IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

Algorithmic Approach for Solving Quadratic Fractional Programming Problem with Multiple Objectives

Huma Akhtar

Assistant Professor and Head, Department of Mathematics, Govt. P.G. College Narsinghgarh, Rajgarh, India.

Abstract: In the framework of this research, we propose a method to convert a multi-objective quadratic fractional programming problem (MOQFPP) into a single objective quadratic fractional programming problem (SOQFPP) using median of maximin and minimax technique. This can be demonstrated using a numerical example.

Keyword: Multi-objective quadratic fractional programming (MOQFP) and median of maximin and minimax.

Introduction: The Multi-Objective Quadratic Fractional Programming Problem (MOQFPP) is a quadratic limiting optimization problem with more than one objective functions. These objective functions are quantities that gauge the effectiveness of the system such as cost/profit, real cost/standard cost, output/employee cost/time, etc. MOQFP finds applications in various real-world scenarios including production planning, agricultural planning, healthcare management etc. In 1983, Chandra Sen [2] introduced the concept of multi-objective linear programming and outlined an approach for constructing multi-objective functions where optimal values of individual problems exceed zero. Sulaiman and Sadiq (2006) [6] investigated multi-objective functions using mean and median values. Subsequently, Najmaddin A. Sulaiman, Hamadameen and Abdul-Qader O. (2008) proposed a novel technique utilizing optimal averages (O_{AV}) for functions, derived from modifications of Chandra Sen's and Sulaiman and Sadiq's approaches [3]. In 2013, Suleiman and Nawkhass [5] introduced an optimal average of maximin and minimax technique to convert MOQFPP into a single QFPP. Nahar Samsun et al. [1] introduced a harmonic average technique to optimize the objective function, synthesizing multiple objectives into a single function. Recently in 2021 Margia Yesmin and Md. Abdul Alim [7] presented an advanced transformation technique for addressing Multi-Objective Optimization Problems (MOOP).

In order to build on this research, we have created a Multi-Objective Quadratic Fractional Programming Problem (MOQFPP) and investigated the algorithm used to solve the Quadratic Fractional Programming Problem for multi-objective functions. The median of the maximin and minimax approaches are used in this algorithm. This can be explained with a numerical example.

2. Quadratic Programming Problem (QPP): Mathematical formulation of QPP is as follows:

$$Max\ Z\ or\ Min\ Z = \delta + C^T\mathrm{X} + \frac{1}{2}\mathrm{X}^T G\mathrm{X} \quad \text{or} \quad ({c_1}^T\mathrm{X} + \alpha)({c_2}^T\mathrm{X} + \beta)$$

subject to:

$$A\mathrm{X} \leq b$$

$$\mathrm{X} \geq 0$$

where,

i) A is an $m \times n$ matrix,

ii) b is an $m \times 1$ vector,

iii) C, c_1, c_2 and X are $n \times 1$ vectors,

iv) X^T is a transpose of X,

v) G is an $n \times n$ symmetric square matrix and

vi) δ, α and β are scalars.

3. Quadratic Fractional Programming Problem (QFPP): The mathematical formulation of QFP problem is as follows:

$$Max\ Z\ or\ Min\ Z = \frac{({c_1}^T\mathrm{X}+\alpha_1)({c_2}^T\mathrm{X}+\alpha_2)}{({d_1}^T\mathrm{X}+\beta_1)}$$

subject to:

$$A\mathrm{X} \leq b$$

$$\mathrm{X} \geq 0$$

where,

i) A is an $m \times n$ matrix,

ii) b is an $m \times 1$ vector,

iii) $c_1,\ c_2, d_1$ and X are $n \times 1$ vectors,

iv) $\alpha_1 \alpha_2$ and β_1 are scalars,

v) ${c_1}^T, {c_2}^T$ and ${d_1}^T$ are transpose of c_1 , c_2 and d_1 respectively.

vi) ${d_1}^T\mathrm{X} + \beta > 0$ and

vii) $\mathrm{S} = \{\mathrm{X}: \mathrm{AX} \leq \mathrm{b},\ \mathrm{X} \geq 0\}$ is non empty and bounded.

4. Multi-Objective Quadratic Fractional Programming Problem (MOQFPP):

The Mathematical formulation of Multi-Objective Quadratic Fractional Programming Problem (MOQFPP) is:

$$\left.\begin{array}{l} Max\ z_1 = \frac{(c_{11}{}^T X+\alpha_1)(c_{21}{}^T X+\beta_1)}{c_1{}^T X+\gamma_1} \\ Max\ z_2 = \frac{(c_{12}{}^T X+\alpha_2)(c_{22}{}^T X+\beta_2)}{c_2{}^T X+\gamma_2} \\ \vdots \\ Max\ z_r = \frac{(c_{1r}{}^T X+\alpha_r)(c_{2r}{}^T X+\beta_r)}{c_r{}^T X+\gamma_r} \\ Min\ z_{r+1} = \frac{(c_{1r+1}{}^T X+\alpha_{r+1})(c_{2r+1}{}^T X+\beta_{r+1})}{c_{r+1}{}^T X+\gamma_{r+1}} \\ \vdots \\ Min\ z_s = \frac{(c_{1s}{}^T X+\alpha_s)(c_{2s}{}^T X+\beta_s)}{c_s{}^T X+\gamma_s} \end{array}\right] \qquad (4.1)$$

subject to:

$$AX \leq b \qquad (4.2)$$

$$X \geq 0 \qquad (4.3)$$

where,

i) A is an $m \times n$ matrix,

ii) b is an $m \times 1$ vector,

iii) C_i, c_{1i}, c_{2i} and X are $n \times 1$ vectors, where $i = 1,2,3, \ldots, s$,

iv) α_i, β_i and γ_i are scalars, where $i = 1,2,3, \ldots, s$,

v) $C_i{}^T, c_{1i}{}^T$ and $c_{2i}{}^T$ are transpose of C_i, c_{1i}, and c_{2i} respectively, where $i = 1,2,3, \ldots, s$

vi) $c_{1i}{}^T X + \alpha_i, c_{1i}{}^T X + \beta_i$ and $c_{1i}{}^T X + \gamma_i$ are positive for all feasible solutions, where $i = 1,2,3, \ldots, s$ and

vii) $S = \{X: AX \leq b,\ X \geq 0\}$ is non empty and bounded.

