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ON FUZZY NEUTROSOPHIC SUPRA REGULARBAIRE SPACES

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Abstract : In this paper the concept of fuzzy neutrosophic supra Regular open (closed) sets, fuzzy neutrosophic supra regular nowhere dense sets are introduced and studied. By means of fuzzy neutrosophic supra regular nowhere dense set, the concepts of fuzzy neutrosophic supra regular Baire space are introduced and characterization of fuzzy neutrosophic supra regular Baire space are studied.

IndexTerms - Fuzzy neutrosophic supra Regular open set, Fuzzy neutrosophic supra Regular closed set, Fuzzy neutrosophic supra regular dense set, Fuzzy neutrosophic supra regular nowhere dense set, Fuzzy neutrosophic supra regular first and second category, Fuzzy neutrosophic supra regular Baire spaces.

1. INTRODUCTION

In 1984, Mashhour.et.al [5] introduced the concept of supra topological spaces. In 1987, Abd E-Monsef et.al. [1] introduced Fuzzy supra topological space and defined basic notion of fuzzy supra topological space.
Arockiarani I, Sumathi, and MartinaJency [2] introduced the idea of fuzzy neutrosophic supra sets with the concept of fuzzy neutrosophic topological spaces. In their study, Poongothai and Thangaraj [6] investigated fuzzy settings on supra σ-Baire spaces, while Poongothai and Priya [7] studied fuzzy supra semi Baire spaces.
Building upon Smarandache.F in 2005 [11] Neutrosophic Supra Set, A Generalization of the Intuitionistic Fuzzy Sets. Poongothai and Celine [8] investigated fuzzy supra regular Baire spaces. Salama and Alblowi [10] investigated neutrosophic supra sets and neutrosophic topological spaces.
Through our investigation of neutrosophic supra Regular Baire spaces on fuzzy neutrosophic supra topological spaces, we aim to contribute to the development of the theory of fuzzy neutrosophic supra Regular Baire spaces and provide a foundation for further research in this area

2. PRELIMINARIES

Definition 2.1. [2] A fuzzy neutrosophic supra set A on the universe of discourse X is defined as, $A = \{x, T_A(x), I_A(x), F_A(x)\}$, $x \in X$ where $T, I, F : X \to [0, 1]$ and $0 \le T_A(x)+I_A(x)+F_A(x) \le 3$. With the condition $0 \le T_{A^*}(x)+I_{A^*}(x)+F_{A^*}(x) \le 2$.

Definition 2.2. [2] A fuzzy neutrosophic supra set A is a subset of a fuzzy neutrosophic supra set B. (i.e.,) $A \subseteq B$ for all x if $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$.

Definition 2.3. [2] Let X be a non-empty set, and $A=\{x, T_A(x), I_A(x), F_A(x)\}$, $B=\{x, T_B(x), I_B(x), F_B(x)\}$ be two fuzzy neutrosophic supra sets. Then

$A \cup B = \{x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\}$

$A \cap B = \{x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\}$

Definition 2.4. [2] The difference between two fuzzy neutrosophic supra sets A and B is defined as $A\backslash B(x)=\{x, \min(T_A(x), F_B(x)), \min(I_A(x), 1-I_B(x)), \max(F_A(x), T_B(x))\}$.

Definition 2.5. [2] A fuzzy neutrosophic supra set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0$, $I_A(x) = 0$, $F_A(x) = 10$ for all $x \in X$. It is denoted by 0_N.

Definition 2.6. [2] A fuzzy neutrosophic supra set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1$, $I_A(x) = 1$, $F_A(x) = 0$ for all $x \in X$. It is denoted by I_N.

Definition 2.7. [2] The complement of a fuzzy neutrosophic set A is denoted by A^c and is defined as $A^c = \{x, T^c{}_A(x), I^c{}_A(x), F^c{}_A(x)\}$, where $T^c{}_A(x) = F_A(x)$, $I_A{}^c(x) = 1 - I_A(x)$, $F_A{}^c(x) = T_A(x)$. The complement of fuzzy neutrosophic set A can also be defined as $A^c = 1_N - A$.

Definition 2.8. [2] A fuzzy neutrosophic topology on a non-empty set X is a τ_N of fuzzy neutrosophic sets in X Satisfying the following axioms.

(i). $0_N, 1_N \in \tau$.

(ii). $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.

(iii). $\cup A_i \in \tau$ for any arbitary family $\{A_i : i \in J\} \in \tau$.

In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic supra set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9. [2] The complement A_c of a fuzzy neutrosophic supra set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set inX.

Definition 2.10. [2] Let (X, T^*) be a fuzzy neutrosophic supra topological space and λ be any fuzzy supra set in (X, T^*). The interior and the closure of λ are defined respectively as follows:

(i). $int^*(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T^*\}$;

(ii). $cl^*(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1-\mu \in T^*\}$.

