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A NEW FORM OF b-OPEN SET VIA FERMATEAN NEUTROSOPHIC SETS

[1] Joyce Brightlin.K, [2] Nirmala Irudayam.F

[1] PG Student, [2] Assistant Professor
[1] Department of Mathematics,
[1]Nirmala college for women, Coimbatore, India

Abstract : This article focuses majorly on introducing a new set named Fermatean Neutrosophic b-open set. This new set is introduced utilizing the key features of b-open sets in Fermatean Neutrosophic Topology. We have also proposed the idea of Fermatean b-interior and b-closure operators and thus proceed to form its corresponding topology. We also putforth the characteristics of this newly framed set along with its continuity.

***IndexTerms* - Fermatean Neutrosophic set; Fermatean Neutrosophic topological space; Fermatean Neutrosophic b-open set.**

INTRODUCTION

The neutrosophic structure, fuzzy sets and intuitionistic fuzzy sets and decision-making are important for many areas of mathematics and applied in the interesting field of study. Smarandache introduced the concept of neutrosophic structure and he extended the concepts of intuitionistic fuzzy sets to a deeper understanding and interpretation in terms of truthfulness, indeterminacy and falsity. He brought flexibility in neutrosophic sets.

NEED OF THE STUDY

The establishment To model the complex situations with high levels of uncertainty, indeterminacy and incomplete information in decision making process. Because it allows for a more nuanced representation within a topological space.

RESEARCH METHODOLOGY

The methodology section is by using algebraic topology techniques to analyze the properties that remain invariant under continuous deformations and representing data as simplical complexes.outline the plan and method that how the study is conducted. This includes Universe of the study, sample of the study,Data and Sources of Data, study's variables and analytical framework. The detailsare as follows;

3.1 Data Representation

Transforming data points allows for topological analysis by studying the connectivity and relationship between data points at different scales.

3.2 Topological invariants

We are Using topological invariants to charecterize the fundamental properties of a data set.

3.3 Fermatean neutrosophic set:

Let $y \neq \phi$. A fermatean neutrosophic set $A_f = \{(s, \phi_A(s), \psi_A(s), \zeta_A(s)) : s$ □ $Y\}$,

where $\phi_A, \psi_A, \zeta_A : Y \rightarrow [0, 1]$ and, for each s □ Y,

$0 \leq \phi_A^3(s) + \psi_A^3(s) \leq 1$ and $0 \leq \zeta_A^3(s) \leq 1$, then for each $s \in Y$, $0 \leq \phi_A^3(s) + \psi_A^3(s) + \zeta_A^3(s) \leq 2$. $\zeta_A(s)$ is an independent component; $\phi_A(s)$ and $\psi_A(s)$ are dependent components.

Interior set (A°):

Given a subset A of X, where X is a topological space, the interior of A is defined as union of all open sets contained in A.

Closure set (Ā):

Given a subset A of X, the closure of A is defined as the intersection of all closed sets containing A.

$Int(A) \leq A \leq Cl(A)$

3.4 Introduction to b-open sets:

Let J_b be a Fermatean neutrosophic subset on (U, τ_b)

- If $J_b \subseteq Cl_b(Int_b(J_b))$, it is said to be a Fermatean neutrosophic semi-open set. Besides, the complement of the Fermatean neutrosophic semi-open set is said to be the Fermatean neutrosophic semi-closed set.
- If $J_b \subseteq Cl_b(Int_b(Cl_b(J_b)))$, it is said to be a Fermatean neutrosophic β-open set. Additionally, the complement of a Fermatean neutrosophic β-open set is said to be a Fermatean neutrosophic β-closed set.
- If $J_b \subseteq Int_b(Cl_b(J_b))$, it is said to be Fermatean neutrosophic pre-open set.
  And if $J_b \subseteq (Cl_b(Int_b(J_b))) \cup (Int_b(Cl_b(J_b)))$ then it is said to be Fermatean neutrosophic b-open set .

3.4.1.Theorem:

Every fermatean neutrosophic open set is fermatean neutrosophic pre-open set.

Proof:

Let (X,τ) be a fermatean neutrosophic topological space and let $A \in \tau$ that is A is a fermatean neutrosophic open set in X.

