IJNRD.ORG ISSN : 2456-4184
INTERNATIONAL JOURNAL OF NOVEL RESEARCH AND DEVELOPMENT (IJNRD) | IJNRD.ORG
An International Open Access, Peer-reviewed, Refereed Journal

Higher Dimensional Cylindrically Symmetric Solutions with Dust matter and Energy Distribution in $f(R)$ *Gravity*

[1]L.S. Ladke,[*2]S.D.Mhashakhetri, [3]V.P. Tripade

[1]*Nilkanthrao Shinde Science and Arts College, Bhadrawati,Dist-Chandrapur,india*

[*2]*Shivaji Junior Science College Gadchiroli, Dist- Gadchiroli,india,*

[3]*Sarvodaya Mahavidyalaya ,Sindewahi, Dist-Chandrapur,india*

Abstract:

We study five dimensional cylindrical symmetric solutions in presence of dust matter in f(R) theory of gravity. To explore energy density for obtained solutions having constant scalar curvature, the generalized Landau-Lifshitz energy momentum complex is used. Stability and constant scalar curvature condition is explored for f(R) models.

Keywords: *Cylindrical symmetric solution, Dust matter, f (R) gravity, Energy momentum complex.*

Introduction:

Einstein general relativity plays an important role in modern physics, but due to missing content problem at all astronomical scales, the problem of moditing general relativity has attained much attention out of many other alternative theories of Einstein's theory of gravity. $f(R)$ theory of gravity has a long history.

This $f(R)$ theory recovers certain well- established GR solutions as the initial value problem and describe the stability of these solutions. $f(R)$ theory of gravity first studied by Weyl and Later Buchdhal put his ideas in the same theory $f(R)$ gravity. By considering higher order curvature in term there does not exist any singularity in f (R) theory of gravity [1]. $f(R)$ theory of gravity is actually a family of theories, each one defined by a different function of Ricci scalar. Weyl [2] and Eddington [3] have discussed various action integrals of $f(R)$ gravity. Nojiri S. and Odintsov S. D. [4, 5, 6] proved that the $f(R)$ theory of gravity provides very natural unification of the early time inflation and late time acceleration. Cylindrical solution in metric $f(R)$ gravity is studied by Azadi A. et al [7] .Many authors [8, 9, 10, 11, 12] have done a remarkable work in $f(R)$ theory of gravity.The energy localization is the unsolved problem in general relativity. Information on the energy of the matter and gravitational field is carries by energy momentum complex. Moller [13] was the first who describe the energy momentum complex in any coordinate system. Weinberg [14], Bergmann and Thomson [15], J.N. Goldberg [16] studied the energy momentum complexes. Different energy momentum complexes differ by a term which related to the boundary conditions of the physical situation is shown by Chang et al [17]. Einstein tried to solve the problem of energy localization but that complex is not symmetric in its indices and therefore conservation law of angular momentum cannot be defined Agguirregabiria et al [18] showed that the different energy momentum complexes may give the same energy distribution for any Kerr-schild space- time . In last few years, many authors have tried o solve energy localization problem using modified theories. Multamiki et al [19] generalized the energy momentum complex in $f(R)$ gravity.

Sharif and S. Arif [20] obtained the energy distribution of static cylindrically symmetric solution in $f(\mathrm{R})$ gravity.

Xulu [21] has investigated, Weinberg, Landau-Lifshitz and papapetrou Energy momentum prescriptions for Bianchi Type-I universe. M. Sharif and M. Farasat Shamir [22] studied Energy distribution in f(R) gravity using Landau-Lifshitz energy momentum compex. Static cylindrically symmetric solution in f(R) gravity using assumption of constant scalar curvature and dust matter is investigated by M. Sharif and Sadia Arif [23]. Energy distribution of the solution is explored by them by applying Landau-Lifshitz energy momentum complex and stability for some f(R) model.