3. FUZZY NEUTROSOPHIC SUPRA REGULAR OPEN SETS

Definition 3.1. Let (X_{N^*}, T_{N^*})be a fuzzy neutrosophic supra topological space. A fuzzy neutrosophic supra set A_{N^*}, is called a fuzzy neutrosophic supra regular open set if $int^*(cl^*(A_{N^*})) = A_{N^*}$ and a fuzzy neutrosophic supra set A_{N^*} is called a fuzzy neutrosophic supra regular closed set if $cl^*(int^*(A_{N^*})) = A_{N^*}$.

Proposition 3.1. Let A_{N^*} be a fuzzy neutrosophic supra set in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then A_{N^*} is a fuzzy neutrosophic supra regular closed set if and only if $A_{N^{*c}}$ is a fuzzy neutrosophic supra regular closet set.

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular open set

$$int^*(cl^*(A_{N^*})) = A_{N^*}$$
$$(int^*(cl^*(A_{N^*})))^c = A_{N^{*c}}$$
$$cl^*(cl^*(A_{N^*}))^c = A_{N^{*c}}$$
$$cl^*(int^*(A_{N^{*c}})) = A_{N^{*c}}$$

$A_{N^{*c}}$ is a fuzzy neutrosophic supra regular closed set.

Conversely, Let $A_{N^{*c}}$ be a fuzzy neutrosophic supra regular closed set

$$cl^*(int^*(A_{N^{*c}})) = A_{N^{*c}}$$
$$(cl^*(int^*(A_{N^{*c}})))^c = (A_{N^{*c}})^c$$
$$int^*(int^*(A_{N^{*c}}))^c = A_{N^*}$$
$$int^*(cl^*(A_{N^{*c}})^c) = A_{N^*}$$
$$int^*(cl^*(A_{N^*})) = A_{N^*}.$$

$A_{N^{*c}}$ is a fuzzy neutrosophic supra regular open set

Proposition 3.2. Every fuzzy neutrosophic supra regular open set is fuzzy neutrosophic supra open set.

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular open set in (X_{N^*}, T_{N^*}) Then $int^*(cl^*(A_{N^*})) = A_{N^*}$. By the definition [2.10] of fuzzy neutrosophic supra interior, $int^*(A_{N^*})$ is the largest fuzzy neutrosophic supra open set contained in A_{N^*}. $int^*(cl^*(A_{N^*}))$ is the largest fuzzy neutrosophic supra open set contained $cl^*(A_{N^*})$. Hence, A_{N^*} is a fuzzy neutrosophic supra open set.

Proposition 3.3. Every fuzzy neutrosophic supra regular closed set is fuzzy neutrosophic supra closed set.

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular closed set in (X_{N^*}, T_{N^*}). Then $cl^*(int^*(A_{N^*})) = A_{N^*}$. By the definition [2.10] of fuzzy neutrosophic supra closure, $cl^*(A_{N^*})$ is the smallest fuzzy neutrosophic supra closed set containing A_{N^*}, $cl^*(int^*(A_{N^*}))$ is the smallest fuzzy neutrosophic supra closed set containing $int^*(A_{N^*})$. Hence, A_{N^*} is a fuzzy neutrosophic supra closed set.

Proposition 3.4.

(1). The fuzzy neutrosophic supra closure of a fuzzy neutrosophic supra open set is a fuzzy neutrosophic supra regular closed set.
(2). The fuzzy neutrosophic supra interior of a fuzzy neutrosophic supra closedset is a fuzzy neutrosophic supra regular open set.

Proof. (1). Let A_{N^*} be a fuzzy neutrosophic supra open set in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}). By the definition [2.10] of fuzzy neutrosophic supra interior,

$$int^*((A_{N^*})) \leq A_{N^*}$$
$$int^*(cl^*(A_{N^*})) \leq cl^*(A_{N^*})$$
$$\Longrightarrow cl^*(int^*(cl^*(A_{N^*}))) \leq cl^*(cl^*(A_{N^*}))$$
$$\Longrightarrow cl^*(int^*(cl^*(A_{N^*}))) \leq cl^*((A_{N^*})) \ldots\ldots(1)$$

Since

$$A_{N^*} \leq cl^*(A_{N^*})$$
$$int^*(A_{N^*}) \leq int^*(cl^*(A_{N^*}))$$
$$A_{N^*} \leq int^*(cl^*(A_{N^*}))$$
$$\Longrightarrow cl^*(A_{N^*}) \leq cl^*(int^*(cl^*(A_{N^*}))) \ldots\ldots(2)$$

From (1) and (2)

$cl^*(int^*(cl^*(A_{N^*}))) = cl^*(A_{N^*})$ fuzzy neutrosophic supra closure of a fuzzy neutrosophic supra open set is a fuzzy neutrosophic supra regular closed set.

