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EFFECT OF ASYMMETRIC SLIP AND NEWTONIAN COOLING ON CONVECTIVE HEAT TRANSFER FLOW OF A NANOFLUID IN VERTICAL CHANNEL WITH VARIABLE VISCOSITY

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Abstract:
An attempt has been made to explore the influence of variable viscosity, thermal radiation, dissipation, non-uniform heat sources on hydromagnetic convective heat transfer flow of Eg based Cu nanofluid through a porous medium confined in a vertical channel with asymmetric slips and convective boundary conditions. After evaluating the governing equations numerically we find that increase in asymmetric slips ($\alpha o,\alpha 1$)leads to an enhancement in velocity while the temperature enhances with αo and decays with $\alpha 1$. in the flow region. Increase in Biot numbers (Bio,Bi1) decays velocity while the temperature enhances with Bio, reduces with Bi1 in the flow region. Increase in viscosity parameter (B) leads to a decay in the thickness of the momentum and thermal boundary layers in Eg based Cu-nanofluid.

Keywords: Variable viscosity, Asymmetric slip, Newtonian cooling, non-uniform heat source, Vertical channel, Nanofluids

1. INTRODUCTION

Much attention has been given in the past decade to the study of nanofluids due to its applications in science, technology and industries. The possible applications are biomedical, antibacterial activity, nanodrug delivery, cancer therapeutic, electromechanical systems, industrial cooling, energy storage, solar absorption. Ultra high-performance cooling is one of the most vital needs of many industrial technologies. However, low thermal conductivity is a limitation in developing energy-efficient heat transfer fluids that is required for ultra high-performance cooling. The cooling applications of nanofluids include silicon mirror cooling, electronics cooling, vehicle cooling, and transformer cooling and so on. Nanofluid technology can help to develop better oils and lubricants. Nanofluids are now being developed for medical applications, including cancer therapy and safe surgery, by cooling. To all the numerous applications must be added that, nanofluids can be used in major process industries, including materials and chemicals, food and drink, oil and gas, paper and printing etc. Choi and Eastman [3] were probably the first to employ a mixture of nanoparticles and base fluid that such fluids were designated as *nano-fluids*. Choi [4] was the first who experimentally verified that the addition of nanoparticles in conventional base fluids appreciably enhanced the thermal conductivity. Experimental results (Choi et al [5]), (Das et al [6]) have illustrated that the thermal conductivity of the nanofluid can be increased considerably via the introduction of a small volume fraction of nanoparticles.

The study of the flow of a viscous fluid with temperature, dependent properties is of great importance in industries such as food processing, coating, and polymer processing, see Macosko and Oron et al. [21]. In industrial systems fluids can be subjected to extreme conditions such as high temperature, pressure, and shear rates. External heating such as the ambient temperature and high shear rates can lead to a high temperature being generated within the fluid.This may have a significant effect on the fluid properties. It is a well-known fact in fluid dynamics studies that the property which ismost sensitive to temperature rise is viscosity. Further more Myers et al. [20] have been studied the fluids used in industries such as polymer fluids have a viscosity that varies rapidly with temperature and this may give rise to strong feedback effects, which can lead to significant changes in theflowstructure of the fluid. Elbashbeshy and Bazid [7] investigated the effect of temperature dependent viscosity on heat transfer over amoving surface. Tshehla [28] has described the flow of a variable viscosity fluid down an inclined plane with a free surface.,

It is significant to investigate the effects of heat generation or absorption when fluids undertaking endothermic or exothermic chemical reactions. The presence of heat generation or absorption can be used in semiconductor wafers and electronic chips. Alam and Ahammad [1] have applied Nachtsheim-Swigert shooting iteration technique with sixth-order Runge-Kutta integration scheme to study the effects of variable chemical reaction and variable electric conductivity on free convective heat and mass transfer flow along an inclined stretching sheet with variable heat and mass fluxes under the influence of Dufour and Soret effects. The y have considered that electrical conductivity is a function of velocity. MHD free convective flow over a vertical cone with variable electric conductivity in the presence of chemical reaction has been delineated by Kumar et al . [11] by applying an efficient finite difference technique of the Crank-Nicolson type.

Natural convection inside channels has been a subject of extended research during the last decades due to its applications in engineering such as electronic cooling systems, nuclear reactors and heat exchangers. The vertical channel is an often encountered configuration in thermal engineering equipment, as an example, collectors of solar power, cooling devices of digital and micro-digital equipments and many others. Gill and Casal [10] have made an analysis on the influence of electrically conducting the case of fully developed mixed convection between horizontal parallel plates with a linear axial temperature distribution. The combined forced and free convective flow in a vertical channel flow of a nanofluid in a vertical channel with viscous dissipation, dual mixed convection and isothermal –isoflux boundary conditions have been studied by Barletta [2]. Maïga et al. [15, 16], Rossi di Schio [25], Sheikholeslami and Ganji [26], Xu et al. [29].