Higher dimensional theories were reinforced in physics to exploit the special properties that supergravity and super string theories. Higher dimensional theories are now an active field of research in general relativity and high energy physics. Weinberg [24] studied the unification of fumdamental forces with gravity which shows that the space-time must be higher dimension. Many researchers [25, 26, 27, 28] encouraged to use the higher dimensional theory. The concept of five dimensional space-time has been introduced by Kaluza and Klein [29, 30] which unified gravitation with electromagnetic interaction. Higher dimensional cosmological models play a vital role in many aspects of early stage of cosmological problems. A numerous authers have done work on higher dimensional space-times.

The concept of symmetry is very important in the development of mathematics and physics. It is believed that the nature of universe can be explained in terms of symmetries rather than detailed dynamics. The spherical symmetric solution is the first nearby solution to the nature and cylindrical symmetry is the next closest approach to universe after spherical symmetry. M. Farasat Shmir[31] studied dust static and cylindrically symmetric solution in f(R) theory using constant and non-constant curvature solution. M. Sharif et. al. [32] studied static cyllindric interior solutions in f(R) gravity.

In this paper, we have studied a five dimensional cylindrically symmetric solutions with dust matter and energy distribution in f(R) gravity. Generalised Landau-Lifshtz energy momentum complex is used to explore energy density of obtained solutions. Stability and constant scalar curvature condition is explored for f(R) models.

$f(R)$ **Theory of Gravity:**

The action of $f(R)$ theory of gravity is expressed as

$$S=\int\sqrt{-g}\left(\frac{1}{16\pi G}f(R)+L_m\right)d^5x \qquad (1)$$

Where $f(R)$ is arbitrary function of Ricci scalar R and L_m is the matter Lagrangian. By varying this action with respect to metric tensor g_{uv}, the corresponding field equations can be derived

$$F(R)R_{\mu\nu}-\frac{1}{2}f(R)g_{\mu\nu}-\nabla_\mu\nabla_\nu F(R)+g_{\mu\nu}\Box F(R)=kT_{\mu\nu} \qquad (2)$$

For $\mu,\nu=1,2,3,4,5$

Where $F(R)=\frac{df(R)}{dR}$, $\Box=\nabla^\mu\nabla_\nu$ (3)

Here ∇_μ represents the covariant derivative, $k=8\pi$ is coupling constant in gravitational units and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from L_m

After contraction of the field equation, we get

$$F(R)R - \frac{5}{2} f(R) + 4 \square F(R) = 8\pi T \qquad (4)$$

In vacuum when $T = 0$, the equation turn out to be

$$F(R)R - \frac{5}{2} f(R) + 4 \square F(R) = 0 \qquad (5)$$

This is the relationship between $f(R)$ and $F(R)$

For $R=R_0$, **Eqns** (4) given as following

$$F(R_0)R_0 - \frac{5}{2} f(R_0) = 8\pi T \qquad (6)$$

$$\square F(R_0) = 0 \quad , \quad F(R_0)=\text{constant}=R_0$$

This condition is constant scalar curvature for the non-vacuum case

From field **Eqns** (4), we get

$$f(R) = \frac{2}{5}[F(R)R + 4\square F(R) - 8\pi T] \qquad (7)$$

Using this value of $f(R)$ in non-vacuum field **Eqns** (2), get

$$F(R)R_{\mu\nu} - \frac{1}{5}[F(R)R + 4\square F(R) - 8\pi T] - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \square F(R) = 8\pi T_{\mu\nu}$$

In above equation, the term on the left hand side is independent of index μ, so we can write

$$K_\mu = \frac{1}{g_{\mu\nu}}\left[F(R)R_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) - 8\pi T_{\mu\nu}\right] \qquad (8)$$

Here K_μ is used to represent the traced quantity.

Metric & Field equations

We consider five dimensional cylindrical space-time as following

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2(d\theta^2 + \alpha^2 dz^2 + \beta^2 du^2) \qquad (9)$$

Where A, B are function of r & α, β has dimension of $\frac{1}{r}$

The Ricci Scalar R has the form

$$R = \frac{\ddot{A}}{AB} - \frac{\dot{A}^2}{2A^2B} - \frac{3\dot{B}}{rB^2} - \frac{\dot{A}\dot{B}}{2AB^2} + \frac{6}{Br^2} + \frac{3\dot{A}}{ABr} \tag{10}$$

Where dot (.) denotes the derivatives with respect to r

Dust energy-momentum tensor is given as

$$T_{\mu\nu} = \rho u_\mu u_\nu \tag{11}$$

Where $u_\mu = \delta_\mu^0$ is five-velocity in co-moving coordinate and ρ is density of dust matter.

