



# **CALCULATION OF TRANSFER FUNCTION AND ANALYSATION OF STABILITY OF THE TRANSFER FUNCTION FROM A SCHEMATIC BLOCK DIAGRAM AND FROM A LINEAR DIFFERENTIAL EQUATION, AS WELL AS CONTROLLABILITY AND OBSERVABILITY ANALYSIS OF A 3X3 SQUARE MATRICS**

[ABIR CHAKRABORTY<sup>1</sup>, JUNIOR SCIENTIST OF ISR.UC.PT ROBOTICS LAB  
UNIVERSITY OF COIMBRA PORTUGAL]

[DR. KAUSHIK SARA KAR<sup>2</sup>, RESEARCH FELLOW, CONTROL AND  
AUTOMATION DEPARTMENT, INDIAN STATISTICAL INSTITUTE (ISI),  
KOLKATA, WEST BENGAL, INDIA]

[ABSTRACT: Here in this journal we want to develop the key concept of transfer function not by using direct mat-lab code, but only from schematic block diagram. For simplicity we have used a single stage negative feedback schematic block diagram as for first option and for the next case we have developed the schematic block diagram from a second order equation. In both the cases after calculating the transfer function we have analysed the stability criterion where we have used two types of plotting techniques 1. NYQUIST PLOT 2. BODE PLOT. Then in the next phase we have brought the concept of state space matrix with the help of observability and controllability in order to discuss the stability analysis of any 3X3 square matrix. So our main focus is to analyse only the stability of a linear control system.]

KEYWORDS: transfer function, nyquist plot, bode plot, observability, controllability

## **1. INTRODUCTION**

Before discussing about the main topic of the paper let's discuss about the basic of control system engineering technology. Transfer function is a key factor of control system programming. The main definition of "Transfer function" is something like that; The **TRANSFER FUNCTION** is defined by

## $C(s)/R(s)$ =LAPLACE TRANSFORM OF OUTPUT FUNCTION/LAPLACE TRANSFORM OF INPUT FUNCTION

$C(s)/R(s)=G(s)/1+G(s)H(s)$  Where  $G(S)$ = forward path gain in laplacian-domain ,  
 $H(S)$ =feedbackpath gain in laplacian domain.

The transfer function has two important terminologies; poles and zeroes,

**POLES:** Let us consider a transfer function

$$C(s)/R(s)=(S-Z1)*(S-Z2)*(S-Z3)*(S-Z4)/(S-P1)*(S-P2)*(S-P3)*(S-P4)$$

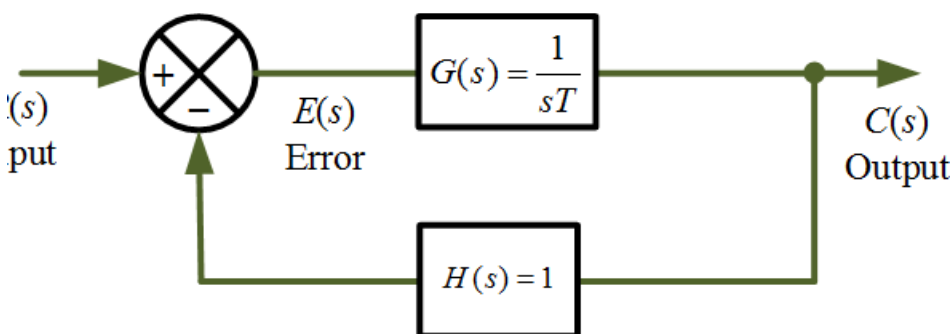
So pole means the value or the values for which the entire numerical value of the transfer function of a control

system unit will tend to infinity.

**ZEROES:** Let us consider a transfer function

$$C(s)/R(s)=(S-Z1)*(S-Z2)*(S-Z3)*(S-Z4)/(S-P1)*(S-P2)*(S-P3)*(S-P4)$$

Here we have used the concept of negative feedback only , so let us realise the concept of negative feedback from a '**BLOCK DIAGRAM**'



Lets talk about the stability analysis of the entire transfer functions which we have used BODE PLOT analysis . Now the question is that there are **NYQUIST PLOT** or **NICHOLE'S CHART ANALYSIS** but still we have used BODE PLOT

Because mat-lab syntaxes are easy to obtain and use in this case as well as we are calculating the **GAIN MARGIN** and **PHASE MARGIN** anaslysis for which we need to know the technology of **BODE PLOT**. **BODE PLOT** is a method of **ABSOLUTE STABILITY**. Which judges whether the system is absolutely stable or unstable . So you need to know the necessity of '**GAIN MARGIN**' and '**PHASE MARGIN**'.

