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INTERVAL VALUED INTUITIONISTIC PENTAGONAL FUZZY MATRIX GAME BY USING (α, β)- CUT

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Abstract: In this paper, a two person zero sum games with fuzzy payoffs are considered. The Payoff elements are taken to be Pentagonal intuitionistic fuzzy numbers. The aim of this paper is to develop a method to solve such games. Each Pentagonal intuitionistic fuzzy numbers using (α, β) – cuts. P. Grezewski's theorem [7] has been used to transform the obtained intervals to Grezewski's intervals. The intervals are then compared using acceptability index to get the saddle point.

Keywords: Two person zero-sum game, fuzzy payoff, Pentagonal Intuitionistic fuzzy number, (α, β) – cuts, Acceptability index, Saddle point.

Introduction:

Game theory is the study of the way in which strategies interactions among rational players produce outcomes with respect to the preference of those players, none of which might have intended by any of them. A two person game where two players are defined as decision makers is a simplest case of game theory. In real game situation, usually players are not able to evaluate exactly the of the game due to lack of information. Therefore, Fuzzy game theory is studied by various researchers now a days.

Atanassov [1] introduced the concept of an intuitionistic fuzzy set characterized by two functions expressing the degree of membership and the degree of non-membership respectively.which is mean to reflect the fact the degree of non-membership is not always equal to 1 minus the degree of membership, while there may be some hesitation degree. Pal and Nayak [12,14,17] worked on application of triangular Intuitionistic fuzzy number in bi-matrix games. In this paper we formulate a matrix game with the payoffs as pentagonal intuitionistic fuzzy numbers. Concept of (α, β) – cuts is used to convert the pentagonal intuitionistic fuzzy to intervals. P. Grezewski's Concept has been used to approximate the interval fuzzy numbers corresponding to pentagonal intuitionistic fuzzy numbers. A solution method is proposed for matrix game with saddle point. Numerical example has also been provided.

Preliminaries:

Definition: Intuitionistic fuzzy set

Let U=$\{x_1, x_2, x_3, \ldots, x_n\}$ be a finite universal set. An intuitionistic fuzzy set $\bar{A}$ in given universal set U is an object having the form

$\bar{A} = \{\langle x_i, \mu_{\bar{A}}(x_i), V_{\bar{A}}(x_i)\rangle : x_i \in U\}$

Where the function

$\mu_{\bar{A}}$: U→[0,1]; i,e $x_i \in U \quad \rightarrow \mu_{\bar{A}}(x_i) \in[0,1]$. And

$V_{\bar{A}}$: U→[0,1]; i,e $x_i \in U \quad \rightarrow V_{\bar{A}}(x_i) \in [0,1]$.

Define the degree of membership and degree of non-membership of an element $x_i \in U$, such that they satisfy the following conditions

$$0 \leq \mu_{\bar{A}}(x) \leq V_{\bar{A}}(x) \leq 1, \forall \; x_i \in U$$

This is known as intuitionistic condition. The degree of acceptance $\mu_{\bar{A}}(x)$ and non-acceptance $V_{\bar{A}}(x)$ can be arbitrary.

Definition: (α, β) – cuts intuitionistic fuzzy set

A set (α, β) – cuts generated by IFS$\bar{A}$, where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \leq 1$ is defined as

$\bar{A}_{\alpha,\beta} = \{x \in U: \mu_{\bar{A}}(x) \geq \alpha, V_{\bar{A}}(x) \leq \beta; \alpha, \beta \in [0,1]\}$

Where (α, β) – cut, denoted by $\bar{A}_{\alpha,\beta}$, is defined as the crisp set of elements x which belongs to $\bar{A}$ at least to the degree α and which does not belong to $\bar{A}$ at most to the degree β.

Definition: (Pentagonal Intuitionistic fuzzy number)

A pentagonal intuitionistic fuzzy number denoted by $\bar{A} = (2l, l, a, r, 2r; w_a, u_a)$ is a special intuitionistic fuzzy set on a real number set $\mathbb{R}$, whose membership function and non-membership function are defined as follows

$$\mu_{\bar{A}}(x) = \begin{cases} w_a(x-a+2l)/l & a-2l \leq x \leq a-l \\ w_a(x-a+l)/l & a-l \leq x \leq a \\ w_a(a+r-x)/r & a \leq x \leq a+r \\ w_a(a+2r-x)/r & a+r \leq x \leq a+2r \\ 0 & otherwise \end{cases}$$

$$V_{\bar{A}}(x) = \begin{cases} [(a-l-x)+u_a(x-a+2l)]/l & a-2l \leq x \leq a-l \\ [(a-x)+u_a(x-a+l)]/l & a-l \leq x \leq a \\ [(x-a)+u_a(a+2r-x)]/r & a \leq x \leq a+r \\ [(x-a-r)+u_a(a+2r-x)]/r & a+r \leq x \leq a+2r \\ 1 & otherwise \end{cases}$$