Merkin [18] was the first to consider a somewhat different but practically relevant driving mechanism for the natural convection boundary layer flow near a vertical surface in which it was assumed that the flow was setup by the Newtonian heating from the bounding surface, Several authors (Madhusudhan Rao et al. [14], Lavanya et al. [12], Rajesh and Chamkha [24], Sulochana and Rama Krishna [27]) have investigated Newtonian cooling on convective heat/mass transfer in different configurations under varied conditions

The fluid flow behavior subject to the slip flow regime greatly differs from the traditional flow. The slip flows under different flow configurations have been studied by many researchers. Falade John [8] has discussed the entropy generation analysis for porous channel flow with asymmetric slip and thermal boundary conditions Fluid flow in an asymmetric channel has been investigated by Muthu and Berhane Tesfahun [19]. Recently Malleswari and Sreenivasa Reddy [17] have investigated the influence assymmeric slip on convective heat transfer flow in vertical channel.

In this paper, at attempt has been made to investigate the effect of viscosity variation on hudromagnetic non-Darcy convective heat transfer flow of Ethylene based Cu nanofluid in a vertical channel with asymmetric slip and convective boundary conditio ns in the presence of irregular heat sources. The non-linear, coupled equations governing the flow, heat transfer have been executed by using Galerkine Finite element method with quadratic interpolation functions. The velocity, temperature, Nusselt and Sherwood number have exhibited through graphs and tables.

2. FORMULATION OF THE PROBLEM:

Consider the steady flow of a viscous electrically conducting fluid through a porous channel opf distance 2L apart. The fluid consisting of a Ethylene Glycol base fluid and small nanoparticles of Copper in a vertical porous channel with thermal radiation. A uniform magnetic field of strength Ho is applied normal to the plate. It is assumed that there is no applied voltage which imp lies the absence of an electric field. The flow is assumed to be in the x-direction which is taken along the plane in an upward direction. The fluid is assumed to be gray, absorbing emitting but not scattering medium. The radiation heat flux in the x-direction is considered negligible in comparison with that in the z-direction. Due to the fully developed assumption, the flow variables are functions of y only. Figure. 1 shows that the problem under consideration and the co-ordinate system.

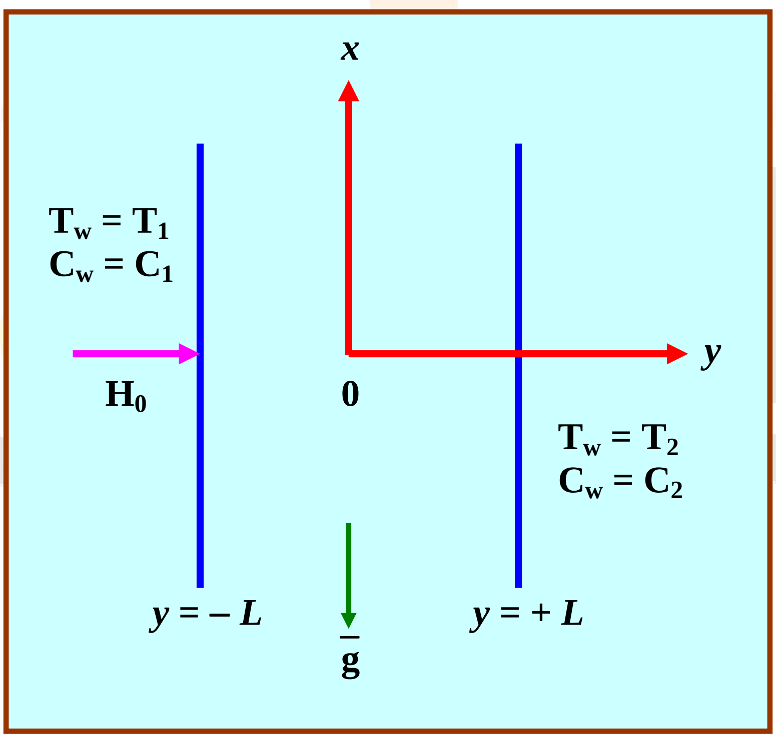

Fig.1. Schematic diagram of the problem under consideration

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Under the above mentioned assumptions, the equation of momentum and thermal energy respectively under Rosseland approximation can be written in dimensional form as :

$$\frac{\partial v}{\partial y} = 0 \qquad (2.1)$$

$$\rho_{nf}(-\mathrm{v}_o \frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}(\mu_{nf}(T)\frac{\partial u}{\partial z}) + (\rho\beta)_{nf} g(T - T_0) -$$

$$-(\sigma_{nf}\mu_e^2 H_o^2)u) - (\frac{\mu_{nf}}{k_p})u - (\frac{C_b}{\sqrt{k_p}})\mathrm{u}^2 \qquad (2.2)$$

$$-v_o \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{nf}}\frac{\partial (q_R)}{\partial y} + \frac{1}{(\rho C_p)_{nf}}(A_{11}^{'}(T_f - T_o)u + B_{11}^{'}(T - T_o)) \qquad (2.3)$$

$$+2\mu_{nf}[(\frac{\partial u}{\partial y})^2 + (\frac{\partial w}{\partial y})^2] + \sigma_{nf}\mu_e^2 H_o^2(u^2) + \mathrm{Q}_1^{'}(C - C_o)$$