**GAIN MARGIN:** The gain margin refers to the amount of gain which can be increased or decreased without making the system unstable. It is expressed in DB.

**PHASE MARGIN:** Phase margin. Phase margin is defined as the amount of change in open-loop phase needed to make a closed-loop system unstable. The phase margin is the difference in phase between  $-180^\circ$  and the phase at the gain cross-over frequency that gives a gain of 0 dB.

So for some extra work we have calculated the phase and gain cross over frequency co related to gain or phase margin respectively.

[NOTE: Before realising the process of programming the reviewer must note that for all four different damping

conditions we have fixed the damping constants in programming for the corresponding transfer functions by using mat-lab syntaxes in each case for each table so that you can understand the transfer functions values based on the nature of damping.

## Controllability

A system with internal state vector  $x$  is called **controllable** if and only if the system states can be changed by changing the system input.

## Observability

Consider a physical system modeled in state-space representation. A system is said to be **observable** if, for every possible evolution of state and control vectors, the current state can be estimated using only the information from outputs (physically, this generally corresponds to information obtained by sensors). In other words, one can determine the behavior of the entire system from the system's outputs. On the other hand, if the system is not observable, there are state trajectories that are not distinguishable by only measuring the outputs.

## CONTROLLABILITY MATRIX

$U=[B:A*B:A^2*B]$ ; [For a 3x3 matrix or linear square matrix with 9 elements ]

## OBSERVABILITY MATRIX

$V=[C^T:A^T*C^T:(A^T)^2*C^T]$  [For a 3x3 matrix or linear square matrix with 9 elements ]

## 2.STEP BY STEP PROGRAMMING METHODOLOGY

**STEP1.** Prepare a schematic block diagram of a linear control system by using negative feedback method

**STEP 2:** Find out the transfer function of the block diagram by using matlab code

**STEP 3:** Find out poles and zeroes of the transfer function along with '**BODE PLOT**' and '**NYQUIST PLOT**' for stability analysis purpose.

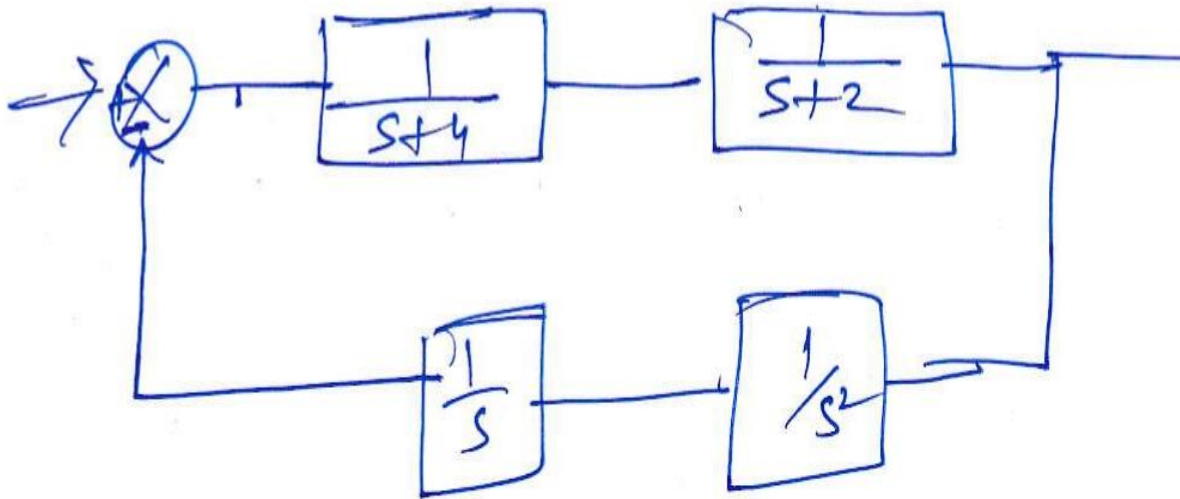
**STEP4:** Find out the transfer function by state space analysis method and then find out the '**BODE PLOT**' and '**NYQUIST PLOT**' for stability analysis purpose.