Where l and r are called spreads and a is called mean value. w_a and u_a represent the maximum degree of non- membership respectively such that they satisfy the condition

$0 \leq w_a \leq 1,\; 0 \leq u_a \leq 1,\; 0 \leq w_a + u_a \leq 1.$

It is easily shown that $\mu_{\bar{A}}(x)$ is convex and $V_{\bar{A}}(x)$ is concave for $x \in U$. Further,

For $a-2l \leq x \leq a-l$, $\mu_{\bar{A}}(x) + V_{\bar{A}}(x) = \frac{w_a(x-a+2l)}{l} + \frac{(a-l-x)+u_a(x-a+2l)}{l} = \frac{(a-x)}{l}(1-w_a-u_a) + (2u_a + 2w_a - 1)$. When $x = a-2l$, $\mu_{\bar{A}}(x) + V_{\bar{A}}(x)$=1, When $x = a-l$,

$\mu_{\bar{A}}(x) + V_{\bar{A}}(x) = w_a + u_a \leq 1.$

For $-l \leq x \leq a$, $\mu_{\bar{A}}(x) + V_{\bar{A}}(x) = \frac{w_a(x-a+2l)}{l} + \frac{(a-l-x)+u_a(x-a+2l)}{l} = \frac{(a-x)}{l}(1-w_a-u_a) + (u_a + w_a)$.When $x = a-l$, $\mu_{\bar{A}}(x) + V_{\bar{A}}(x)$=1, When $x = a$, $\mu_{\bar{A}}(x) + V_{\bar{A}}(x) = w_a + u_a \leq 1$.

For $a \leq x \leq a+r$, When $x = a$, $\mu_{\bar{A}}(x) + V_{\bar{A}}(x) = w_a + u_a \leq 1$. When $x = a+r$,

$\mu_{\bar{A}}(x) + V_{\bar{A}}(x) = \frac{a+r-a}{r}(1-w-u) + u + w$ =1.

For $a+r \leq x \leq a+2r$, $\mu_{\bar{A}}(x) + V_{\bar{A}}(x) = \frac{(x-a)}{r}(1-w-u) + (2w+2u-1)$, When $x = a + r$, $\mu_{\bar{A}}(x) + V_{\bar{A}}(x) = w_a + u_a \leq 1$. When $x = a + 2r$, $\mu_{\bar{A}}(x) + V_{\bar{A}}(x)=1$.

The quantity $\pi_{\bar{a}}(x) = 1 - \mu_{\bar{A}}(x) - V_{\bar{A}}(x)$, is called the measure of uncertainty.

Definition:(α, β) – cuts of a pentagonal intuitionistic fuzzy number.

A (α, β)- cut set of $\bar{A} = \langle 2l, l, a, r, 2r; w_a, u_a \rangle$ is a crisp subset of $\mathfrak{R}$ which is defined as

$\bar{A}_{\alpha,\beta} = \{x \in U: \mu_{\bar{A}}(x) \geq \alpha, V_{\bar{A}}(x) \leq \beta\}$

$0 \leq \alpha \leq w_a$, $u_a \leq \beta \leq 1$ and $0 \leq \alpha + \beta \leq 1$. A α- cut set of pentagonal fuzzy number $\bar{A}$ is a crisp subset of $\mathfrak{R}$, which is defined as

$\bar{A}_\alpha = \{x: \mu_{\bar{A}}(x) \geq \alpha\}$; $0 \leq \alpha \leq w_a$

According to the definition of pentagonal intuitionistic number it can be easily shown that

$\bar{A}_\alpha = \{x: \mu_{\bar{A}}(x) \geq \alpha\}$ is a closed interval, defied by $\bar{A}_\alpha = [a_{\mathbb{L}}(\alpha), a_{\mathbb{R}}(\alpha)]$

Where $a_{\mathbb{L}}(\alpha) = (a - 2l) + \frac{\alpha l}{w_a}$ and $a_{\mathbb{R}}(\alpha) = (a + 2r) - \frac{\alpha r}{w_a}$

Similarly a β cut set of a pentagonal intuitionistic fuzzy number $\bar{A} = \langle 2l, l, a, r, 2r; w_a, u_a \rangle$ is a crisp subset of $\mathfrak{R}$