The boundary conditions are(Ostrach[1954], Ozotop and Abu-nada[2004]):

$$u(-L) = \alpha_o^{'}\frac{\partial u}{\partial y} \quad , k_{nf}\frac{\partial T}{\partial y} = \gamma_o(T - T_o) \; on \;\; y = -L$$

$$u(+L) = -\alpha_1^{'}\frac{\partial u}{\partial y} \; , -k_{nf}\frac{\partial T}{\partial y} = \gamma_1(T - T_f) \;\; on \;\; y = +L \qquad (2.4)$$

where ($\alpha_o^{'}, \alpha_1^{'}$) are the Navier slip coefficients at the walls, μ_f is the viscosity of the base fluid, u is the axial velocity, P is the fluid pressure, ρ_f is the density of the nanoparticle, v_o is the channel porosity due to suction and injection, Cp is the specific heat at constant pressure,(T,kf) are the nanofluid temperature and thermal conductivity of the material respectively,(T_o,T_1) are referenced fluid temperatures and $\gamma_{o,1}$ measures the Newtonian cooling rate at the walls. μ_{nf} is the effective dynamic viscosity, ρ_{nf} is the effective density of the nanofluid,, σ_{nf} is the effective electrical conductivity of the nanofluid, k_f , k_{nf} are the thermal conductivity of the base fluid and nanoparticles respectively. $(\rho C_p)_{nf}$ is the effective heat capacitance of the nanofluid, $(\rho\beta)_{nf}$ is the effective thermal expansion of the nanofluid. which are given by[Choi and Eastman[1995], Gebhar[1962].

$$\mu_{nf} = \mu_f / (1-\varphi)^{2.5} \qquad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \qquad \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s$$

$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_P)_f + \varphi(\rho C_P)_s \qquad (\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s$$

$$k_{nf} = \frac{k_f(k_s + 2k_f - 2\varphi(k_f - k_s))}{(k_s + 2k_f + 2\varphi(k_f - k_s))}, \sigma_{nf} = (\sigma_f + \frac{3(\sigma_f - \sigma_s)\phi}{(\sigma_s + 2\sigma_f)}),$$

where the subscripts nf, f and s represent the thermo physical properties of the nanofluid, base fluid and the nanosolid particles respectively and ϕ is the solid volume fraction of the nanoparticles. The thermo physical properties of the nanofluid are given in Table 1.
The dynamic viscosity of the nanofluids is assumed to be temperature dependent as follows:

$$\mu_f(T) - \mu_o Exp(-m(T - T_o))$$

where μ_o is the nanofluid viscosity at the ambient temperature T_o m is the viscosity variation parameter which depends on the particular fluid.

The thermo physical properties of the nanofluids are given in Table 1 (See ***Oztop and Abu-Nada*** (2004)).

Table – 1 : Physical Properties of nanofluids

Physical properties	Fluid phase (Ethylene Glycol)	**Cu nanofluid**
C_p(j/kg K)	2430	**385**
ρ(kg m^3)	1115	**8933**
k(W/m K)	0.253	**401**
β$x10^{-5}$ 1/k)	5.7	1.67
σ	**10.7**	**1**

The dynamic viscosity of the nanofluids is assumed to be temperature dependent as follows:

$$\mu_f(T) - \mu_o Exp(-m(T - T_o)) \qquad (2.5)$$

where μ_o is the nanofluid viscosity at the ambient temperature T_o m is the viscosity variation parameter which depends on the particular fluid.

By using Rosseland approximation for radiative heat flux, qr is simplified as(Gebhar [1962] Gill and Del Casal[1962])

$$q_r = -\frac{4\sigma^{\bullet}}{3\beta_R}\frac{\partial T'^4}{\partial y} \tag{2.6}$$

where $\sigma^{\bullet}$=5.6607x10^{-8} Wm^{-2}K^{-4} is the Stefan –Boltzman constant and β_R is the Rosseland mean absorption coefficient. In the case of nanofluid ,herein (optically thick)the thermal radiation travels only a short distance before being scattered or absorbed .If the temperature differences within the fluid flow are sufficiently small, T'^4 may be expressed as a linear combination of temperature .This is done by expanding T'^4 in a Taylor series about top wall temperature Ti as follows:

$$T'^4 \cong T_o^4 + 3T_o^3(T-T_o) + 6T_o^2(T-T_o)^2 + ----- \tag{2.7}$$

Neglecting higher order terms in the above equation beyond the first order in $(T-T_o)$,we get(Gebhar [1962], Gill and Del Casal[1962])

$$T'^4 \cong 4T_o^3 T - 3T_o^4 \tag{2.8}$$

In view of the equations(2.5) and (2.7),equation(2.2) becomes

$$0 = \frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho C_p)_{nf}}\frac{16\sigma^* T_o^3}{3\beta_R}\frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho C_p)_{nf}}(A_{11}'(T_f - T_o)u + B_{11}'(T-T_o))$$
$$+2\mu_{nf}[(\frac{\partial u}{\partial y})^2 + (\frac{\partial w}{\partial y})^2] + \sigma_{nf}\mu_e^2 H_o^2(u^2) + Q_1'(C-C_o) \tag{2.9}$$

