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A STUDY ON LIMIT OF A FUNCTION OF SINGLE VARIABLE

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ABSTRACT:

This article would give theoretical knowledge, applicable knowledge and constructive knowledge to students by using different methods, formulae of limit of a function of single variable. Students can improve problem solving skills. In mathematics, limit of a function which is used in various fields such as Physics, Engineering, Biology and so on. The primary purpose of the limit of a function is the study of some properties of limit of a function of single variable and we can find the limits of different functions like, functions of two variables, complex functions, trigonometric functions, exponential functions, by applying different methods etc….

Key words: Definition of limit of a function of single variable, Types of limit of a function, Limit of a function properties, Indeterminate forms, L' Hospital Rules, Formulae and Methods etc.,

INTRODUCTION:

My aim of this study is about formation of limit of a function of single variable, from where the students' learning of limits could be described. Learning is measured by the students' development as seen through their responses to various tasks. In the theory of limit of a function derivatives depend on the notion of limits. Study of limit of a function is one of the most important topic in mathematics due to its widespread application in multidisciplinary fields, like measuring the strength of magnetic field, electric field etc. Let the function y=f(x) where x is independent variable y is dependent variable.

DEFINITION OF LIMIT OF A FUNCTION OF SINGLE VARIABLE

- A function f is said to be tends to a limit a, as x tends to a, if for each positive integer ε, there corresponds a positive number δ, such that for every x∈[a- δ , a+ δ] that implies │f(x)-a│<Ɛ

It is denoted by $\lim_{x\to a} f(x)$=A

- The right hand limit of a function is the value of the function tends to when the variable tends to its *lim*it from right.

It is denoted by $\lim_{x\to a+f(x)}$ =A^{+}

- The left hand limit of a function is the value of the function tends to when the variable tends to its limit from left.

It is denoted by $\lim_{x\to a-f(x)}$ =A^{-}

- The limit of a function exists if the left-hand limit is equals to the right-hand limit.

i.e., $\lim_{x\to a-f(x)}$ = $\lim_{x\to a+f(x)}$ =A

INDETERMINATE FORM

Indeterminate form is a mathematical expression that can be obtained by any value depending on circumstances. In limit of a function, it is usually possible to compute the limit of sum, difference, product, quotient or power of two functions by taking the combination of the separate limits of each respective function.

INDETERMINATE FORMS LIST

INDETERMINATE FORMS
• $0/0$ • 0^0 • ∞/∞ • $0 \times \infty$ • $\infty - \infty$ • 1^∞ • ∞^0

L'HOSPITAL RULE

L'Hospital Rule is a general method of evaluating indeterminate forms like 0/0, , etc. To evaluate the limits of indeterminate forms for the derivation in calculus, L'Hospital Rule can be applied more than once. We can apply this rule but still it holds any indeterminate form every time after its application. If the problem is out of the indeterminate form, we can't be able to apply L'Hospital Rule.

LIMIT OF A FUNCTION PROPERTIES

If limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exists then,

- **Law of Addition :** $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

- **Law of Subtraction :** $\lim_{x\to a}(f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
- **Law of Multiplication :** $\lim_{x\to a}(f(x).g(x)) = \lim_{x\to a} f(x) . \lim_{x\to a} g(x)$
- **Law of Division :** $\lim_{x\to a}(f(x)/g(x)) = \lim_{x\to a}$ f(x)/$\lim_{x\to a} g(x)$ where $\lim_{x\to a} g(x)\neq 0$
- **Law of Constant :** $\lim_{x\to a} c = c$
- **Law of Root :** $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$
- **Law of Power :** $\lim_{x\to a} f(x)^n = (\lim_{x\to a} f(x))^n$ Where n is an integer.

METHODOLOGY:

The important concept of limit of a function will be introduced for the continuity of function f at a, the values of a function f for the values of x near a is f(a). Now we are going to discuss the various methods used in obtaining limits of a given function. Each method will be accompanied by some examples illustrating methods such as direct substitution method, factorization method, rationalization method etc.

TYPES OF LIMIT OF A FUNCTON:

➢ **LIMIT OF A FUNCTION OF SINGLE VARIABLE**

A function of single variable x, that is f(x) is said to be tends to a limit A, as x tends to a, if for each positive integer ε, there corresponds a positive number δ, such that for every x∈[a- δ , a+ δ] that implies │f(x)-a│<Ɛ

It is denoted by $\lim_{x\to a} f(x)$=A

➢ **LIMIT OF A FUNCTION OF TWO VARIABLES**

A function of two variables x,y that is f(x,y) this function depends on two variables x and y.Then this function has the limit A as (x,y) →a for each ϵ>0 ,∃ δ >0 such that |f(x,y)−A| < ϵ when ever |(x,y)−a| < δ.

It is denoted by $\lim_{(x,y)\to a} f(x,y) = A$

➢ **LIMIT OF TRIGONOMETRIC FUNCTIONS**

There are important limit properties that are involved in trigonometric functions

- $\lim sinx = sina$
- $\lim_{x\to a} cosx = cosa$
- $\lim_{x\to a} tanx = tana$
- $\lim_{x\to a} cotx = cota$
- $\lim_{x\to a} secx = seca$ where a is real number

METHODS:

By applying different methods we can solve problems of limit of a function. Now here we are going to discus some methods and problems.