$\bar{A}_\beta = \{x: V_{\bar{A}}(x) \leq \beta\}$; $u_a \leq \beta \leq 1$

It follows from the definition that $\bar{A}_\beta$ is a closed interval, denoted by $\bar{A}_\beta = [a_{\mathbb{L}}(\beta), a_{\mathbb{R}}(\beta)]$

Where $a_{\mathbb{L}}(\beta) = (a - 2l) + \frac{(1-\beta)l}{1-u_a}$ and $a_{\mathbb{R}}(\beta) = (a + 2r) - \frac{(1-\beta)r}{1-u_a}$.

It can be easily proven that for $\bar{A} = \langle 2l, l, a, r, 2r; w_a, u_a \rangle$ belonging to pentagonal intuitionistic fuzzy number $\mathfrak{R}$. And for any $\alpha \in [0, w_a]$ and $\beta \in [u_a, 1]$ where $0 \leq \alpha + \beta \leq 1$.

$\bar{A}_{\alpha,\beta} = \bar{A}_\alpha \wedge \bar{A}_\beta$

Transformation of PIFNs to Intervals.

To Study the transformation of triangular intuitionistic fuzzy numbers to intervals we use the definition of interval numbers which was proposed by moore[11], and defined as follows.

Definition: Interval Number

An interval number $\bar{A}$ is closed interval defined by

$\bar{A} = [a_{\mathbb{L}}, a_{\mathbb{R}}] = \{x \in \mathfrak{R}: a_{\mathbb{L}} \leq x \leq a_{\mathbb{R}}$; $\mathfrak{R}$ be the set of real numbers$\}$

The number $a_{\mathbb{L}}$ and $a_{\mathbb{R}}$ are called the lower and upper limit of the interval $\bar{A}$ respectively. An interval $\bar{A}$ alternatively represented in mean-width and center-radius form as

$\bar{A} = \langle \mathbb{M}_1(\bar{A}), \mathbb{M}_2(\bar{A}), \mathbb{M}_3(\bar{A}), \mathbb{M}_4(\bar{A}), \omega(\bar{A}) \rangle = \{x \in \mathfrak{R}: \mathbb{M}_1(\bar{A}) - \omega(\bar{A}) \leq x \leq \mathbb{M}_4(\bar{A}) + \omega(\bar{A})\}$

Where $\mathbb{M}_1(\bar{A}) = a_{\mathbb{L}} + w$, $\mathbb{M}_2(\bar{A}) = a_{\mathbb{L}} + \frac{w}{2}$, $\mathbb{M}_3(\bar{A}) = a_{\mathbb{R}} - \frac{w}{2}$, $\mathbb{M}_4(\bar{A}) = a_{\mathbb{R}} - w$, $\omega(\bar{A}) = \frac{a_{\mathbb{R}} - a_{\mathbb{L}}}{5}$ are respectively the mean points and one- fifth width of the interval $\bar{A}$. Actually, each real number can be regarded as an interval, such as $\forall\, x \in \mathfrak{R}$ an interval can be written as $[x, x]$, which has zero length.

The set of all interval number in $\mathfrak{R}$ denoted by $I(\mathfrak{R})$.As defined earlier, (α, β)- cuts are taken a pentagonal intuitionistic fuzzy number. Then, intervals corresponding $\alpha -$ cut and $\beta -$ cut are transformed to P.Grzegorzewski's intervals.

Let $\bar{A}$ be a pentagonal intuitionistic fuzzy number, I_α and I_β be the (α, β)- cuts of $\bar{A}$ then,

$I_\alpha = \langle \mathbb{M}_1, \mathbb{M}_2, \mathbb{M}_3, \mathbb{M}_4, \omega \rangle$, $I_\beta = \langle \mathbb{M}_1{}^I, \mathbb{M}_2{}^I, \mathbb{M}_3{}^I, \mathbb{M}_4{}^I, \omega^I \rangle$