<u>To prove</u>: A is an fermatean neutrosophic pre-open set.

We know that, the interior of A is defined as the union of all open sets contained in A. That is $A_0 \subseteq A$

Since A is a fermatean neutrosophic open set , $int_b(A)=A$

By the property of closure , $A \subseteq cl_b(A)$

Taking interior , $int_b(A) \subseteq int_b(cl_b(A))$

Therefore, $A \subseteq int_b(cl_b(A))$

That is A is an fermatean neutrosophic pre-open set.

Hence the proof.

Example:

Let (X,τ) be a fermatean neutrosophic topological space

$X=\{0,0.5,0.4,0.3,0.1,1\}$ and $\tau= X,\phi,\{0,1\},0.3,0.1$

Let $A=\{0,1,0.1\}$

To find $int_b(cl_b(A))$

$Cl_b(A)=X \cap \{0,0.1,0.4,0.5,1\}$

$=\{0,0.1,0.4,0.5,1\}$

Then $int_b(cl_b(A))=int_b(0,0.1,0.4,0.5,1)$

$=\phi,0,1,0.1$ which is contained in A

That is $A \subseteq int_b[cl_b(A)]$

Therefore A is an fermatean neutrosophic pre-open set.

Remark: Every fermatean neutrosophic pre-open set need not be fermatean neutrosophic open set.

3.4.2.Theorem:

Every fermatean neutrsophic open set is fermatean neutrosophic semi-open set.

Proof:

Let (X,τ) be a fermatean neutrosophic topological space and U□τ that is U is a fermatean neutrosophic open set in X.

To prove: U is an fermatean neutrosophic semi-open set.

Consider A is a subset of X , if U□A since cl(A)□cl_b(U)

Then,A□cl(A)

And hence U□cl_b(U)

Since U is an feramtean neutrosophic open set, then int_b(U)=U

Therefore, U□cl_b(int_b(U))

U is an feramtean neutrosophic semi-open set.

Example:

Let (X,τ) be a fermatean neutrosophic topological space

X={0,0.8,0.4,0.3,,1} and τ= X,ϕ,{0,1},0.8

Let U={0.8,0.3,0.4}

To find cl_b(int_b (U))

int_b(U)=ϕ,0.8

Complement of τ= ϕ,X,{0.3,0.4,0.8},{0,0.4,0.3,1}

Then cl_b(int_b (U))=cl_b(ϕ,0.8)

=X∩{0.3,0.4,0.8}

={0.3,0.4,0.8}

That is U□ cl_b(int_b (U))

Therefore U is an fermatean neutrosophic Semi-open set.

Remark: Every fermatean neutrosophic semi-open need not be fermatean neutrosophic open set.

3.4.3.Theorem:

Every fermatean Neutrosophic open set is a Fermatean Neutrosophic b-open set.

Proof:

Consider (X,τ) be a Fermatean Neutrosophic Topological space, U ϵτ (ie) U is a Fermatean Neutrosophic open set

To prove: U is a Fermatean Neutrosophic b-open set

We know that every fermatean neutrosophic open set is fermatean neutrosophic semi-open set, then

U□int_b[cl_b(U)]

Also, every fermatean neutrosophic open set is fermatean neutrosophic pre-open set, then

U□cl_b[int_b(U)]

Therefore, U □ (cl_b[int_b(U)]) □ (int_b[cl_b(U)])

That is U is an fermatean neutrosophic b-open set.

Hence the proof.

Example:

Let (X,τ) be a fermatean neutrosophic topological space and U□X such that U={u_1,u_2}

X={0,0.1,0.7,0.9,0.8,0.4,0.3,1} and

τ= {X,ϕ,{0,1},0,0.1,0.9,{1,0.7},{0,1,0.4}

Let A={(u_1,0.8,0.4,0.1),(u_2,0,1,0.4)}

Here $0\leq 0.8^3+0.4^3\leq 1$ and $0\leq 0^3+1^3\leq 1$

Also $0\leq 0.8^3+0.4^3+0.1^3\leq 2$ and $0\leq 0^3+1^3+0.4^3\leq 2$

int_b(A)=ϕ,0,{0,1},0.1,{0,1,0.4}

To find cl_b(U)