**STEP 5:** prepare a 3x3 square matrices and then test the '**CONTROLLABILITY**' and '**OBSERVABILITY**' by using matlab code.

**STEP 6:** Prepare a second order differential equation then find out transfer function by using matlab codes or syntaxes. Then find out the '**BODE PLOT**' and '**NYQUIST PLOT**' for stability analysis purpose.

### 3. MATLAB PROGRAMMING CODES

#### 3.1 BLOCK DIAGRAM USED FOR TRANSFER FUNCTION CALCULATION PURPOSE



#### 3.2 DIFFERENTIAL EQUATION USED FOR TRANSFER FUNCTION CALCULATION PURPOSE

$$y'' + 5y' + 6y = 2x' + x$$

or,  $s^2 y + 5s y + 6y = 2s x + x$  [BY USING LAPLACE TRANSFORMATION]

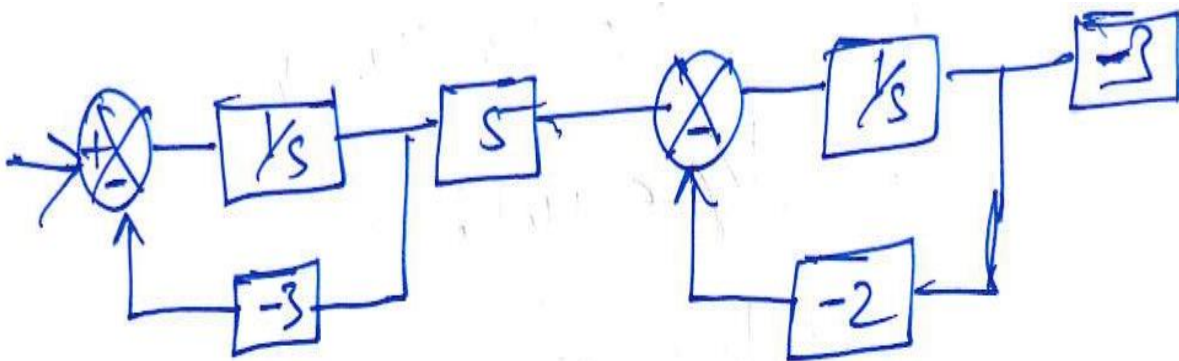
$$\text{or, } (y/x) = (2s + 1) / (s^2 + 5s + 6)$$

$$= (2s + 1) / (s + 3)(s + 2)$$

$$= 5 / (s + 3) - 3 / (s + 2)$$

$$= (5/s) / (1 + 3/s) - (3/s) / (1 + 2/s)$$

#### 3.3 BLOCK DIAGRAM GENERATED FROM THE ABOVE MENTIONED DIFFERENTIAL EQUATION



#### 3.4 TRANSFER FUNCTION BY USING BLOCK DIAGRAM

syms x1 x2 x3 x4 k s

$$x1 = 1 / (s + 4);$$

$$x2 = 1 / (s + 2);$$

$$x3 = 1 / s;$$

$$x4 = 1 / (s * s);$$

```
k=(x1*x2)/(1+(x1*x2*x3*x4));
```

```
k
```

```
k =
```

```
1/((s + 2)*(s + 4)*(1/(s^3*(s + 2)*(s + 4)) + 1))
```

```
poles(k)
```

```
ans =
```

```
0.26692609584625265070057988430579 - 0.38261056354607409247992634909349i
```

```
0.26692609584625265070057988430579 + 0.38261056354607409247992634909349i
```

```
-0.59309637164942851760461587065643
```

```
-1.9330184152862076930553113040771
```

```
-4.007737404756869090741232593878
```

```
bode/(k)
```

```
nyquist/(k)
```

### 3.5 TRANSFER FUNCTION BY USING STATE SPACE ANALYSIS

```
syms s A B C
```

```
A=[2 3;6 8];
```

```
B=[1;1];
```

```
C=[2 3];
```

```
I=[1 0;0 1];
```

```
y=C*inv(s*I-A)*B;
```

```
y
```

```
y =
```

```
- 24/(- s^2 + 10*s + 2) - (3*(s - 2))/(- s^2 + 10*s + 2) - (2*(s - 8))/(- s^2 + 10*s + 2)
```

```
poles(y)
```

```
ans =
```

```
3*3^(1/2) + 5
```

```
5 - 3*3^(1/2)
```

```
bode/(y)
```

```
nyquist/(y)
```