We consider the solution of equation(2.1) as:

$$v=-v_0 \tag{2.10}$$

Introducing the following dimensionless variables:

$$y' = \frac{y}{L},\ u' = \frac{u}{U},\ p' = \frac{p}{\rho_f U^2},\ \theta = \frac{T-T_0}{T_f - T_0} \tag{2.11}$$

we obtain the following ordinary differential equations with appropriate boundary conditions

$$1 + A_2 S\frac{\partial u}{\partial y} + (\frac{\partial^2 u}{\partial y^2} - B\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y}) + e^{B\theta}[A_1 A_3 G(\theta) - A_6 M^2(u) - \Delta(u^2)] = 0 \tag{2.12}$$

$$(A_5 + \frac{4Rd}{3})\frac{\partial^2\theta}{\partial y^2} + (A_4 S\Pr)\frac{\partial \theta}{\partial y} + A_{11}u + B_{11}\theta + Ec\Pr[e^{-B\theta}(\frac{\partial u}{\partial y})^2 + A_6 M^2(u^2)] = 0 \tag{2.13}$$

The transformed boundary conditions (2.3) reduce to

$$u(-1) = \alpha_0\frac{\partial u}{\partial y}(-1), u(+1) = -\alpha_1\frac{\partial u}{\partial y}(+1),$$

$$\frac{\partial\theta}{\partial y}(-1) = (\frac{Bi_o}{A_5})\theta(-1), \frac{\partial\theta}{\partial y}(+1) = -(\frac{Bi_1}{A_5})\theta(+1)$$

where u is the dimensionless fluid velocity θ is the dimensionless fluid temperature,$\alpha_{0,1}$ are the dimensionless slip parameters at the walls and S is the fluid suction/injection parameter due to channel porosity,$Bi_{0,1}$ are the Biot numbers.

$G = \frac{\beta g(T_f - T_o)L^2}{\mu_f U}$ (Grashof number), $S = \frac{v_0 L}{U}$ (Suction parameter), $M = \frac{\sigma\mu_e^2 H_0^2 L^2}{\rho_f U \mu_f}$ (Magnetic parameter), $\Delta = \frac{C_b UL}{\sqrt{k_p}}$ (Forchheimer parameter), $A_{11} = \frac{L^2 A_{11}'}{\rho C_p}, B_{11} = \frac{L^2 B_{11}'}{\rho C_p}$ (Space /temperature dependent heat source), $Rd = \frac{4\sigma^{\bullet} T_0^3}{\beta_R k_f}$ (Radiation parameter), $\Pr = \frac{\mu_f C_p}{k_f}$ (Prandtl number), $Ec = \frac{U^2}{C_p(T_f - T_o)}$ (Eckert parameter), $B = m(T_f - T_0)$ (Viscosity parameter)

$$A_1 = (1-\varphi)^{2.5},\ A_2 = 1 - \varphi + \varphi(\frac{\rho_s}{\rho_f}),\ A_3 = 1 - \varphi + \varphi((\frac{(\rho\beta)_s}{(\rho\beta)_f}), A_4 = 1 - \varphi + \varphi\frac{(\rho C_P)_s}{(\rho C_P)_f}$$

$$A_5 = \frac{k_{nf}}{k_f}, A_6 = (1 + \frac{3(1-\sigma)\phi}{(\sigma+2)}), \sigma = \frac{\sigma_s}{\sigma_f}$$

The limiting case $\alpha_{0,1} \to \infty$ corresponds to the perfect lubricated plate surface.

3. FINITE ELEMENT ANALYSIS

The finite element analysis with quadratic polynomial approximation functions is carried out along the axial distance across the vertical channel. The behavior of the velocity, temperature and concentration profiles has been discussed computationally for different variations in governing parameters. The Galerkin method has been adopted in the variational formulation in each element to obtain the global coupled matrices for the velocity, temperature and concentration in course of the finite element analysis.

Choose an arbitrary element e_k and let u^k, v^k θ^k and C^k be the values of u, v,θ and C in the element e_k. We define the error residuals as

$$E_u^k = \frac{d}{dy}\left(\frac{du^k}{dy}\right) - B\frac{du^k}{dy}\frac{d\theta^k}{dy} + A_1A_3G(\theta^k) + Su^k + A_1 - A_1A_6M^2(u^k) - \Delta(u^k)^2 \quad (3.1)$$

$$E^k{}_\theta = \frac{A_5}{\Pr}\frac{d}{dy}\left(\frac{d\theta^k}{dy}\right) - S\theta^k A_4u^k + A_{11}u^k + B_{11}\theta^k + Ec[(\frac{du^k}{dy})^2] + \quad (3.2)$$

$$+ \mathrm{EcM}^2\mathrm{A}_6(u^k)^2$$

where u^k, θ^k are values of u, θ in the arbitrary element e_k. These are expressed as linear combinations in terms of respective local nodal values.

$$u^k = u_1^k\psi_1^k + u_2^k\psi_1^k + u_3^k\psi_3^k,$$

$$\theta^k = \theta_1^k\psi_1^k + \theta_2^k\psi_2^k + \theta_3^k\psi_3^k \quad (3,3)$$

where ψ_1^k, ψ_2^k --------- etc are Lagrange's quadratic polynomials.

