859958	-3.014357	-3.045666	-2.831915
SC	-3.737375	-3.020511	-9.315141	-2.367916	-2.986220	-2.809882	-2.848867	-3.005619	-3.031553	-2.822092

*Note: This table presents the estimations of the GARCH(1,1) model, described by the following equation: GARCH = C_0 + C_1*RESID (-1)² + C_2*GARCH (-1). The probabilities are indicated in parentheses.*

APPENDIX 4: Results of the Estimation of the TGARCH Model

Dependent Variable: R

Method: ML-ARCH (Marquardt) Normal Distribution

Coeff	BTC	ETH	USDT	SOL	XRP	ADA	DOGE	LTC	BCH	XLM
Variance Equation										
C0	7.96E-05	0.000264	1.15E-07	0.000924	0.000419	0.000175	0.000268	0.000120	0.000104	0.000283
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
C1	0.115013	0.155259	0.183389	0.218347	0.526153	0.131839	0.409859	0.099335	0.159321	0.226320
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
C2	0.044394	0.022089	0.360993	-0.002999	-0.172120	-0.015606	0.227898	-0.041591	0.019051	-0.076399
	(0.0000)	(0.0777)	(0.0000)	(0.9283)	(0.0000)	(0.1275)	(0.0000)	(0.0000)	(0.1115)	(0.0000)
Log Likelihood	0.827340	0.763033	0.811029	0.640919	0.566171	0.840059	0.621546	0.890366	0.821459	0.765195
AIC	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
SC	6620.670	4099.397	13356.74	1194.110	5151.550	2709.617	4738.317	5332.796	3022.046	4354.886

Note: This table presents the estimations of the TGARCH model, described by the following equation: $GARCH = C_0 + C_1{*}RESID(-1)^2 + C_2{*}RESID(-1)^2{*}(RESID(-1) < 0) + C_3{*}GARCH(-1)$. *Probabilities are indicated in parentheses.*

APPENDIX 5: Results of the Estimation of the EGARCH (1,1) Model

Dependent Variable: R

Method: ML-ARCH (Marquardt) Normal Distribution

Coeff	BTC	ETH	USDT	SOL	XRP	ADA	DOGE	LTC	BCH	XLM
Variance Equation										
C0	7.96E-05	0.000264	1.15E-07	0.000924	0.000419	0.000175	0.000268	0.000120	0.000104	0.000283
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
C1	0.115013	0.155259	0.183389	0.218347	0.526153	0.131839	0.409859	0.099335	0.159321	0.226320
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
C2	0.044394	0.022089	0.360993	-0.002999	-0.172120	-0.015606	0.227898	-0.041591	0.019051	-0.076399
	(0.0000)	(0.0777)	(0.0000)	(0.9283)	(0.0000)	(0.1275)	(0.0000)	(0.0000)	(0.1115)	(0.0000)
Log Likelihood	0.827340	0.763033	0.811029	0.640919	0.566171	0.840059	0.621546	0.890366	0.821459	0.765195
AIC	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
SC	6620.670	4099.397	13356.74	1194.110	5151.550	2709.617	4738.317	5332.796	3022.046	4354.886

*Note: This table presents the estimations of the EGARCH model, described by the following equation:LOG(GARCH) = C_0 + C_1*ABS (RESID (-1)/@SQRT (GARCH (-1))) + C_2*RESID (-1)/@SQRT (GARCH (-1)) + C_3*LOG (GARCH (-1)). Probabilities are indicated in parentheses.*

REFERENCES

Aalborg, H. A., Molnár, P., & de Vries, J. E. (2019). What can explain the price, volatility and trading volume of Bitcoin? Finance Research Letters, 29, 255–265.

Akhtaruzzaman M. et al. (2022). Systemic risk-sharing framework of cryptocurrencies in the COVID–19 crisis, Finance Research Letters Volume 58, Part B, December 2023, 104334

Alam, S., (2017). Testing the weak form of efficient market in cryptocurrency. Journal of Engineering & Applied Sciences, 12(9), P.2285-2288.

BaekC. et al. (2015). Bitcoins as an investment or speculative vehicle?, Applied Economics Letters 22(1)

Bartos, J., (2015). Does Bitcoin follow the hypothesis of efficient market. International, Journal of Economic Sciences, 4(2), P.10-23.

Baur, D. G., & Dimpfl, T. (2018). Asymmetric volatility in cryptocurrencies. Economics Letters, 173, 148–151.

Bouri, E., Saeed, T., Vo, X. V., & Roubaud, D. (2021). Quantile connectedness in the cryptocurrency market. Journal of International Financial Markets, Institutions and Money, 71, 101302.

Bouri, E., Shahzad, S. J. H., Roubaud, D., Kristoufek, L., & Lucey, B. (2020). Bitcoin, gold, and commodities as safe havens for stocks: New insight through wavelet analysis. The Quarterly Review of Economics and Finance, 77, 156–164.

Briere et al. (2015). Virtual Currency, Tangible Return: Portfolio Diversification with Bitcoin Journal of Asset Management (2015) 16 (6), pp. 365–373

Chen, W., Guo, X., Chen, Z., Zheng, Z., & Lu, Y. (2020). Phishing Scam Detection on Ethereum: Towards Financial Security for Blockchain Ecosystem. International Joint Conferences on Artificial Intelligence Organization.