Its components

$$T_{11} = T_{22} = T_{33} = T_{44} = 0 \quad T_{55} = \rho$$

Eqns (8) is independent of index μ. So $K_\mu - K_\nu = 0$ for all $\mu, \nu = 1,2,3,4,5$

We get following two independent equations

$$\frac{3\dot{A}F}{2ABr} + \frac{3\dot{B}F}{2B^2r} - \frac{\ddot{F}}{B} + \frac{\dot{A}\dot{F}}{2AB} + \frac{\dot{B}\dot{F}}{2B^2} - \frac{8\pi\rho}{A} = 0 \tag{12}$$

$$\frac{\ddot{A}F}{2AB} - \frac{\dot{A}\dot{B}F}{4AB^2} + \frac{\dot{A}F}{ABr} - \frac{\dot{A}^2F}{4A^2B} - \frac{2F}{Br^2} + \frac{\dot{B}F}{2B^2r} + \frac{\dot{A}\dot{F}}{2AB} - \frac{\dot{F}}{Br} - \frac{8\pi\rho}{A} = 0 \tag{13}$$

Solution of the field equation:

In this section, we solve field equation with the assumption of constant scalar curvature. For dust matter, the conservation law of energy momentum tensor $T^{\nu}_{\mu;\nu} = 0$ gives $A = A_0$ = constant. Thus the system of field **Eqns** (12) & (13) is reduced to unknown's values of F, ρ and B with the following two non-linear differential equation

$$\frac{3\dot{B}F}{2B^2r} - \frac{\ddot{F}}{B} + \frac{\dot{B}\dot{F}}{2B^2} - \frac{8\pi\rho}{A_0} = 0 \tag{14}$$

$$-\frac{2F}{Br^2}+\frac{\dot{B}F}{2B^2r}-\frac{\dot{F}}{Br}-\frac{8\pi\rho}{A_0}=0 \qquad (15)$$

Now system of equation reduces to two with two unknowns. Subtracts **Eqns** (15) from (14), it follows that

$$\frac{3\dot{B}F}{2B^2r}-\frac{\ddot{F}}{B}+\frac{\dot{B}\dot{F}}{2B^2}+\frac{2F}{Br^2}-\frac{\dot{B}F}{2B^2r}+\frac{\dot{F}}{Br}=0 \qquad (16)$$

For constant curvature $R=R_0$, $F(R_0)$=constant=R_0 it is apparent that the first and second derivative of $F(R)=\frac{df(R)}{dR}$ will always reduce to $\dot{F}(R_0)=\ddot{F}(R_0)=0$

yields the field equation as

$$\frac{\dot{B}}{B}+\frac{2}{r}=0 \qquad (17)$$

Integrate **Eqns** (17), we get

$$B=\frac{a}{r^2} \qquad (18)$$

Where a are constant,

Inserting in **Eqns** (14), we obtain

$$\rho=\frac{-3F(R_0)A_0}{8\pi a} \quad , \quad A_0>0 \;\; a>0 \qquad (19)$$

Which is purely constant,

The solution with constant curvature takes the forms

$$ds^2=A_0(r)dt^2-\frac{a}{r^2}dr^2-r^2(d\theta^2+\alpha^2dz^2+\beta^2du^2) \qquad (20)$$

By using **Eqns** (10), the scalar curvature R_0 becomes

$$R_0 = \frac{12}{a} \tag{21}$$

Substituting the values of R_0 *and* ρ in **Eqns** (6), we get Ricci scalar curvature function $f(R)$

$$f(R_0) = \frac{6}{a}\dot{f}(R_0) \tag{22}$$

Generalized Landau–Lifshitz energy momentum complex:

We explore energy density for obtained solution having constant scalar curvature.
The generalized landau–lifshitz EMC will be used. Here energy momentum complex is used only for these solution exhibiting constant scalar curvature.