➢ DIRECT SUBSTITUTION METHOD

In this method for a continuous function, the limit can be obtained by direct substitution. If given function is in the form of polynomial that is linear polynomial, second degree polynomial, third degree polynomial and so on by direct substitution method we can calculate the limit of given function.

PROBLEM

- **FIND THE LIMIT OF A FUNCTION** $\lim_{x\to 5} 20x^2 + 3x + 1$

SOLUTION:

Given function is

$\lim_{x\to 5} 20x^2 + 3x + 1$ ________________(1)

By direct substitution method

Substitute x=5 in equation (1)

= $20(5)^2+3(5)+1$

= 20(25)+15+1

= 500+15+1

= 516

There fore limit of a given is 516.

➢ FACTORIZATION METHOD

If the function in the form of f(x)/g(x), then the limit is in indeterminate form i.e., 0/0,∞/∞, 0*∞, ∞ − ∞, 1^∞, 0^0 etc. Factorization leads to cancellation of that common factor and reduction of the limit determinate form.

PROBLEM

- **FIND THE LIMIT OF A FUNCTION:** $\lim_{X\to 1} \frac{X^4-3X+2}{X^5-4X+3}$

SOLUTION:

Given function is

= $\lim_{X\to 1} \frac{X^4-3X+2}{X^5-4X+3}$ ______________(1)

Substitute x=1 in equation (1)

= $\frac{(1)4-3(1)+2}{(1)^5-4(1)+3}$

= $\frac{1-3+2}{1-4+3}$

= $\frac{-2+2}{-3+3}$ $=\frac{0}{0}$ which is an indeterminate form

It can be solved by using factorization method OR by using L'Hospital Rule

If (X-1) is a factor of Both numerator and denominator by factor theorem of equation (1)

$$\lim_{X\to 1}\frac{X^4-3X+2}{X^5-4X+3} = \lim_{X\to 1}\frac{(x-1)(x^3+x^2+x-2)}{(x-1)(X^4+x^3+x^2+x-3)}$$

$$= \lim_{X\to 1}\frac{(x^3+x^2+x-2)}{(X^4+x^3+x^2+x-3)} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

Substitute x=1 in equation (2)

$$=\frac{(1^3+1^2+1-2)}{(1^4+1^3+1^2+1-3)}$$

$$= \frac{3-2}{4-3} = \frac{1}{1} = 1$$

$$= 1$$

There fore the limit of a given function is 1

- **RATIONALIZATION METHOD**

In this method, the rationalization of indeterminate form leads to determinate.

PROBLEM

- **FIND THE LIMIT OF A FUNCTION:** $\lim_{X\to O}\frac{cotxx^3}{1-cosx}$

SOLUTION:

Given function is

$$= \lim_{X\to O}\frac{cotxx^3}{1-cosx} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

Substitute x=0 in equation (1)

$$= \frac{cotx(0)(0)}{1-cos(0)} = \frac{0}{o}$$ which is an indeterminate form

Simplify the function by applying rationalization method

Rationalize equation (1) by (1+cosx)

$$= \frac{cotxx^3}{1-cosx}$$

$$=\frac{cotxx^3}{1-cosx}\left(\frac{1+cosx}{1+cos}\right)$$

$$= \frac{cosxx^3(1+cosx)}{sinx(1-cos^2x)}$$ **[$\sin^2$x+$\cos^2$x=1]**

$$= \frac{cosxx^3(1+cosx)}{sinxsin^2x}$$

$$= \frac{cosxx^3(1+cosx)}{sin^3x}$$

Now apply the limit to above equation

$$=\lim_{x\to o}\frac{cosxx^3(1+cosx)}{sin^3x}$$

By using the formula $\lim_{x\to o}(\frac{sinx}{x})^3$ **=1**

$$=\lim_{x\to o}\frac{x^3}{sin^3x}\lfloor\lim_{X\to O}(cosx+cos^2x)\rfloor$$

$$=1\lfloor cos(0)+cos^2(0)\rfloor(\text{as x is tends to 0})$$

$$=1\lfloor 1+1\rfloor$$

$$=2$$

Therefore limit of a given function is 2.

CONCLUSION:

By observing above methods we can calculate limit of a function of single variable. Students can remember the definition and formulae and by understanding concepts they will be able to apply formulae to evaluate the limit of a function of single variable problems, limit of a function definitions represent the functions approach to a limit but not equal to the limit. Students can improve their knowledge concept wise and application wise and they can identify difference between the structure of mathematics and attainment of the concepts via cognitive processes.

REFERENCES:

- https://byjus.com/jee/limits-of-functions/
- https://www.cuemath.com/jee/techniques-of-evaluating-limits-limits-continuity-differentiability/
- Shanti Narayan, P.K. Mittal Differential Calculus, S.Chand, New Delhi;
- https://sathee.prutor.ai/article/maths/maths-indeterminate-forms/
- Mathematical Analysis by S.C.Malik; Savita Arora, New Age International Publishers.