### 3.6 CONTROLLABILITY BY USING MATLAB CODE

```
A=[2 3 4;6 8 9;7 5 3];
```

```
B=[1 0 1];
```

```
C=[2 3 4];
```

```
TT=A*B';
```

```
TT
```

```

TT =
    6
   15
   10
TK=A*A*B';
TK
TK =
    97
   246
   147
S=[B;TT';TK'];
S
S =
    1    0    1
    6   15   10
    97  246  147
det(S)
ans
-234.0000

```

### 3.7 OBSERVABILITY BY USING MATLAB CODE

```

J=C';
K=A'*A'*C';
L=A'*C';
J'
ans =
    2    3    4
K'
ans =
   729   785   791
L'
ans =
   50   50   47
Q=[J';L';K'];
Q
Q =
    2    3    4
   50   50   47
   729   785   791

```

```
det(Q)
```

```
ans =
```

```
649.0000
```

### 3.8 TRANSFER FUNCTION OF A DIFFERENTIAL EQUATION BY USING MATLAB CODE

```
syms x1 x2 x3 x4 x5 x6 y s
```

```
x1=1/s;
```

```
x2=-3;
```

```
x3=5;
```

```
x4=1/s;
```

```
x5=-2;
```

```
x6=-3;
```

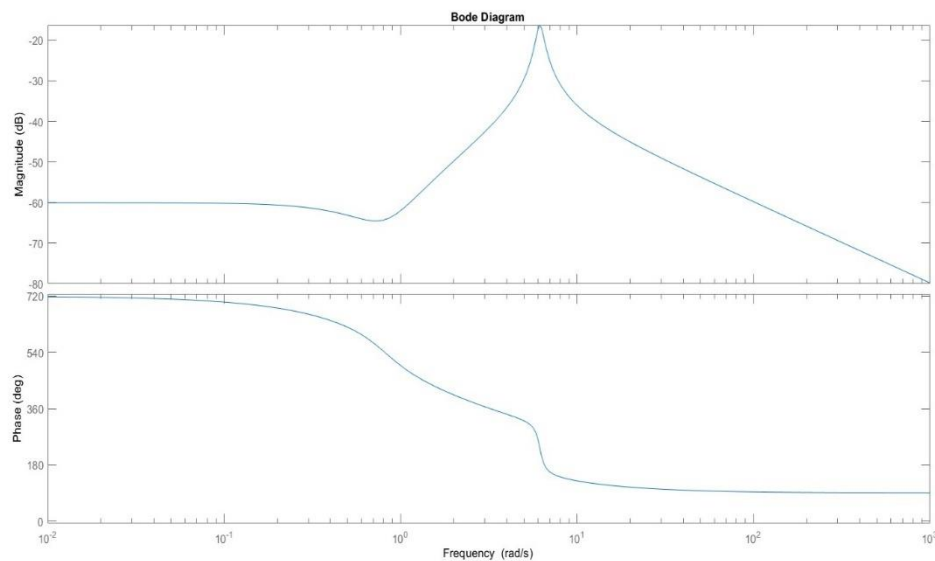
```
y=(x3*x1)/(1-x1*x2)+(x4*x6)/(1-x4*x5);
```

```
y =
```