The generalized Landau–Lifshits EMC is given by

$$\tau^{\mu\nu} = \dot{f}(R_0)\tau_{LL}^{\mu\nu} + \frac{1}{6k}\{\dot{f}(R_0)R_0 - f(R_0)\}\frac{\partial}{\partial x^i}\left(g^{\mu\nu}x^i - g^{\mu i}x^\nu\right) \tag{23}$$

We take $\mu, \nu = 5$

Energy density is represented by 55–component and it is as following,

$$\tau^{55} = \dot{f}(R_0)\tau_{LL}^{55} + \frac{1}{6k}\{\dot{f}(R_0)R_0 - f(R_0)\}\left(\frac{\partial}{\partial x^\lambda}g^{55}x^\lambda + 3g^{55}\right) \tag{24}$$

$$\tau^{55} = \dot{f}(R_0)\tau_{LL}^{55} + \frac{1}{6k}\{\dot{f}(R_0)R_0 - f(R_0)\}\left(\frac{r\dot{A}}{A^2} + \frac{3}{A}\right)$$

where $x^\lambda \frac{\partial}{\partial x^\lambda}g^{55} + 3g^{55} = r\frac{\partial}{\partial r}\left(\frac{1}{A}\right) + \frac{3}{A} = \frac{r\dot{A}}{A^2} + \frac{3}{A}$

Where τ_{LL}^{55} represents the sum of energy–momentum tensor and the energy–momentum pseudo tensor is given by

$$\tau_{LL}^{55} = (-g)\left(T^{55} + t_{LL}^{55}\right) \tag{25}$$

and

$$t^{55}_{LL} = \frac{1}{2k}\begin{bmatrix} \left(2\Gamma^{\gamma}_{\alpha\beta}\Gamma^{\sigma}_{\gamma\sigma} - \Gamma^{\gamma}_{\alpha\sigma}\Gamma^{\sigma}_{\beta\gamma} - \Gamma^{\gamma}_{\alpha\gamma}\Gamma^{\sigma}_{\beta\sigma}\right)\left(g^{5\alpha}g^{5\beta} - g^{55}g^{\alpha\beta}\right) \\ + g^{5\alpha}g^{\beta\gamma}\left(\Gamma^{5}_{\alpha\sigma}\Gamma^{\sigma}_{\beta\gamma} + \Gamma^{5}_{\beta\gamma}\Gamma^{\sigma}_{\alpha\sigma} - \Gamma^{5}_{\sigma\gamma}\Gamma^{\sigma}_{\alpha\beta} - \Gamma^{5}_{\alpha\beta}\Gamma^{\sigma}_{\gamma\sigma}\right) \\ + g^{5\alpha}g^{\beta\gamma}\left(\Gamma^{5}_{\alpha\sigma}\Gamma^{\sigma}_{\beta\gamma} + \Gamma^{5}_{\beta\gamma}\Gamma^{\sigma}_{\alpha\sigma} - \Gamma^{5}_{\sigma\gamma}\Gamma^{\sigma}_{\alpha\beta} - \Gamma^{5}_{\alpha\beta}\Gamma^{\sigma}_{\gamma\sigma}\right) \\ + g^{\alpha\beta}g^{\gamma\sigma}\left(\Gamma^{5}_{\alpha\gamma}\Gamma^{5}_{\beta\sigma} - \Gamma^{5}_{\alpha\beta}\Gamma^{5}_{\sigma\gamma}\right) \end{bmatrix}$$

(26)

Solving **Eqns** (26) , t^{55}_{LL} takes the form

$$t^{55}_{LL} = \frac{-3}{4\pi A_0 r^4}\left[\frac{\alpha^2\beta^2 + \alpha^2 + \beta^2}{\alpha^2\beta^2}\right] \qquad (27)$$

Substitute values of T^{55} & t^{55}_{LL} in **Eqns** (25),

$$\tau^{55}_{LL} = A_0 r^4\, a\, \alpha^2\beta^2\left[\frac{3}{4\pi A_0 r^4}\left(\frac{\alpha^2\beta^2 + \alpha^2 + \beta^2}{\alpha^2\beta^2}\right) - \frac{\rho}{A_0^2}\right] \qquad (28)$$