(2). Let A_{N^*} be a fuzzy neutrosophic supra closed set in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}). By the definition [2.10] of fuzzy neutrosophic supra closure,

$$A_{N^*} \leq cl^*(A_{N^*})$$
$$int^*(A_{N^*}) \leq cl^*(int^*(A_{N^*}))$$
$$\Longrightarrow int^*(int^*(A_{N^*})) \leq int^*(cl^*(int^*(A_{N^*})))$$
$$\Longrightarrow int^*(A_{N^*}) \leq int^*(cl^*(int^*(A_{N^*}))) \ldots\ldots(3)$$

Since,

$$int^*(A_{N^*}) \leq A_{N^*}$$
$$cl^*(int^*(A_{N^*})) \leq cl^*(A_{N^*})$$
$$\Longrightarrow cl^*(int^*(A_{N^*})) \leq A_{N^*}$$
$$\Longrightarrow int^*(cl^*(int^*(A_{N^*}))) \leq int^*(A_{N^*}) \ldots\ldots(4)$$

From (3) and (4), $int^*(cl^*(int^*(A_{N^*}))) = int^*(A_{N^*})$ fuzzy neutrosophic supra interior of a fuzzy neutrosophic supra closed set is a fuzzy neutrosophic supra regular open set.

Definition 3.2. Let (X_{N^*}, T_{N^*}) be a fuzzy neutrosophic supra topological space and A_{N^*} be a fuzzy neutrosophic supra set in X_{N^*}, then the fuzzy neutrosophic supra regular closure and fuzzy neutrosophic supra regular interior of A_{N^*} is denoted and defined respectively as

$$rcl^*(A_{N^*}) = \wedge \{B_{N^*} | B_{N^*} \text{ is a fuzzy neutrosophic supra regular closed set in } X_{N^*} \text{ and } A_{N^*} \leq B_{N^*}\},$$
$$rint^*(A_{N^*}) = \vee \{B_{N^*} | B_{N^*} \text{ is a fuzzy neutrosophic supra regular open set in } X_{N^*} \text{ and } B_{N^*} \leq A_{N^*}\}.$$

Proposition 3.5. For any fuzzy neutrosophic supra set A_{N^*} in a fuzzy neutrosophicsupra topological space (X_{N^*}, T_{N^*}). Then

(1). $[rint^*(A_{N^*})]^c = rcl^*(A_{N^*}{}^c)$
(2). $[rcl^*(A_{N^*})]^c = rint^*(A_{N^*}{}^c)$

Proof.

(1). Consider $rint^*(A_{N^*}) = \vee\{B_{N^*} | B_{N^*}$ is a fuzzy neutrosophic supra regular set in $A_{N^*}\}$
$[rint^*(A_{N^*})]^c = 1 - \vee\ \{B_{N^*} | B_{N^*}$ is a fuzzy neutrosophic supra regular open set in X_{N^*} and $B_{N^*} \le A_{N^*}\}$
$= \wedge\ \{B_{N^*}{}^c | B_{N^*}{}^c$ is a fuzzy neutrosophic supra regular closed set in X_{N^*} and $B_{N^*}{}^c \ge A_{N^*}{}^c\}$
$= rcl^*(A_{N^*}{}^c)$.

(2). $rcl^*(A_{N^*}) = \wedge\ \{B_{N^*} | B_{N^*}$ *is* a fuzzy neutrosophic supra regular closed set in X_{N^*} and $A_{N^*} \le B_{N^*}\}$
$[rint^*(A_{N^*})]^c = 1 - \wedge\ \{B_{N^*} | B_{N^*}$ *is* a fuzzy neutrosophic supra regular closed set in X_{N^*} and $A_{N^*} \le B_{N^*}\}$
$= \vee\{B_{N^*}{}^c | B_{N^*}{}^c$ is a fuzzy neutrosophic supra regular open set in X_{N^*} and $A_{N^*}{}^c \ge B_{N^*}{}^c\} = rint^*(A_{N^*}{}^c)$.

Proposition 3.6. For any two fuzzy neutrosophic supra set A_{N^*} and B_{N^*} in a fuzzy neutrosophicsupra topological space (X_{N^*}, T_{N^*}) Then if $A_{N^*} \le B_{N^*}$ implies

(1). $rint^*(A_{N^*}) \le rint^*(B_{N^*})$
(2). $rcl^*(A_{N^*}) \le rcl^*(B_{N^*})$

Proof.