5. Method for solving MOQFPP:

5.1. Median of Maximin and Minimax Technique:

Step1: First of all, we will solve each objective function of (4.1) by using the modified simplex technique (MST) [4].

Step2: Next, we assign a name to the optimum/optimal value of each objective function $Max\ z_i$ say φ_i, where $i = 1,2, \ldots . r$ and Min z_i say φ_i, where $i = r+1, r+2, \ldots . s$.

Step3: Choose $m_1 = min\{\varphi_i\}$, $\forall\ i = 1, 2, \ldots, r$ and $m_2 = max\{\varphi_i\}$, $\forall\ i = r+1, \ldots, s$ then calculate

$$\text{Md} = \text{Median}\ (|\ m_j\ |), \qquad j = 1, 2$$

Step4: Optimize the combined objective function by using MST [4] with the same constraints (4.2) & (4.3) as:

$$\text{Max. Z} = \frac{(\sum_{i=1}^{r} Max\ z_i - \sum_{i=r+1}^{s} Min\ z_i)}{\text{Md}} \quad (5.1)$$

6. Numerical Example:

Example 6.1.

$$\text{Max. } z_1 = \frac{(2x_1+x_2+1)(2x_1+x_2+2)}{(2x_1+2x_2+2)}$$

$$\text{Max. } z_2 = \frac{(4x_1+2x_2+2)(6x_1+3x_2+6)}{(3x_1+3x_2+3)}$$

$$\text{Max. } z_3 = \frac{(4x_1+2x_2+2)(6x_1+3x_2+6)}{(6x_1+6x_2+6)}$$

$$\text{Min. } z_4 = \frac{(-8x_1-4x_2-4)(6x_1+3x_2+6)}{(5x_1+5x_2+5)}$$

$$\text{Min. } z_5 = \frac{(-4x_1-2x_2-2)(10x_1+5+10)}{(2x_1+2x_2+2)}$$

Subject to:

$$x_1 + 2x_2 \leq 4, \qquad 3x_1 + x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

Solution: After computing the value of each individual objective function by using the modified simplex technique, the results are displayed in table 1:

Table 1

i	z_i	x_i	φ_i	Md
1	5	(2,0)	5	14.5
2	20	(2,0)	20	
3	10	(2,0)	10	
4	-24	(2,0)	-24	
5	-50	(2,0)	-50	

Formulate the combined objective function as:

$$\text{Max. Z} = \frac{(\sum_{i=1}^{r} Max\ z_i - \sum_{i=r+1}^{s} Min\ z_i)}{\text{Md}} \quad \text{where} \quad \text{Md} = \text{Median}\ (|\ m_j\ |), \qquad j = 1, 2$$

$$\text{Max. } Z = \frac{(22x_1+11x_2+11)(18x_1+9x_2+18)}{(145x_1+145x_2+145)}$$

Subject to:

$$x_1 + 2x_2 \leq 4, \qquad 3x_1 + x_2 \leq 6,$$

$$x_1, x_2 \geq 0$$

Hence, the optimal solution is:

Max. Z = 6.83, $x_1 = 2$, $x_2 = 0$.

9. References:

[1] Nahar Samsun and Alim Md. Abdul, “A new geometric average technique to solve multi-objective linear fractional programming problem and comparison with new arithmetic average technique”, IOSR Journal of Mathematics, Vol. 13, No. 3, PP. 39-52, 2017.

[2] Sen. Chandra, “A new approach for multi-objective rural development planning”, The Indian Economic Journal, Vol. 30, No. 4, PP. 91-96, 1983.

[3] Sulaiman, N. A., Hamadameen and abdul-Qader O., “Optimal transformation technique to solve multi-objective linear programming problem (MOLPP)”, Journal of Kirkuk University-Scientific Studies, Vol.3, No.2, 2008.

[4] Sulaiman, N. A. and Nawkhass M.A,” A new modified simplex method to solve quadratic fractional programming problem and compared it to a traditional simplex method by using pseudoaffinity of quadratic fractional functions”, Applied Mathematical Sciences, Vol. 7 issue 76, PP. 3749-3764, 2013.

[5] Sulaiman, N. A. and Nawkhass M.A., “Transforming and solving multi-objective quadratic fractional programming problems by optimal average of maximin & minimax techniques”, American Journal of Operational Research, Volume 3, issue 3, PP. 92-98, 2013.

[6] Sulaiman N.A. and Sadiq G.W., “Solving the linear multi-objective programming problems”, using mean and median values”, Al-Rafiden Journal of Computer Sciences and Mathematical, University of Mosul, Vol. 3, No. 1, PP. 69-83, 2006.

[7] Yesmin Margia and Abdul Md. Alim, “Advanced Transformation Technique to Solve Multi-Objective Optimization Problems”, American Journal of Operations Research, 11, 166-180 2021, https://www.scirp.org/journal/ajor ISSN Online: 2160-8849 ISSN Print: 2160-8830.