Problem in consideration

If the player I has m strategies available to him and the player II has n strategies available to him. By the expression $\bar{A}_{ij} = \langle 2l, l, a, r, 2r; w_{a,} u_a \rangle$ we mean the pay-off that the player I play the row pure strategies $'i'$ and the player II play the column pure strategies $'j'$. Then the pay off for various strategies is represented by $m \times n$ pay-off matrix whose entries are pentagonal intuitionistic fuzzy numbers as

$$\bar{A}=\begin{bmatrix} \langle 2l_{11}, l_{11}, a_{11}, r_{11}, 2r_{11}; w_{a_{11}}, u_{a_{11}} \rangle & \langle 2l_{12}, l_{12}, a_{12}, r_{12}, 2r_{12}; w_{a_{12}}, u_{a_{12}} \rangle & \dots & \langle 2l_{1n}, l_{1n}, a_{1n}, r_{1n}, 2r_{1n}; w_{a_{1n}}, u_{a_{1n}} \rangle \\ \langle 2l_{21}, l_{21}, a_{21}, r_{21}, 2r_{21}; w_{a_{21}}, u_{a_{21}} \rangle & \langle 2l_{22}, l_{22}, a_{22}, r_{22}, 2r_{22}; w_{a_{22}}, u_{a_{22}} \rangle & \dots & \langle 2l_{2n}, l_{2n}, a_{2n}, r_{2n}, 2r_{2n}; w_{a_{2n}}, u_{a_{2n}} \rangle \\ \dots & \dots & \dots & \dots \\ \langle 2l_{m1}, l_{m1}, a_{m1}, r_{m1}, 2r_{m1}; w_{a_{m1}}, u_{a_{m1}} \rangle & \langle 2l_{m2}, l_{m2}, a_{m2}, r_{m2}, 2r_{m2}; w_{a_{m2}}, u_{a_{m2}} \rangle & \dots & \langle 2l_{mn}, l_{mn}, a_{mn}, r_{mn}, 2r_{mn}; w_{a_{mn}}, u_{a_{mn}} \rangle \end{bmatrix}$$

Thus we need a methodology too solve such a problem.

Methodology

Step: 1

Reduce all the pentagonal intuitionistic fuzzy numbers to intervals I_α, I_β by taking (α, β)- cuts.

Step: 2

Thus the given pay-off matrix is reduced to two pay-off matrices α- cut payoff matrix and β- cut payoff matrix, entries of which are I_α intervals and I_β intervals respectively, and then entries of α- cut payoff matrix and β- cut payoff matrix are transformed to mean-width or center-radius form.

Step: 3

For each of two payoff matrices, row minimum and column maximum is obtained by using acceptability index or otherwise.

Step: 4

In step-3, if there exist a saddle point for each of two payoff matrices obtained in step-2,then the saddle points must corresponds to the same point in the original game. The corresponding point is then called the saddle point of the game under the consideration.

Step: 5

If saddle point does not exist. The problem can be solved by converting to a linear programming problem.

Numerical Problem

Consider the problem $\bar{A}=\begin{bmatrix} \langle 8,4,120,6,12; 0.8,0.1 \rangle & \langle 10,5,160,4,8; 0.5,0.3 \rangle \\ \langle 20,10,80,3,6; 0.8,0.2 \rangle & \langle 30,15,100,2,4; 0.4,0.5 \rangle \end{bmatrix}$

Solution:

Let $\alpha = 0.4$ and $\beta = 0.5$

$a_{\mathbb{L}}(\alpha)=(a-2l)+\frac{\alpha l}{w_a}$,$a_{\mathbb{R}}(\alpha)= (a+2r)-\frac{\alpha r}{w_a}$

α – cut for pentagonal fuzzy number

$\bar{A}_\alpha=\begin{bmatrix} [114,129] & [154,164.8] \\ [65,84.5] & [85,102] \end{bmatrix}$

β – cut for pentagonal fuzzy number

$\bar{A}_\beta=\begin{bmatrix} [114.2,128.6] & [153.57,165.14] \\ [66.25,84.12] & [85,102] \end{bmatrix}$

After changed α – cut payoff matrix and β – cut payoff matrix are transformed to mean-width and center radius form as follows.

$\bar{A}_\alpha = \begin{bmatrix} \langle 129,121.5,121.5,114,15\rangle & \langle 156.16,155.08,163.72,162.64,2.16\rangle \\ \langle 68.9,66.95,82.55,80.6,3.9\rangle & \langle 88.4,86.7,100.3,98.6,3.4\rangle \end{bmatrix}$ and

$\bar{A}_\beta = \begin{bmatrix} \langle 117.08,115.64,127.16,125.72,2.88\rangle & \langle 155.88,154.72,163.98,162.82,2.314\rangle \\ \langle 69.824,68.03,82.33,80.54,3.57\rangle & \langle 88.4,86.7,100.3,98.6,3.4\rangle \end{bmatrix}$

For α – cut pay off matrix, $\bar{a}_{11}$ and $\bar{a}_{21}$ are the minimum of row 1 and 2 respectively and $\bar{a}_{11}$ and $\bar{a}_{21}$ are the maximum of column 1 and 2 respectively. Therefore $\bar{a}_{11}$ is the saddle point. So the corresponding value $\langle 129,121.5,121.5,114,15\rangle$ is the fuzzy game value of the matrix $\bar{A}_\alpha$.