(1). $rint^*(A_{N^*}) = \vee\ \{C_{N^*} | C_{N^*}$ *is a fuzzy neutrosophic supra regular open set in* X_{N^*} *and* $C_{N^*} \le C_{N^*}\}$
since $rint^*(A_{N^*})$ *is the largest fuzy neutrosophic supra regular open set cointained in* A_{N^*}
$rint^*(A_{N^*}) \le A_{N^*}$, $A_{N^*} \le B_{N^*}$ implies $rint^*(A_{N^*}) \le A_{N^*} \le B_{N^*}$ implies $rint^*(A_{N^*}) \le B_{N^*}$, $rint^*(A_{N^*})$ is a fuzzy neutrosophic supra regular open setcontained in B_{N^*}. But $rint^*(B_{N^*})$ is the largest fuzzy neutrosophic supra regular open set contained in B_{N^*}, $rint^*(A_{N^*}) \le rint^*(B_{N^*})$.

(2). $A_{N^*} \le B_{N^*}$ if and only if $A_{N^*}{}^c \ge B_{N^*}{}^c$. Consider $B_{N^*}{}^c \le A_{N^*}{}^c$, $rint^*(B_{N^*})^c \le rint^*(A_{N^*})^c$, $(rcl^*(B_{N^*}))^c \le (rcl^*(A_{N^*}))^c$, $rcl^*(A_{N^*}) \le rcl^*(B_{N^*})$

4. FUZZY NEUTROSOPHIC SUPRA REGULAR NOWHERE DENSE SETS

Definition 4.1. Let (X_{N^*}, T_{N^*}) be a fuzzy neutrosophic supra topological Space. A fuzzy neutrosophic supra set A_{N^*} in (X_{N^*}, T_{N^*}) is called a fuzzy neutrosophic supra regular nowhere dense set if there exists no non-zero fuzzy neutrosophic supra regular open set B_{N^*} in (X_{N^*}, T_{N^*})such that $B_{N^*} < cl^*(A_{N^*})$. That is, $int^*cl^*(A_{N^*}) = 0$, in (X_{N^*}, T_{N^*}).

Example 4.1. Let X_{N^*} = {a, b, c}. The fuzzy neutrosophic supra sets A_{N^*}, B_{N^*}, C_{N^*} *and* D_{N^*} are defined on X_{N^*} as follows:

$A_{N^*} : X_{N^*} \to [0,1]$ defined as $A_{N^*}(a)$=0.5; $A_{N^*}(b)$=0.7; $A_{N^*}(c)$=0.3

$B_{N^*} : X_{N^*} \to [0,1]$ defined as $B_{N^*}(a)$=0.6; $B_{N^*}(b)$=0.5; $B_{N^*}(c)$=0.8

$C_{N^*} : X_{N^*} \to [0,1]$ defined as $C_{N^*}(a)$=0.4; $C_{N^*}(b)$=0.4; $C_{N^*}(c)$=0.7

$D_{N^*} : X_{N^*} \to [0,1]$ defined as $D_{N^*}(a)$=0.7; $D_{N^*}(b)$=0.3; $D_{N^*}(c)$=0.6

Then $T_{N^*} = \{0, A_{N^*}, B_{N^*}, C_{N^*}, D_{N^*}, (A_{N^*} \vee B_{N^*}), (C_{N^*} \vee D_{N^*}), (A_{N^*} \vee D_{N^*}), (A_{N^*} \vee C_{N^*}), (B_{N^*} \vee D_{N^*}), (A_{N^*} \wedge B_{N^*}), (C_{N^*} \wedge D_{N^*}),$ $(A_{N^*} \wedge D_{N^*}), (A_{N^*} \wedge C_{N^*}), (B_{N^*} \wedge D_{N^*}), 1\}$ is a fuzzy neutrosophic supra topological on X_{N^*}. The non-zero fuzzy neutrosophic supra regular open sets in (X_{N^*}, T_{N^*}) are A_{N^*}, B_{N^*}, C_{N^*}, D_{N^*}, $(A_{N^*} \vee B_{N^*})$, $(C_{N^*} \vee D_{N^*})$, $(A_{N^*} \vee D_{N^*})$, $(A_{N^*} \vee C_{N^*})$, $(B_{N^*} \vee D_{N^*})$, $(A_{N^*} \wedge B_{N^*})$, $(C_{N^*} \wedge D_{N^*})$, $(A_{N^*} \wedge D_{N^*})$, $(A_{N^*} \wedge C_{N^*})$, $(B_{N^*} \wedge D_{N^*})$. Now the fuzzy neutrosophic supra sets $int^*cl^*(A_{N^*}) = A_{N^*} \neq 0$ and $int^*cl^*(C_{N^*}) = int^*[1 - (A_{N^*} \vee B_{N^*})] = 0$ in (X_{N^*}, T_{N^*}). Hence A_{N^*} is not a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}) and C_{N^*} in a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*})

Remark 4.1.

1. If A_{N^*} and B_{N^*} are fuzzy neutrosophic supra regular nowhere dense sets in a fuzzy neutrosophic supra topological Space (X_{N^*}, T_{N^*}) then $A_{N^*} \vee B_{N^*}$ need not be a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

2. The complement of a fuzzy neutrosophic supra regular nowhere dense set ina fuzzy neutrosophic supra topological Space need not be a fuzzy neutrosophic supra regular nowhere dense set. For consider the following example.