Galarkin's method is used to convert the partial differential Equations (3.1) – (3.2) into matrix form of equations which results into 3x3 local stiffness matrices. All these local matrices are assembled in a global matrix by substituting the global nodal values and using inter element continuity and equilibrium conditions. The resulting global matrices have been solved by iterative procedure until the convergence i.e $|u_{i+1}-u_i|<10^{-6}$ is obtained.

COMPARISON

In the absence of convection(G=0),heat sources(A11=0=B11),variable viscosity(B=0)the results are in good agreement with Malleswari and Sreenivasa Reddy [17]

Parameter	**Malleswari and Sreenivasa Reddy [17]**				**Present results**			
	τ(-1)	τ(+1)	Nu(-1)	Nu(+1)	**τ(-1)**	**τ(+1)**	**Nu(-1)**	**Nu(+1)**
Ec	-----------	-----------	0.3328474 0.399166 0.448919	0.858343 1.010345 1.011167	-----------	-----------	**0.332821 0.399189 0.448926**	**0.858343 1.010345 1.011167**
Pr	-----------	-----------	0.026509 0.069259 0.119915	0.067910 0.177437 0.286915	-----------	-----------	**0.026517 0.069263 0.119923**	**0.067915 0.177434 0.286912**
αo	1.07617 0.97718 0.89849	0.93078 1.01156 1.07580	0.264871 0.311924 0.349923	0.678585 0.807821 0.912144	**1.07616 0.97722 0.89853**	**0.93073 1.01157 1.07577**	**0.264877 0.311911 0.349915**	**0.678585 0.807821 0.912144**
α1	1.12749 1.20336 1..26929	0.88869 0.82622 0.77210	0.265259 0.3406268 0.380440	0.752466 0.862845 0.959779	**1.12754 1.20331 1..26933**	**0.88854 0.82869 0.77193**	**0.265266 0.340624 0.380442**	**0.752469 0.862844 0.959778**
Bio	-----------	-----------	0.264863 0.541769 1.016420	0.678586 0.955269 2.205509	-----------	-----------	**0.264865 0.541767 1.016418**	**0.678584 0.955267 2.205502**
Bi1	----------	----------	0.1186056 0.045987 0.015634	0.5324554 0.459879 0.429549	----------	----------	**0.118608 0.045988 0.015631**	**0.532444 0.459881 0.429554**

5.RESULTS AND DISCUSSION:

In this analysis an attempt has been made to investigate the effect of thermal radiation, non-uniform heat source, asymmetric slip on the convective heat transfer flow Eg based u nanofluid in vertical channel with Newtonian cooling. The velocity and temperature has been discussed for different parametric variations.

Figs.2a-2b represent the nanofluid velocity with thermal radiation parameter(Rd) and Eckert number (Ec). Higher the thermal radiative heat flux larger the magnitude of velocity in Eg-Cu nanofluid. Higher the dissipative energy (Ec) smaller the magnitude of the velocity. The temperature decelarates with increasing Rd and accelerates the increase in Ec in the flow region. This may be due to the fact that increases in Rd lead to a decay and growth with Ec in the thickness of the thermal boundary layer.

Figs.3a-3b depicts the behaviour of the velocity and temperature with heat source parameters A11&B11 It is found that magnitude of axial velocity accelerates with increase in the strength of the space/temperature dependent heat sources. This is due to the fact that heat is generates in space/temperature dependent heat source with the buoyancy forces increases which grows the flow rate and there by gives rise to a increment in the velocity profile in Eg-Cu nanofluid. In the presence of space/heat generating source(A11,B11) heat is absorbed in the boundary layer which leads to a reduction in temperature with rise in A11 and B11.The thickness of the momentum boundary layer grows while the thermal boundary layer decay with higher values of A11 and B11.

Figs.4a-4b display the effect of nanoparticle volume fraction(ϕ) and Activation energy parameter(B) on the nanofluid velocity and temperature. It is found that an increase in the nanoparticle volume fraction upsurges the magnitude of nanofluid velocity in Eg-Cu nanofluid. These figures illustrate this agreement with the physical behaviour. When the volume of the nanoparticle increases the thermal conductivity and hence enhances the momentum boundary layer thickness.Fig.4b shows that the variation of temperature with ϕ. It can be seen from the profiles that an increase in the nanoparticle volume fraction decreases the temperature in thermal boundary layer in Eg-Cu nanofluid. This is due to the fact that the thickness of the thermal boundary layer decays with increase in ϕ.The effect of viscosity parameter(B) on u,θ shows that the effect of variable viscosity is to reduces the velocity(u) and temperature in the flow region in Eg-Cu nanofluid..This may be attributed to the fact that increase in viscosity parameter(B) leads to a decay in the thickness of the momentum and thermal boundary layers in Eg based Cu-nanofluid.