Complement of τ= ϕ,X,{0.1,0.4,0.7,0.8,0.9,1},{0,0.4,0.8,0.7,0.9,1},{0,0.1,0.4,0.7,0.8,1},{0.1,0.4,0.9,0.7,0.8},{0.8,0.9,0.1,0,0.4},{0.1,0.7,0.8,0.9}

Cl_b(A)=X∩{0,0.1,0.4,0.7,0.8,1}

={0,0.1,0.4,0.7,0.8,1}

Then cl_b(int_b (A))=cl_b(ϕ,0,0.1,{0,1},{0,1,0.4})

=X∩{0,0.1,0.4,0.7,1,0.8}
={0,0.1,0.4,0.7,1,0.8}

Int_b(cl_b(A))=int_b(0,0.1,0.4,0.7,0.8,1)

=ϕ,0,0.1,{0,1},{0,1,0.4}

That is cl_b(int_b (A))□ int_b(cl_b(A))=ϕ,0,0.1,{1,0,0.4},0.7,0.8

A□ cl_b(int_b (A))□ int_b(cl_b(A))

Therefore A is an fermatean neutrosophic b-open set.

Remark:

Every Fermatean Neutrosophic b-open set need not be a Fermatean Neutrosophic open set**.**

3.4.4. Theorem:

Every fermatean neutrosophic pre-open set is fermatean neutrosophic b-open set.

Proof:

Let X be a fermatean neutrosophic topological space and

let A be a subset of X

Consider, A is fermatean neutrosophic pre-open set

(ie.,) A□ int_b[cl_b(A)]

To prove: A is fermatean neutrosophic b-open set

For every x□A therexists an open set U such that

x□U□ cl_b[int_b(A)] since every fermatean neutrosophic open set is fermatean neutrosophic semi-open set

also U□ int_b[cl_b(A)] since every fermatean neutrosophic open set is fermatean neutrosophic pre-open set

then, U□(cl_b[int_b(A)])□ (int_b[cl_b(A)])

since x□A , A□(cl_b[int_b(A)])□ (int_b[cl_b(A)])

so, A is a fermatean neutrosophic b-open set

Therefore, every fermatean neutrosophic pre-open set is fermatean neutrosophic b-open set.

Example:

Let X={ 0,0.5,0.1,0.4,0.3,1} τ={X,ϕ,{0,1},0.3,0.1}

Consider A= {0,1,0.1} which is fermatean neutrosophic pre-open set

To prove: A is fermatean neutrosophic b-open set

$int_b[cl_b(A)]=\phi,0,1,0.1$

$int_b(A)=\phi,\{0,1\},0.1$

$cl_b[int_b(A)] = cl_b\{\phi,\{0,1\},0.1\}$

$= X\cap\{0,1,0.1,0.4,0.5\} = \{0,1,0.1,0.4,0.5\}$

A□($cl_b[int_b(A)]$)□ ($int_b[cl_b(A)]$)

Therefore, A is fermatean neutrosophic b-open set.

Remark:

Every fermatean neutrosophic b-open set need not be fermatean neutrosophic pre-open set.

3.4.5. Theorem:

The intersection of fermatean neutrosophic open and fermatean neutrosophic b-open set is fermatean neutrosophic b-open set.

Proof:

Let (X,τ) be a topological space , U is an open set in τ and let B be a fermatean neutrosophic b-open set

(ie,.) B□($cl_b[int_b(B)]$)□ ($int_b[cl_b(B)]$)

U∩B=U∩ □($cl_b[int_b(B)]$)□ ($int_b[cl_b(B)]$)

To prove: U∩B□($cl_b[int_b(U\cap B)]$)□ ($int_b[cl_b(U\cap B)]$)

Since U is open , U=int_bU

And B is fermatean neutrosophic b-open ,

B□($cl_b[int_b(B)]$)□ ($int_b[cl_b(B)]$)

$int_b(U\cap B)$□ $int_b(U)\cap int_b(B)$ [property of intersection]

□ U∩ $int_b(B)$

So, $int_b(U\cap B)$□U and

$int_b(U\cap B)$□ $int_b(B)$ [B contains its interior]

applying closure, $cl_b[int_b(U\cap B)]$□ $cl_b[int_b(B)]$

since B is fermatean neutrosophic b-open,

and because of U∩B□B

Therefore, U∩B□($cl_b[int_b(B)]$)□ ($int_b[cl_b(B)]$)

So, U∩B is a fermatean neutrosophic b-open set.