Example 4.2. Let X_{N^*} = {a, b, c}. The fuzzy neutrosophic supra sets A_{N^*}, B_{N^*}, *and* C_{N^*} are defined on X_{N^*} as follows:
$A_{N^*} : X_{N^*} \to [0,1]$ defined as $A_{N^*}(a)$=0.3; $A_{N^*}(b)$=0.1; $A_{N^*}(c)$=0.2
$B_{N^*} : X_{N^*} \to [0,1]$ defined as $B_{N^*}(a)$=0.7; $B_{N^*}(b)$=0.4; $B_{N^*}(c)$=0.1
$C_{N^*} : X_{N^*} \to [0,1]$ defined as $C_{N^*}(a)$=0.1; $C_{N^*}(b)$=0.2; $C_{N^*}(c)$=0.7
Then $T_{N^*} = \{0, A_{N^*}, B_{N^*}, C_{N^*}, (B_{N^*} \vee C_{N^*}), (B_{N^*} \wedge C_{N^*}), 1\}$ is a fuzzy neutrosophic supra topological on X_{N^*} . On Computation, one can see that fuzzy neutrosophic supra regular nowhere dense sets in (X_{N^*}, T_{N^*}) are $1 - B_{N^*}$, $1 - C_{N^*}$ and $1 - (B_{N^*} \vee C_{N^*})$. Now $(1 - B_{N^*}) \vee (1 - C_{N^*}) = 1 - (B_{N^*} \wedge C_{N^*})$ and $int^*cl^*[(1 - B_{N^*}) \vee (1 - C_{N^*})] = int^*(1 - (B_{N^*} \wedge C_{N^*}) = A_{N^*} \neq 0$ and hence $(1 - B_{N^*}) \vee (1 - C_{N^*})$ is not a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}) and $1 - B_{N^*}$ is a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}) where as A_{N^*} is not afuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

Definition 4.2. A fuzzy neutrosophic supra set A_{N^*} in a fuzzy neutrosophic supra topological Space $(X_{N^*}, T_{N^*})$is called a fuzzy neutrosophic supra regular dense if there exists no fuzzy neutrosophic supra regular closed set B_{N^*} in (X_{N^*}, T_{N^*}) suchthat $A_{N^*} < B_{N^*} < 1$. That is, $cl^*(A_{N^*}) = 1$, in (X_{N^*}, T_{N^*})

Proposition 4.1. If A_{N^*} is a fuzzy neutrosophic supra regular nowhere dense setin a fuzzy neutrosophic supra topological Space (X_{N^*}, T_{N^*}), then $1 - A_{N^*}$ is a fuzzy neutrosophic supra regular dense set in (X_{N^*}, T_{N^*}).

Proof. Let A_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Then $int^*cl^*(A_{N^*}) = 0$ in (X_{N^*}, T_{N^*}). Now $1 - int^*cl^*(A_{N^*}) = 1 - 0 = 1$ and hence $cl^*int^*(1 - A_{N^*}) = 1$ in (X_{N^*}, T_{N^*}). But $cl^*int^*(1 - A_{N^*}) \leq cl^*(1 - A_{N^*})$ implies that $1 \leq cl^*(1 - A_{N^*})$. That is, $cl^*(1 - A_{N^*}) = 1$ in (X_{N^*}, T_{N^*}). Therefore $1 - A_{N^*}$ is a fuzzy neutrosophic supra dense set in (X_{N^*}, T_{N^*}).

Proposition 4.2. If A_{N^*} is a fuzzy neutrosophic supra regular closed set with $int^*(A_{N^*}) = 0$, in a fuzzy neutrosophic supra topological Space (X_{N^*}, T_{N^*}), then A_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set (X_{N^*}, T_{N^*}).

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular closed set with $int^*(A_{N^*}) = 0$, in (X_{N^*}, T_{N^*}). Then $cl^*(A_{N^*}) = A_{N^*}$ and $int^*(A_{N^*}) = 0$ and hence $int^*cl^*(A_{N^*}) = int^*(A_{N^*}) = 0$ in (X_{N^*}, T_{N^*}),. Therefore A_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set (X_{N^*}, T_{N^*}).

Proposition 4.3. If A_{N^*} is a non-zero fuzzy neutrosophic supra set in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then $A_{N^*} \vee int^*cl^*int^*A_{N^*} \leq cl^*A_{N^*}$, in (X_{N^*}, T_{N^*}).