Inserting the 55-component of **Eqns** (23), it follows

$$\tau^{55} = \dot{f}(R_0)A_0 r^4\, a\, \alpha^2\beta^2\left[\frac{3}{4\pi A_0 r^4}\left(\frac{\alpha^2\beta^2 + \alpha^2 + \beta^2}{\alpha^2\beta^2}\right) - \frac{\rho}{A_0^2}\right] + \frac{3}{48\pi A_0}\left\{\dot{f}(R_0)R_0 - f(R_0)\right\} \qquad (29)$$

Is the equation for Energy density satisfying the condition of constant $R = R_0$.

We calculate energy density for different well known $f(R)$ models with the validity and stability condition.

First Model: $f(R) = R + \varepsilon R^2$ (Noakes 1983)

Where ε as a positive real number.it is also called the inflation model realized by R^2

The model has stability criteria which is bounded to $\varepsilon > 0$ i.e. $\ddot{f}(R) = 2\varepsilon > 0$.Einsteins theory is retrieved if $\varepsilon = 0$.

The 55-component of the generalized EMC becomes

$$\tau^{55} = \left(1 + \frac{24\varepsilon}{a}\right)A_0 r^4\, a\, \alpha^2\beta^2\left[\frac{3}{4\pi A_0 r^4}\left(\frac{\alpha^2\beta^2 + \alpha^2 + \beta^2}{\alpha^2\beta^2}\right) - \frac{\rho}{A_0^2}\right] + \frac{9\varepsilon}{a^2\pi A_0} \qquad (30)$$

This model fulfilled the condition for constant $R = R_0, \frac{1}{2\varepsilon} > 0$

Second Model: we assume the model given as $f(R) = R - \frac{\mu^2}{R}$ (Capozziello et.al 2003)

Where μ is a positive integer.This model is first dark energy model, called the Carroll-Duvuri-Tordden-Turnner model.

The 55-component of the dark energy takes form

$$\tau^{55} = A_0 r^4 a\, \alpha^2\beta^2\left(1+\frac{a^2\mu^4}{144}\right)\left[\frac{3}{4\pi A_0 r^4}\left(\frac{\alpha^2\beta^2+\alpha^2+\beta^2}{\alpha^2\beta^2}\right)-\frac{\rho}{A_0^2}\right]+\frac{a^2\mu^4}{1152\pi A_0} \qquad (31)$$

This model also satisfies the constant scalar curvature and stability condition $a^2=\frac{12}{\mu^2}$ and $\mu^2>0$

Third Model: $f(R)=R-\frac{a_0}{R}-b_0R^2$ (Nojiri and Odintsov 2003)

Where a_0 and b_0 are real number. This model holds the negative power of the curvature which lead to the cosmic acceleration of the expanding universe. However these terms might not be able to satisfy the stability condition and can be significantly improved by adding square terms of the scalar curvatur

$$\tau^{55} = A_0 r^4 a\, \alpha^2\beta^2\left(1+\frac{a^2a_0}{144}-\frac{24b_0}{a}\right)\left[\frac{3}{4\pi A_0 r^4}\left(\frac{\alpha^2\beta^2+\alpha^2+\beta^2}{\alpha^2\beta^2}\right)-\frac{\rho}{A_0^2}\right]+\frac{1}{16\pi A_0}\left[\frac{a\,a_0}{6}-\frac{144\,b_0}{a^2}\right]$$

(32)

The condition for stability, i.e. $\ddot{f}(R)\le 0$ yields $a^2a_0+1728b_0\ge 0$ for $a_0\ge b_0$ above becomes model acceptable.

Discussion and Conclusion

In this paper, we investigated five dimensional Cylindrically Symmetric Solutions with dust matter in f(R) theory gravity. We solved field equations in cylindrical symmetry with the assumption of constant scalar curvature and obtained the solution. Landau-Lifshitz energy momentum complex is used to study the energy density of the obtained solution. We calculate energy density for three different well known models with the validity and stability condition. Model having negative power of the scalar curvature support to cosmic acceleration. These models may be useful to study dark energy and dark matter stages of the universe.

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