CoinMarketCap.“Cryptocurrencies”.https://coinmarketcap.com/

Conlon, T., & McGee, R. (2020). Safe haven or risky hazard? Bitcoin during the Covid-19 bear market. Finance Research Letters, 35, 101607.

Corbet, S., Hou, G., Hu, Y., Larkin, C. J., Lucey, B. M., & Oxley, L. (2022). Cryptocurrency liquidity and volatility interrelationships during the COVID-19 pandemic. SSRN Electronic Journal.

Corbet, S., Larkin, C. J., Lucey, B. M., Meegan, A., & Yarovaya, L. (2018). The volatility generating effects of macroeconomic news on cryptocurrency returns. SSRN Electronic Journal.

Corbet, S., Larkin, C., & Lucey, B. (2020). The contagion effects of the COVID-19 pandemic: Evidence from gold and cryptocurrencies. Finance Research Letters, 35, 101554.

Dyhrberg, A.H., (2016). Bitcoin, gold and the dollar–A GARCH volatility analysis. Finance Research Letters, 16(1), P.85-92.

Foley, S., Gandal, N., Jentzsch, N., & Moore, T. (2019). Cryptocurrencies as a hedge in financial markets. Journal of Financial Stability, 41, 56-67

Geuder, J., Kinateder, H., & Wagner, N. F. (2019). Cryptocurrencies as financial bubbles: The case of Bitcoin. Finance Research Letters, 31.

Gomes, O., & Gubareva, M. (2020). Complex Systems in Economics and Where to Find Them. Journal of Systems Science and Complexity.

Guesmi K, Saadi S, Abid I, et al. (2019). Portfolio diversification with virtual currency: evidence from bitcoin. Int Rev Financial Anal. 2019; 63:431–437.

Hayes, A.S., (2017). Cryptocurrency value formation: An empirical study leading to a cost of production model for valuing bitcoin. Telematics & Informatics, 34(7), P.1308-1321.

Haroon, O., & Rizvi, S. A. R. (2020). COVID-19: Media coverage and financial markets behavior—A sectoral inquiry. Journal of Behavioral and Experimental Finance, 27, 100343.

Jeffrey Chu, Stephen Chan, Saralees Nadarajah, Joerg Osterrieder (2017), “GARCH modelling of cryptocurrencies.”Journal of Risk and Financial Management, 10 (17), pp. 1-15

Ji, Q., Zhang, D., & Zhao, Y. (2020). Searching for safe-haven assets during the COVID-19 pandemic. International Review of Financial Analysis, 71,

Katsiampa, P., (2017). Volatility estimation for Bitcoin: A comparison of GARCH models. Economics Letters, 158(1), P.3-6.

Katsiampa, P. (2017). Volatility estimation for Bitcoin: A comparison of GARCH models. Economics Letters, 158, 3–6.

Katsiampa, P., Corbet, S., & Lucey, B. (2019). High frequency volatility co-movements in cryptocurrency markets. Journal of International Financial Markets, Institutions and Money, 62, 35– 52.

Kristoufek, L. (2013). BitCoin meets Google Trends and Wikipedia: Quantifying the relationship between phenomena of the Internet era. Scientific Reports, 3, 7.

Lahmiri S, Bekiros S. Chaos, randomness and multi-fractality in Bitcoin market. Chaos, SolitonsFract. 2018;106:28–34.

Liu, J. & Serletis, A. (2019). Volatility in the cryptocurrency market. Open Economies Review, 30(4), P.779-811.

Liu, Y., & Tsyvinski, A. (2021). Risks and returns of cryptocurrency. The Review of Financial Studies, 34(6), 2689–2727.

Mnif, E., Jarboui, A., & Mouakhar, K. (2020). How the cryptocurrency market has performed during COVID 19? A multifractal analysis. Finance Research Letters, 36, 101647.

Poon, Ser-Huang. & Granger, Clive.W. J. (2003). Forecasting Volatility in Financial Markets: A Review. Journal of Economic Literature, 41(2), pp. 478-539.

Poon, Ser-Huang; Granger, Clive. (2005) Practical issues in forecasting volatility. Financial Analysts Journal; 61, pp. 45-56.

Romanchenko, O., Shemetkova, O., Piatanova, V. & Kornienko, D., (2018). Approach of estimation of the fair value of assets on a cryptocurrency market. In The 2018 International Conference on Digital Science, P. 245-253.

Salisu, A. A., & Ogbonna, A. E. (2021). The return volatility of cryptocurrencies during the COVID19 pandemic: Assessing the news effect. Global Finance Journal, 100641.

Stavroyiannis, S. (2018). A note on the Nelson-Cao inequality constraints in the GJR-GARCH model: Is there a leverage effect? International Journal of Economics and Business Research, 16(4), 442.

Trucíos C. (2019). Forecasting Bitcoin risk measures: A robust approach, International Journal of ForecastingVolume 35, Issue 3, Pages 836-847

Wang, C. (2021). Different GARCH models analysis of returns and volatility in Bitcoin. Data Science in Finance and Economics, 1(1), 37–59.

Zhang, D., Hu, M., & Ji, Q. (2020). Financial markets under the global pandemic of COVID-19. Finance Research Letters, 36, 101528.