Proof. Since $cl^*(A_{N^*})$ is a fuzzy neutrosophic supra regular closed set in (X_{N^*}, T_{N^*}), $int^*cl^*int^*(cl^*(A_{N^*})) \leq cl^*(A_{N^*})$. Then $int^*cl^*int^*(A_{N^*}) \leq int^*cl^*int^*(cl^*(A_{N^*}))$, $A_{N^*} < cl^*(A_{N^*})$ and hence $A_{N^*} \vee int^*cl^*int^*(A_{N^*}) \leq cl^*(A_{N^*})$ in (X_{N^*}, T_{N^*}).

Proposition 4.4. If A_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set in a fuzzy neutrosophic supra regular topological space (X_{N^*}, T_{N^*}), then $int^*(A_{N^*}) = 0$ in (X_{N^*}, T_{N^*}).

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}) . Then $int^*cl^*(A_{N^*})$ in (X_{N^*}, T_{N^*}). Now $int^*(A_{N^*}) \leq int^*cl^*(A_{N^*})$, implies that $int^*(A_{N^*}) \leq 0$, in (X_{N^*}, T_{N^*}). That is $int^*(A_{N^*}) = 0$ in (X_{N^*}, T_{N^*}).

Proposition 4.5. If $A_{N^*} \leq B_{N^*}$ and B_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set in fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then A_{N^*} is also a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

Proof. Now $A_{N^*} \leq B_{N^*}$, implies that $cl^*(A_{N^*}) \leq int^*cl^*(B_{N^*})$ in (X_{N^*}, T_{N^*}). Since $B_{N^*}$is a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). That is $int^*cl^*(A_{N^*}) = 0$. Then $int^*cl^*(A_{N^*}) \leq 0$ in (X_{N^*}, T_{N^*}). That is $int^*cl^*(A_{N^*}) = 0$ in (X_{N^*}, T_{N^*}). Therefore A_N* is a fuzzy neutrosophic supra regular nowhere denseset in (X_{N^*}, T_{N^*}).

Proposition 4.6. If either A_{N^*} or B_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set in fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then $A_{N^*} \wedge B_{N^*}$ is a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

Proof. $int^*cl^*(A_{N^*} \wedge B_{N^*}) \leq int^*[cl^*(A_{N^*}) \wedge cl^*(B_{N^*})] = int^*cl^*(A_{N^*}) \wedge int^*cl^*(B_{N^*})$ in (X_{N^*}, T_{N^*}). If either A_{N^*} or B_{N^*} is a fuzzy neutrosphic supra regular nowhere dense set in (X_{N^*}, T_{N^*}), then either $int^*cl^*(A_{N^*}) = 0$ or $int^*cl^*(B_{N^*}) = 0$. Then either $int^*cl^*(A_{N^*} \wedge B_{N^*}) \leq 0 \wedge int^*cl^*(B_{N^*}) = 0$ (or) $int^*cl^*(A_{N^*} \wedge B_{N^*}) \leq int^*cl^*(A_{N^*}) \wedge 0 = 0$. Therefore, $A_{N^*} \wedge B_{N^*}$ is a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

Proposition 4.7. If A_{N^*} is a fuzzy neutrosophic supra regular dense set in fuzzy neutrosophic supra topological space(X_{N^*}, T_{N^*}), than A_{N^*} is a fuzzy neutrosophic supra regular open set in(X_{N^*}, T_{N^*}).

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular dense set in (X_{N^*}, T_{N^*}). Then $cl^*(A_{N^*}) = 1$, in (X_{N^*}, T_{N^*}). Since $cl^*(A_{N^*}) \leq cl^*(A_{N^*}), 1 \leq cl^*(A_{N^*})$. That is $cl^*(A_{N^*}) = 1$, in (X_{N^*}, T_{N^*}). Now $cl^*int^*cl^*(A_{N^*}) = cl^*int^*(A_{N^*}) = cl^*(A_{N^*}) = 1$ and hence $A_{N^*} \leq cl^*int^*cl^*(A_{N^*})$ in (X_{N^*}, T_{N^*}). Therefore A_{N^*} is a fuzzy neutrosophic supra regular open set in (X_{N^*}, T_{N^*}).

Definition 4.3. A fuzzy neutrosophic supra set A_{N^*} in a fuzzy neutrosophic supratopological space (X_{N^*}, T_{N^*}). is called a fuzzy neutrosophic supra regular first category set if $A_{N^*} = \vee_{i=1}^{\infty}(A_{N_i^*})$ where $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Any other fuzzy neutrosophic supra set in (X_{N^*}, T_{N^*}). Is said to be fuzzy neutrosophic supra regular second category.

Definition 4.4. Let A_{N^*} be a fuzzy neutrosophic supra regular first category set in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}). Then $1 - A_{N^*}$ is called a fuzzy neutrosophic supra regular residual set in (X_{N^*}, T_{N^*}).