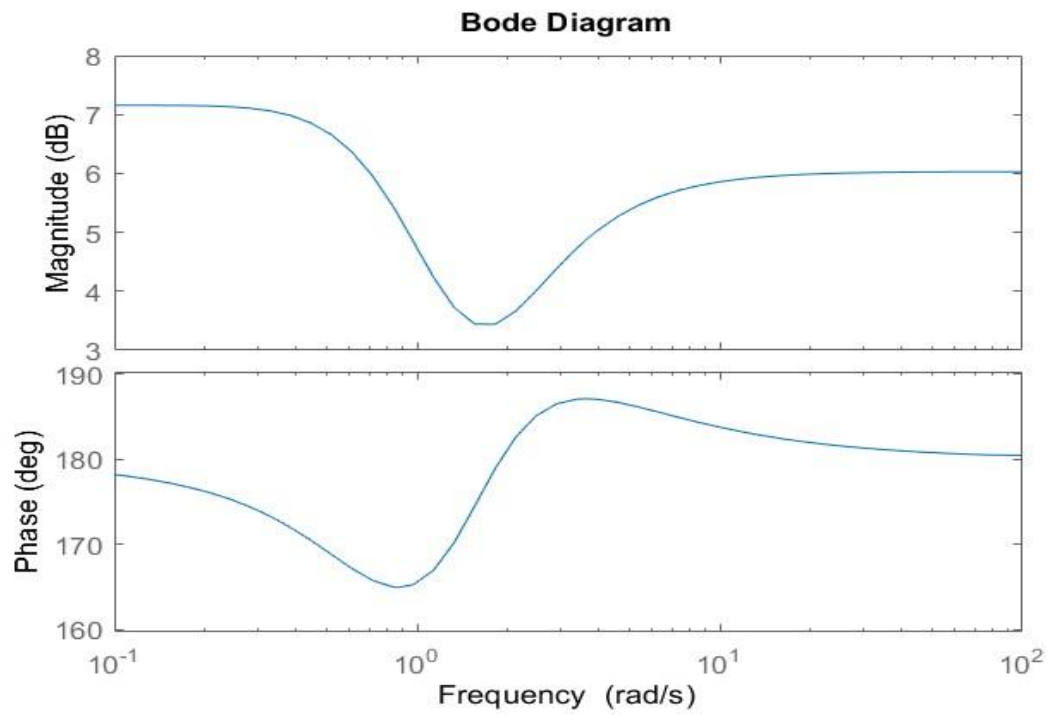
```
5/(s*(3/s + 1)) - 3/(s*(2/s + 1))
```