Hence the proof.

Remark: The intersection of two fermatean neutrosophic b-open set need not be fermatean neutrosophic b-open set.

Lemma:

Let (U, τ_b) be a Fermatean neutrosophic topological space, and J_b and k_b be Fermatean neutrosophic b-open subsets of U. However, $J_b\cap k_b$ is not a Fermatean neutrosophic b-open set.

Proof:

It is clearly obtained if J_b and k_b are chosen such that J_b □ $Cl_b(Int_b(J_b))$ □ $Int_b(Cl_b(J_b))$, k_b □ $Cl_b(Int_b(k_b))$□ $Int_b(Cl_b(k_b))$ and $J_b\cap S_b$ is not a Fermatean neutrosophic b-open set in a Fermatean neutrosophic topological space.

Definition:

- A function $f:(X,\tau)\rightarrow(Y,\square)$ is continuous if $f^{-1}(v)$ is closed in X for every closed set v in Y.
- A function f: $:(X,\tau)\rightarrow(Y,\square)$ is b-continuous if $f^{-1}(v)$ is b-closed in X for every closed set v in Y.

3.4.6.Theorem :

Every fermatean neutrosophic continuous map is fermatean neutrosophic b-continuous.

Proof:

Let v be closed set in $(Y,\square\square)$

Since f is fermatean neutrosophic continuous map, then

$f^{-1}(v)$ is fermatean neutrosophic closed set in (X,τ)

We know that every fermatean neutrosophic closed set is fermatean neutrosophic b-closed set.

Then, $f^{-1}(v)$ is fermatean neutrosophic b-closed in (X,τ)

Hence, f is fermatean neutrosophic b-continuous.

Definition:

Let J_b be a Fermatean neutrosophic subset on (U, τ_b). If $J_b \square Cl_b(Int_b(J_b)) \square Int_b(Cl_b(J_b))$ it is named a Fermatean neutrosophic b-open set. In addition, the complement of a Fermatean neutrosophic b-open set is said a Fermatean neutrosophic b-closed set.

Lemma:

Let J_b be a Fermatean neutrosophic subset on (U, τ_b)

- For all $i \in N$, if $\{J_{bi}\}$ is a Fermatean neutrosophic b-open set, then $(\cup J_{bi})$ is a Fermatean neutrosophic b-open set.
- For all $i \in N$, if $\{J_{bi}\}$ is a Fermatean neutrosophic b-closed set, then $(\cap J_{bi})$ is a Fermatean neutrosophic b-closed set.

Proof:

Let J_{bi} be a family of Fermatean neutrosophic b-open sets. $J_{bi} \square Cl_b(Int_b(J_{bi}))\square Int_b(Cl_b(J_{bi})) \square \square J_{bi} \square Cl_b(Int_b(\square J_{bi}))\square Int_b(Cl_b(\square J_{bi}))$, where all $i \square N$. So, $\square J_{bi}$ is a Fermatean neutrosophic b-open set.

Note:

A subset of X is b-open if and only if it is a neighbourhood of each of its points.

Proposition:

If S_b be a Fermatean neutrosophic semi-open set on (U, τ_b),then this set is a Fermatean neutrosophic b-open set.

Proof:

Let (U,τ_b) be the fermatean neutrosophic topological space and to prove that S_b be a fermatean neutrosophic b-open set

that is $S_b\square Cl_b(Int_b(S_b))\square Int_b(Cl_b(S_b))$

That is the union of fermatean neutrosophic semi-open and fermatean neutrosophic pre-open set

Where $Cl_b(Int_b(\square J_{bi}))$ is the fermatean neutrosophic semi-open set

We know that every fermatean neutrosophic semi-open set is fermatean neutrosophic b-open set but the converse may or may not be true.