5.FUZZY NEUTROSOPHIC SUPRA REGULAR BAIRE SPACES

Definition 5.1. Let (X_{N^*}, T_{N^*}) be a fuzzy neutrosophic supra topological space. Then (X_{N^*}, T_{N^*}) is called a fuzzy neutrosophic supra regular Baire space $int^*(\vee_{i=1}^{\infty} (A_{N^*})_i) = 0$ where $(A_{N^*})_i$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

Example 5.1. Let $X_{N^*} = \{a, b, c\}$. The fuzzy neutrosophic supra sets $A_{N^*}, B_{N^*}, C_{N^*}, E_{N^*}, F_{N^*}, D_{N^*}$ *and* G_{N^*} are defined on X_{N^*} as follows:

$A_{N^*} : X_{N^*} \rightarrow [0,1]$ defined as $A_{N^*}(a)$=0.5; $A_{N^*}(b)$=0.4; $A_{N^*}(c)$=06
$B_{N^*} : X_{N^*} \rightarrow [0,1]$ defined as $B_{N^*}(a)$=0.4; $B_{N^*}(b)$=0.6; $B_{N^*}(c)$=0.3
$C_{N^*} : X_{N^*} \rightarrow [0,1]$ defined as $C_{N^*}(a)$=0.6; $C_{N^*}(b)$=0.5; $C_{N^*}(c)$=0.7
$D_{N^*} : X_{N^*} \rightarrow [0,1]$ defined as $D_{N^*}(a)$=0.5; $D_{N^*}(b)$=0.6; $D_{N^*}(c)$=0.6
$E_{N^*} : X_{N^*} \rightarrow [0,1]$ defined as $E_{N^*}(a)$=0.3; $E_{N^*}(b)$=0.1; $E_{N^*}(c)$=0.7
$F_{N^*} : X_{N^*} \rightarrow [0,1]$ defined as $F_{N^*}(a)$=0.6; $F_{N^*}(b)$=0.6; $F_{N^*}(c)$=0.7
$G_{N^*} : X_{N^*} \rightarrow [0,1]$ defined as $G_{N^*}(a)$=0.5; $G_{N^*}(b)$=0.5; $G_{N^*}(c)$=0.6

Then $T_{N^*} = \{0, A_{N^*}, B_{N^*}, C_{N^*}, E_{N^*}, F_{N^*}, D_{N^*}, G_{N^*}, 1\}$ is a fuzzy neutrosophic supra topological on X_{N^*} . On computations, one can see that the non-zero fuzzy neutrosophic supra regular open set in (X_{N^*}, T_{N^*}) are $A_{N^*}, C_{N^*}, 1 - B_{N^*}, E_{N^*}, 1 - E_{N^*}, D_{N^*}, G_{N^*}, 1$. Now the fuzzy neutrosophic supra regular nowhere dense set in(X_{N^*}, T_{N^*}) are $1 - D_{N^*}, 1 - G_{N^*}, 1 - F_{N^*}, B_{N^*}, 1 - A_{N^*}$. On computations, $int^*[(1 - D_{N^*}) \vee (1 - G_{N^*}) \vee (1 - F_{N^*}) \vee (B_{N^*}) \vee (1 - A_{N^*})] = int^*(1 - A_{N^*}) = 0$. Hence the fuzzy neutrosophic supra topological Space (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire Space.

Proposition 5.1. If $int^*(\vee_{i=1}^{\infty} (A_{N^*})_i) = 0$, where $(A_{N^*})_i = 0$ and $(A_{N^*})_i$'s are fuzzy neutrosophic supra regular closed sets in a fuzzy neutrosophic supra topologicalspace (X_{N^*}, T_{N^*}), then (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire space.

Proof. Let $(A_{N^*})_i$'s fuzzy neutrosophic supra regular closed sets in (X_{N^*}, T_{N^*}). Since $int^*(A_{N^*})_i = 0$, by preposition 4.2, the $(A_{N^*})_i$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Therefore $int^*(\vee_{i=1}^{\infty} (A_{N^*})_i) = 0$, where$(A_{N^*})_i$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}), implies that (X_{N^*}, T_{N^*}), is a fuzzy neutrosophic supra regular Baire Space.

Proposition 5.2. If $cl^*(\wedge_{i=1}^{\infty} (A_{N^*})_i) = 1$, where $(A_{N^*})_i$'s are fuzzy neutrosophic supra regular dense and fuzzy neutrosophic supra regular open sets in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire Space.