## 4. PLOT OF RESULTS OF IMAGES OBTAINED THROUGH MATLAB CODE

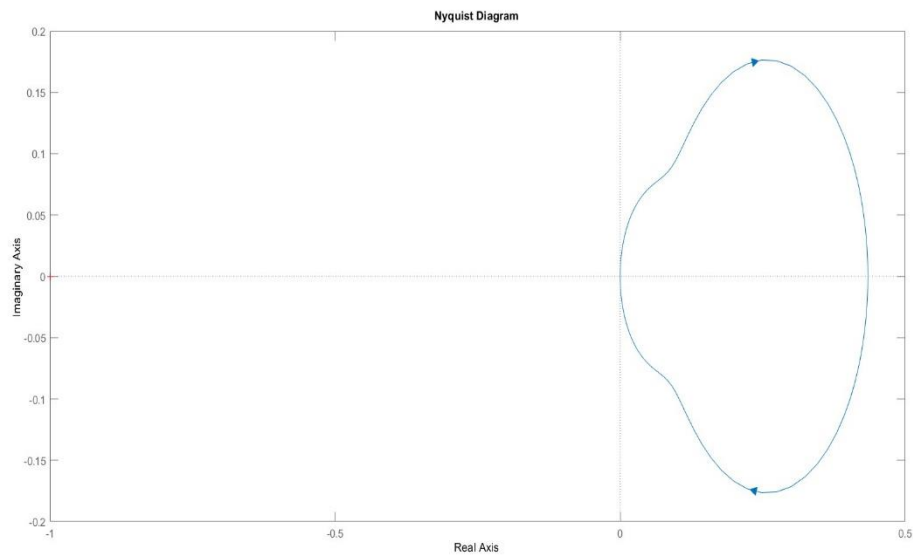
### 4.1 BODE PLOT OF TRANSFER FUNCTION OBTAINED FROM BLOCK DIAGRAM



## 4.2 BODE PLOT OF TRANSFER FUNCTION OBTAINED FROM STATE SPACE MATRICES

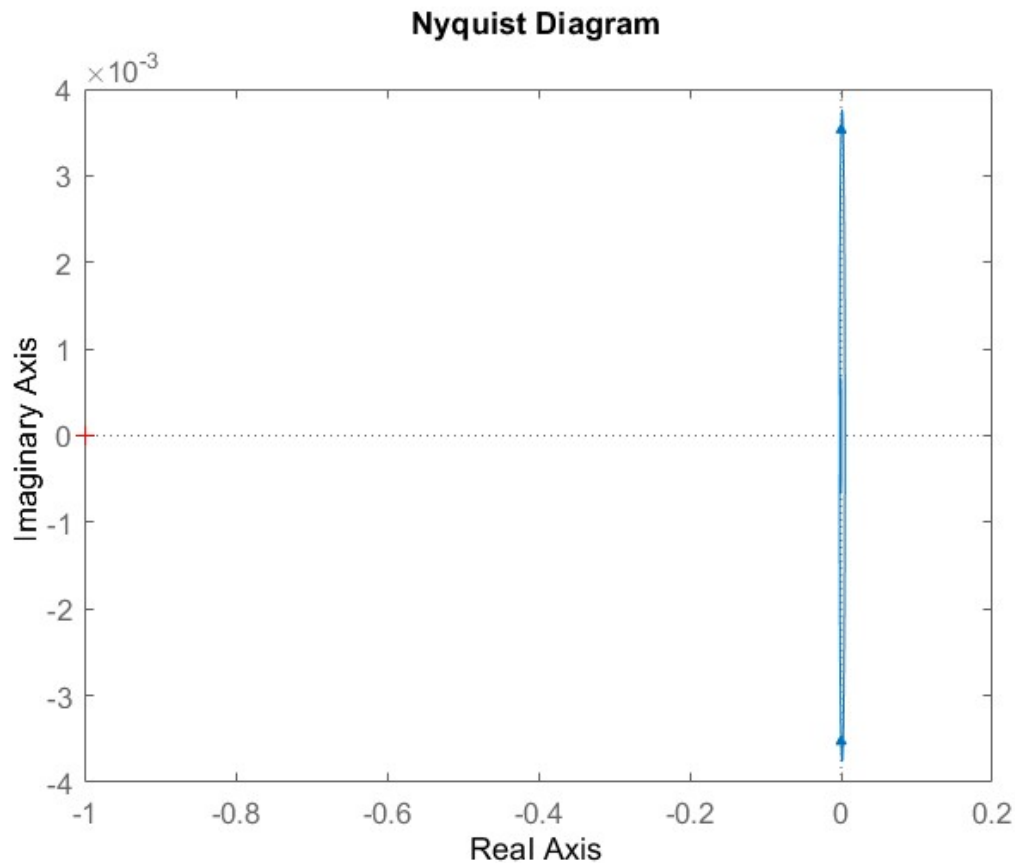


## 4.3 NYQUIST PLOT OF TRANSFER FUNCTION OBTAINED FROM BLOCK DIAGRAM





#### 4.4 NYQUIST PLOT OF TRANSFER FUNCTION OBTAINED FROM STATE SPACE ANALYSIS



#### 5. DISCUSSIONS ON PROGRAMMING

As we have seen that obtained poles are negative and they lie on the left half of the 'S' plane that's why in the '**NYQUIST PLOT**' the critical point  $(-1+j*0)$  has not been encircled at any point of time. Therefore the system is stable what we have got from the '**NYQUIST PLOT**'. There-after if we examine the determinants of '**CONTROLLABILITY**' AND '**OBSERVABILITY**' MATRICES easily we can conclude that in both the cases determinants are non-zero. So our linear and square (3x3) matrices are '**CONTROLLABLE**' as well as '**OBSERVABLE**'.

#### 6. FUTURE SCOPE OF WORK

In future we want to apply or use '**NICHOL'S CHART**' or we can use the same block diagram concept for non-linear stability analysis (like **LYAPUNOV STABILITY** and **THERMOSTAT CONTROLLED HEATING**) in order to generate transfer function and stability conclusion related techniques.

## REFERENCES

1. **‘LINEAR CONTROL SYSTEM’-with mat-lab application- Prof. B.S.Manke**
2. **‘CONTROL SYSTEM ENGINEERING’- Professor Nagrath and Gopal**
3. **“COMPARISON OF GAIN MARGIN AND PHASE MARGIN ALONG WITH SEVERAL OTHER PARAMETERIC COEFFICIENTS OF A SECOND ORDER SYSTEM UNDER DIFFERENT DAMPING CONDITIONS- A MATLAB SYNTAX BASED APPLICATION OF CONTROL SYSTEM ENGINEERING”- ABIR CHAKRABORTY<sup>1</sup>, DR.ABHIJIT BHOWMIK<sup>2</sup>, Vol-8 Issue-6 2022, IJARIE-ISSN(O)-2395-4396**
4. **“BASIC CONCEPT OF CONTROL SYSTEM AND IMAGE PROCESSING BY USING MATLAB”- A LABORATORY HANDBOOK FOR GRADUATE ENGINEERING STUDENT, ISBN-978-93-5989-177-4**