Fig.5a-5b exhibits the effect of asymmetric slips on velocity and temperature. From the profiles we find that the asymmetric slip on the walls demonstrates an increasing tendency in the magnitude of the velocity. This may be due to the fact that increase in asymmetric slips (αo, α1) leads to a growth in the thickness of the momentum boundary layer. The temperature experiences an enhancement with rising values of asymmetric slip (αo) while it depreciates with asymmetric slip at the right wall(α1). Thus the thermal boundary layer becomes thicker with higher values of slip(α0) and decays with slip(α1) .

The effect of convective boundary conditions (Bio,Bi1) on flow variables can be seen from figs.6a&6b. From the profiles we find that increase in Biot number (Bio) on the left wall (η=-1) depreciates the magnitude of the velocity while the Biot number (Bi1)on the right wall(η=+1) reduces the velocity in magnitude. This may be due to the fact that increases in Bio and Bi1 lead to decay in the thickness of the momentum boundary layer (fig.6a). Fig.6b demonstrates the effect of Biot numbers on the temperature (θ). Increase in Bio leads to a growth in the thickness of the thermal boundary layer while thermal boundary layer becomes thinner with Biot number (Bi1). This results in an augmentation in temperature with Bi0 and depreciation with Bi1 in temperature in the thermal boundary layer.

The Shear stress(τ) is exhibited in table.2 at the walls $\eta=\pm 1$ for different parameters Rd, Ec, A11, B11, B, ϕ, α0, α1, Bio and Bi1. Increase in space / temperature dependent heat source(A11,B11)/radiation parameter (Rd) upsurges stress at $\eta=\pm 1$ in Eg-Cu nanofluid. Higher the Eckert number (Ec) smaller the stress at $\eta=\pm 1$ in Eg-Cu nanofluid. Higher the nanoparticle volume fraction(ϕ) results in an enhancement in stress at $\eta=\pm 1$ in Eg-Cu nanofluid. Higher the Viscosity parameter(B) smaller stress at both the walls in Eg-Cu nanofluid. An increase in asymmetric slip parameter (αo) decays the stress at η=-1 and grows at η=+1while it exhibits opposite behaviour with rising values of asymmetric slip(α1) at the right wall. Increase in Biot number(Bio) leads to a fall in stress at both the walls while stress augments with higher values of Biot number (Bi1) at the walls.

The local Nusselt number (Nu) at the walls $\eta=\pm 1$ is shown in table 2 for different parametric variations. Higher the space/temperature dependent heat sources (A11, B11)/ nanoparticle volume fraction(ϕ)/viscosity parameter(B) smaller the Nusselt number at both the walls. Higher the dissipative energy larger the Nusselt number at $\eta=\pm 1$. Increase in asymmetric slip(α0) at the left wall and asymmetric slip(α1) at the right wall upsurges Nu at both the walls .The rate of heat transfer augments Nu with rising values of Biot number(Bio) at the walls and decays with Biot number(Bi1) at walls.

6.CONCLUSIONS:

The effect of variable viscosity, thermal radiation, and dissipation on convective heat transfer flow of Eg based Cu nanofluid in a vertical channel with asymmetric slips and convective boundary conditions in the presence of irregular heat sources. The non-linear, coupled equations have been executed by using Finite element method with quadratic interpolation functions. The important findings of the analysis are:

- The velocity enhances and temperature reduces with Rd and opposite effect is noticed with higher values of Eckert number(Ec) in the flow region. Stress increases and Nu decays with Rd .Stress decays and Nu grows with Ec on the walls.
- Increase in space/temperature dependent heat source(A11,B11) upsurges the velocity and decays temperature in the flow region. Stress grows and Nu decays with higher values of A11 and B11 on the walls..
- Increase in nanoparticle volume fraction(ϕ) enhances the velocity and decays temperature .Stress grows and Nu decays with increase in ϕ.
- Increase in asymmetric slips(αo,α1) results in an enhancement in velocity while the temperature enhances with αo and decays with α1. in the flow region. Increase in αo reduces stress, enhances Nu on the walls while Nu increases on both the walls , stress enhances on the left wall, reduces on the right wall.
- Increase in Biot numbers(Bio,Bi1)decays velocity while the temperature enhances with Bio, reduces with Bi1 in the flow region. Stress decays and Nusselt number enhances with increase in Bio while stress grows ,Nu decays with Bi1 on the walls.