$\bar{a}_{11}$=$\langle 129,121.5,121.5,114,15\rangle$ corresponds to $\langle 8,4,120,6,12; 0.8,0.1\rangle$ in the original game.

For β – cut pay off matrix, $\bar{a}_{11}$ and $\bar{a}_{21}$ are the minimum of row 1 and 2 respectively and $\bar{a}_{11}$ and $\bar{a}_{21}$ are the maximum of column 1 and 2 respectively. Therefore $\bar{a}_{11}$ is the saddle point. So the corresponding value $\langle 117.08,115.64,127.16,125.72,2.88\rangle$ is the fuzzy game value of the matrix $\bar{A}_\beta$.

$\bar{a}_{11}$=$\langle 117.08,115.64,127.16,125.72,2.88\rangle$ corresponds to $\langle 8,4,120,6,12; 0.8,0.1\rangle$ in the original game

So, the two game values are same and hence $\langle 8,4,120,6,12; 0.8,0.1\rangle$ is the saddle point of the considered game. And, For the different values of α and , we will get the same saddle point.

Example

Consider the problem $\bar{A} = \begin{bmatrix} \langle 20,10,100,5,10; 0.8,0.2\rangle & \langle 10,5,60,6,12; 0.6,0.3\rangle \\ \langle 14,7,90,8,16; 0.4,0.6\rangle & \langle 30,15,150,10,20; 0.5,0.3\rangle \end{bmatrix}$

Solution:

Let $\alpha = 0.3$ and $\beta = 0.7$

$a_{\mathbb{L}}(\alpha)=(a-2l)+\frac{\alpha l}{w_a}$,$a_{\mathbb{R}}(\alpha)= (a+2r)-\frac{\alpha r}{w_a}$

α – cut for pentagonal fuzzy number

$\bar{A}_\alpha = \begin{bmatrix} [83.75,108.12] & [52.5,69] \\ [81.25,100] & [129,164] \end{bmatrix}$

β – cut for pentagonal fuzzy number

$\bar{A}_\beta = \begin{bmatrix} [83.75,108.12] & [52.14,69.43] \\ [81.25,100] & [120.64,169.6] \end{bmatrix}$

After changed α – cut payoff matrix and β – cut payoff matrix are transformed to mean-width and center radius form as follows.

$\bar{A}_\alpha = \begin{bmatrix} \langle 88.62,86.18,105.68,103.25,4.87\rangle & \langle 55.8,54.15,67.35,65.7,3.3\rangle \\ \langle 85,83,125,98.12,96.25,3.75\rangle & \langle 136,132.5,160.5,157,7\rangle \end{bmatrix}$ and

$\bar{A}_\beta = \begin{bmatrix} \langle 88.62,86.18,105.68,103.25,49\rangle & \langle 55.59,53.86,67.7,65.98,3.45\rangle \\ \langle 85,83.12,98.12,96.25,3.75\rangle & \langle 130.43,125.53,164.71,159.8,9.79\rangle \end{bmatrix}$

For α – cut pay off matrix, $\bar{a}_{12}$ and $\bar{a}_{21}$ are the minimum of row 1 and 2 respectively and $\bar{a}_{11}$ and $\bar{a}_{21}$ are the maximum of column 1 and 2 respectively. Therefore saddle point does not exist.

For β – cut pay off matrix, $\bar{a}_{12}$ and $\bar{a}_{21}$ are the minimum of row 1 and 2 respectively and $\bar{a}_{11}$ and $\bar{a}_{22}$ are the maximum of column 1 and 2 respectively. Therefore saddle point does not exist.

Conclusion: A matrix game with payoff as pentagonal intuitionistic fuzzy numbers is considered. A new Approach to solve such a problem proposed. The proposed method finds the saddle point of the game matrix by converting the pentagonal intuitionistic fuzzy number to interval by taking (α, β)- cuts, hich are then used to convert the given problem to two different problem with entries as crisp intervals. Saddle point of the two problem is obtained by finding the row minimum and column maximum by acceptability index approach. The two obtained saddle point corresponds to same pentagonal intuitionistic fuzzy number.it is also noted that different values of α and β will corresponds to the same saddle point.

The proposed method can be applied extensively in the field of game theory as much of the information in the real world is rarely precisely known.

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