Proof. Now, $cl^*(\wedge_{i=1}^{\infty} (A_{N^*})_i) = 1$, implies $1 - cl^*(\wedge_{i=1}^{\infty} (A_{N^*})_i) = 0$. Then $int^*[1 - \wedge_{i=1}^{\infty} (A_{N^*})_i] = 0$ in (X_{N^*}, T_{N^*}). This implies that $int^*[\vee_{i=1}^{\infty} (1 - A_{N^*})_i] = 0$. since $(A_{N^*})_i$'s are fuzzy neutrosophic supra regular dense in (X_{N^*}, T_{N^*}), $cl^*(A_{N^*})_i = 1$ and $int^*((1 - A_{N^*})_i) = 1 - cl^*(A_{N^*})_i = 1 - 1 = 0$. Hence $int^*(\vee_{i=1}^{\infty} (1 - A_{N^*})_i) = 0$, where $int^*((1 - A_{N^*})_i) = 0$ and $(1 - A_{N^*})_i$'s are fuzzy neutrosophic supra regular closed sets in (X_{N^*}, T_{N^*}). Then by proposition 5.1, the fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire Space.

Proposition 5.3. Let (X_{N^*}, T_{N^*}) fuzzy neutrosophic supra topological space. Then the following are equivalent.

(1). (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire space.
(2). $int^*(A_{N^*}) = 0$ for every fuzzy neutrosophic supra regular first category set A_{N^*} in (X_{N^*}, T_{N^*}).
(3). $cl^*(B_{N^*}) = 1$ for every fuzzy neutrosophic supra regular residual set B_{N^*} in (X_{N^*}, T_{N^*}).

Proof. (1) ⇒ (2) Let A_{N^*} be a fuzzy neutrosophic supra regular first category set in (X_{N^*}, T_{N^*}). Then $A_{N^*} = \vee_{i=1}^{\infty}(A_{N^*_i})$ where $(A_{N^*})_i$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Now $int^*(A_{N^*}) = int^*(\vee_{i=1}^{\infty}(A_{N^*})_i)$. (Since$(X_{N^*}, T_{N^*})$. is a fuzzy neutrosophic supra regular Baire space). Therefore $int^*(A_{N^*}) = 0$, in(X_{N^*}, T_{N^*}).

(2) ⇒ (3) Let B_{N^*} be a fuzzy neutrosophic supra regular residual set in (X_{N^*}, T_{N^*}). Then, $1 - B_{N^*}$ is a fuzzy neutrosophic supra regular first category set in(X_{N^*}, T_{N^*}). By hypothesis, $int^*(1 - B_{N^*}) = 0$, in (X_{N^*}, T_{N^*}). This implies that $1 - cl^*(B_{N^*}) = 0$ and hence $cl^*(B_{N^*}) = 1$, in (X_{N^*}, T_{N^*}).

(3) ⇒ (1) Let A_{N^*} be a fuzzy neutrosophic supra regular first category set in(X_{N^*}, T_{N^*}). Then $A_{N^*} = \vee_{i=1}^{\infty}(A_{N^*_i})$, where $(A_{N^*})_i$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Since A_{N^*} is a fuzzy neutrosophic supra regular first category set in (X_{N^*}, T_{N^*}), $1 - A_{N^*}$ is a fuzzy neutrosophic supra regular residual set in (X_{N^*}, T_{N^*}). By hypothesis, $cl^*(1 - A_{N^*}) = 1$. Then $1 - int^*(A_{N^*}) = 1$, in (X_{N^*}, T_{N^*}). This implies that $int^*(A_{N^*}) = 0$, in (X_{N^*}, T_{N^*}). Hence $int^*(\vee_{i=1}^{\infty}(A_{N^*})_i) = 0$, where $(A_{N^*})_i$'s are fuzzy neutrosophic −supra regular nowhere dense in (X_{N^*}, T_{N^*}), implies that (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire space.

Definition 5.2. A fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}) is said to be of fuzzy neutrosophic supra regular first category space if $\vee_{i=1}^{\infty}(A_{N^*})_i = 1_X$, where $(A_{N^*})_i$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Otherwise (X_{N^*}, T_{N^*})will be called a fuzzy neutrosophic supra regular second category space.

Proposition 5.4. If a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire space, then $(X_{N^*}, T_{N^*})$is a fuzzy neutrosophic supra regular second category space.

Proof. Let (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire space. Then, $int^*(\vee_{i=1}^{\infty}(A_{N^*})_i) = 0$ where $(A_{N^*})_i$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Then $\vee_{i=1}^{\infty}(A_{N^*})_i \neq 1_X$ [otherwise $\vee_{i=1}^{\infty}(A_{N^*})_i = 1_X$, implies that where, $int^*(\vee_{i=1}^{\infty}(A_{N^*})_i) = , int^*(1_X) = 1_X \neq 0$, a contradiction]. Hence (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular second category space.

6. Conclusion

In this paper, we introduced and studied a new concept fuzzy neutrosophic supra Regular open set , fuzzy neutrosophic supra Regular closed set, fuzzy neutrosophic supra Regular nowhere dense set, fuzzy neutrosophic supra Regular dense set, fuzzy neutrosophic supra Regular Baire space, in fuzzy neutrosophic supra topological space. Some characteristics of fuzzy neutrosophic supra Regular Baire spaces are established

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