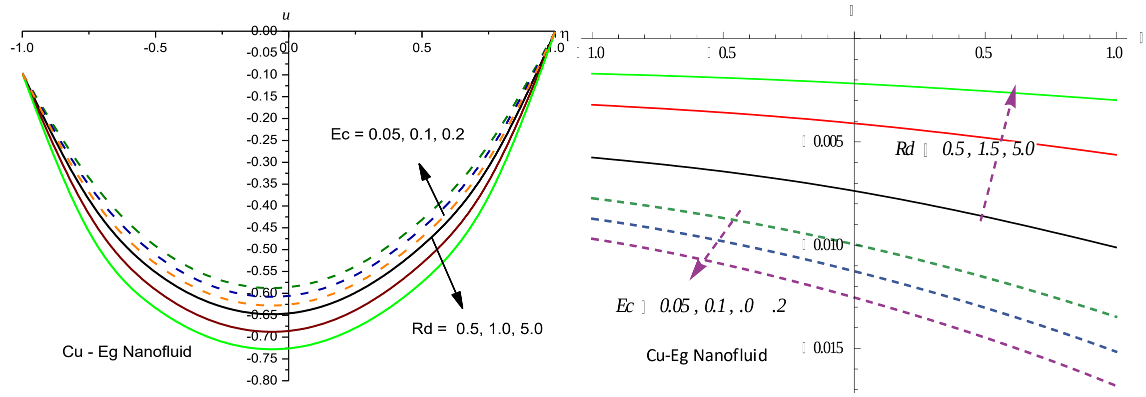

Fig.2 : Variation of [a] Velocity, [b] Temperature(θ) with Rd & Ec
A11=0.1, B11=0.2, B=0.2, ϕ=0.05, α1=0.2, α_0=0.1, Bi_1=0.5, Bi_0=0.5

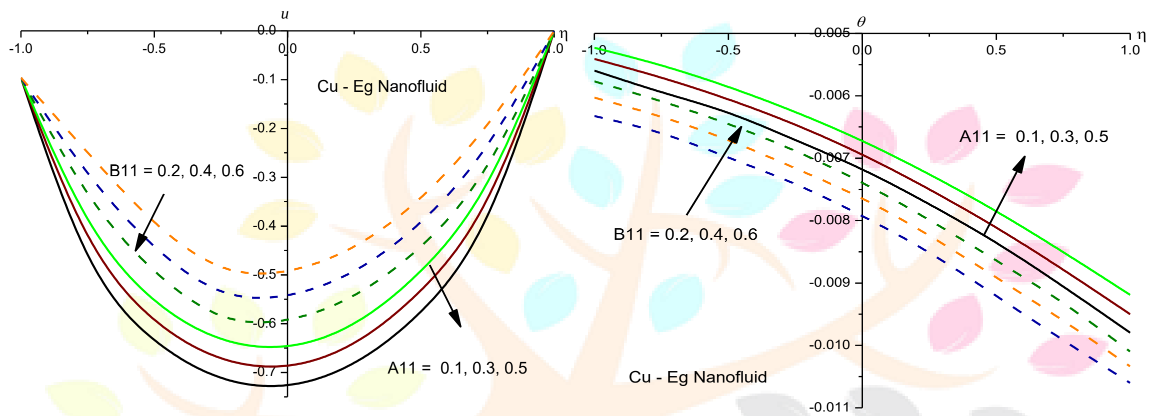

Fig.3 : Variation of [a] Velocity, [b] Temperature(θ) with A11 & B11

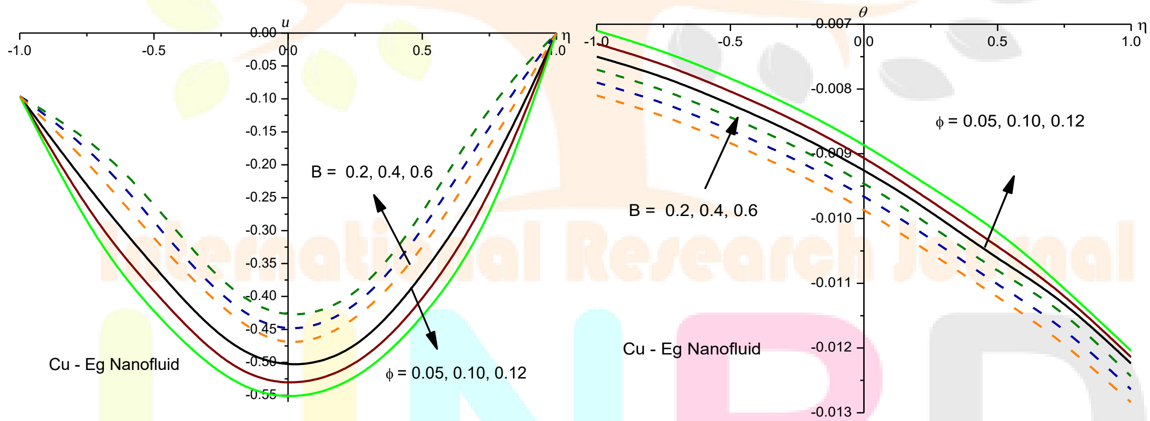

Fig.4 : Variation of [a] Velocity, [b] Temperature(θ) with B & ϕ

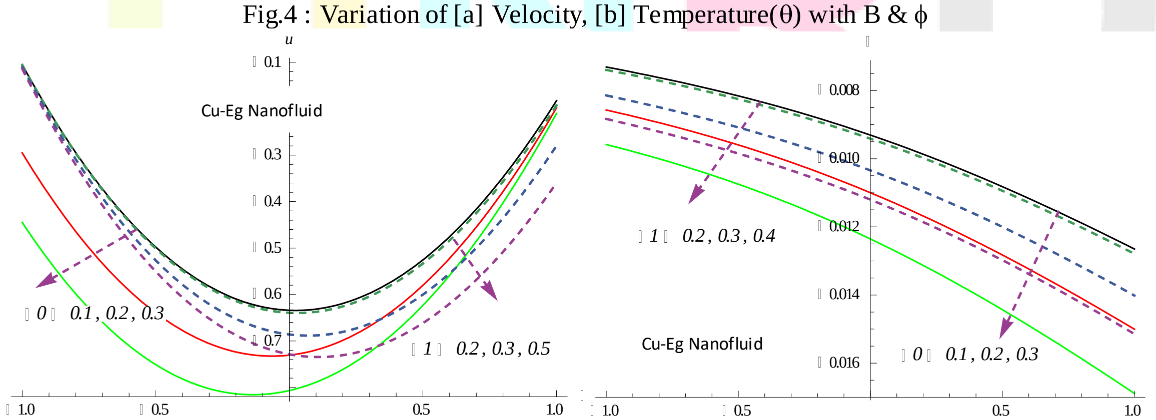

Fig.5 : Variation of [a] Velocity, [b] Temperature(θ) with α1 & α_0

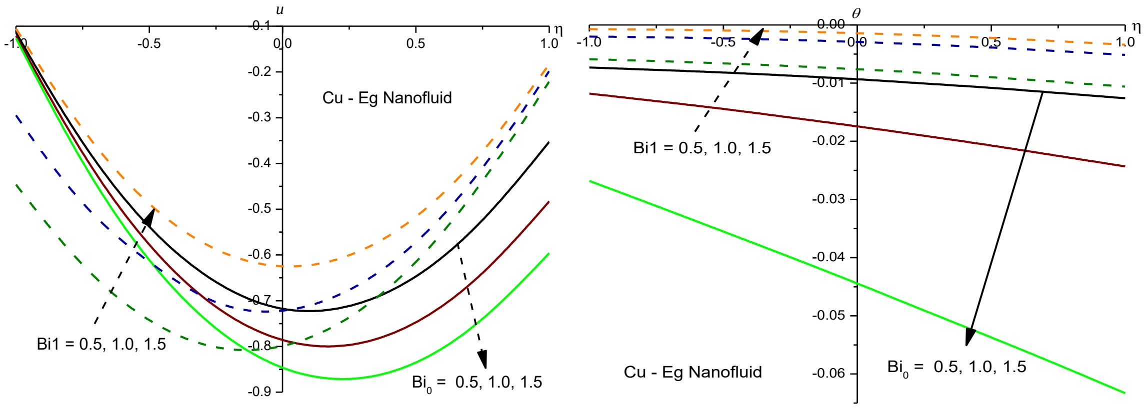

Fig.6 : Variation of [a] Velocity, [b] Temperature(θ) with Bi_1 & Bi_0

Table 2 : Skin friction (τ ±1), Nusselt number (Nu ± 1)

Parameter		τ(-1)	τ(+1)	Nu(-1)	Nu(+1)
Rd	0.5	-0.960955	0.99859	0.00117719	0.00310098
	1.5	-0.961365	0.999213	0.000655009	0.00172535
	5	-0.961607	0.999581	0.000346787	0.00091344
Ec	0.05	-0.960646	0.998128	0.00158067	0.00413197
	0.1	-0.960491	0.997897	0.00178332	0.00465018
	0.15	-0.960339	0.99767	0.00198174	0.00515733
A11	0.1	-0.960955	0.99859	0.00117719	0.00310098
	0.3	-0.960984	0.998634	0.00114014	0.0030031
	0.5	-0.961012	0.998676	0.00110485	0.00290988
B11	0.2	-0.960956	0.998591	0.00117628	0.00309865
	0.4	-0.960958	0.998592	0.00117552	0.00309671
	0.6	-0.960962	0.998594	0.0011734	0.00309133
ϕ	0.05	-0.960955	0.99859	0.00117719	0.00310098
	0.1	-0.962539	1.00589	0.00115379	0.00308803
	0.12	-0.963269	1.01191	0.00113069	0.00307279
B	0.25	-0.959369	0.996356	0.00117475	0.00309464
	0.5	-0.957791	0.994135	0.00117233	0.00308833
	1	-0.956536	0.992369	0.00117044	0.00308332
α0	0.2	-1.05292	0.914314	0.00149484	0.00388044
	0.4	-0.95208	0.990892	0.00175318	0.00460254
	0.6	-0.87248	1.051312	0.00196019	0.00518078
α1	0.2	-1.05795	0.909707	0.00151233	0.00392339
	0.4	-1.10178	0.869553	0.00166542	0.00429933
	0.6	-1.14193	0.832795	0.00180647	0.00464588
Bi_0	0.2	-1.05292	0.914314	0.00149484	0.00388044
	0.4	-1.05211	0.912955	0.00506049	0.00743829
	0.6	-1.04942	0.908473	0.0168406	0.0191927
Bi_1	0.5	-1.05316	0.914558	0.00120269	0.00358973
	1	-1.0538	0.915224	0.000407595	0.00279858
	1.5	-1.054	0.915438	0.000152773	0.00254502

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