4. Definition:

Let (U_1, τ_b), (U_2, τ_b) be Fermatean neutrosophic topological spaces. If the inverse image of all Fermatean neutrosophic open sets on U_2 is a Fermatean neutrosophic b-open set on U_1, then $ð_b : U_1 \rightarrow U_2$ is called a Fermatean neutrosophic b-continuous mapping.

4.1 Properties of fermatean neutrosophic b-continuous mapping:

- A map $f_b:(X,\tau_b)\rightarrow(Y,\tau_b)$ is called fermatean neutrosophic b-continuous map if the inverse image of every closed set in Y is b-closed in X.
- Every fermatean neutrosophic continuous map is fermatean neutrosophic b-continuous map. But not conversely.

- If $f_\Psi :(X,\eth,0)\rightarrow(S,v,\Delta)$ is fermatean neutrosophic b-continuous, then following are equivalent
  - i. f_Ψ is fermatean neutrosophic b-continuous
  - ii. Inverse image of each fermatean neutrosophic b-closed set is fermatean neutrosophic b-closed
  - iii. $Cl_b\,(f_\Psi^{-1}(H,\Delta))\subseteq f_\Psi^{-1}(Cl_b(H,\Delta))$
  - iv. $f_\Psi(Cl_b(F,0))\subseteq Cl_b(f_\Psi(F,0))$
  - v. $f_\Psi^{-1}(int_b(H,\Delta))\subseteq int_b(f_\Psi^{-1}(H,\Delta))$

Note: pre-image of b-continuous mapping is also b-open.

4.1.1 Lemma:

Let (J,τ_b) and (K,τ_b) be fermatean neutrosophic topological space and

$\eth_b : J\rightarrow K$. If $\eth_b$ is a fermatean neutrosophic b-continuous mapping then for every A_b□ J, $\eth_b[int_b(cl_b(A_b)) \cap cl_b(int_b(A_b))]$□$cl_b(\eth_b(A_b)) \cap int_b(\eth_b(A_b))$

Proof:

For a fermatean neutrosophic subset of J,

A_b and $\eth_b$ are fermatean neutrosophic b-continuous mapping

$cl_b(\eth_b(A_b))$ is fermatean neutrosophic closed set on K then

$\eth_b^{-1}[\, cl_b(\eth_b(A_b))]$ is fermatean neutrosophic b-closed set on J

$int_b(cl_b(A_b)) \cap cl_b(int_b(A_b))= cl_b\,(int_b\,(cl_b(cl_b(A_b)))) \cap int_b(\, cl_b\,(int_b(int_b(A_b))))$

□ $cl_b\,(int_b\,(cl_b\,\eth_b^{-1}\,(cl_b\,\eth_b(A_b)))) \cap int_b(\, cl_b\,(int_b\,\eth_b^{-1}\,(int_b\,\eth_b(A_b))))$

□ $\eth_b^{-1}\,(cl_b\,(\eth_b(A_b))) \cap \eth_b^{-1}\,(int_b\,(\eth_b(A_b)))$

Thus $\eth_b[int_b(cl_b(A_b)) \cap cl_b(int_b(A_b))]$□$cl_b(\eth_b(A_b)) \cap int_b(\eth_b(A_b))$

4.2 Definition:

- Let (J,τ_b) ,(K,τ_b) be Fermatean neutrosophic topological spaces. If the inverse image of all Fermatean neutrosophic b-open sets on K is a Fermatean neutrosophic b-open in J, then $\eth_b : J\rightarrow K$ is called a Fermatean neutrosophic b-irresolute mapping.
- Let (J,τ_b) ,(K,τ_b) be Fermatean neutrosophic topological spaces. If the inverse image of all Fermatean neutrosophic b-open sets in K is a Fermatean neutrosophic open on J, then $\eth_b : J\rightarrow K$ is called a Fermatean neutrosophic strongly b-irresolute mapping.

4.2.1 Proposition:

Let (J,τ_b) ,(K,τ_b) be Fermatean neutrosophic topological spaces and

$\eth_b$: J□K. If $\eth_b$ is a Fermatean neutrosophic strongly b-irresolute mapping then for every fermatean neutrosophic b-closed set on K , $\eth_b^{-1}$ is a fermatean neutrosophic closed set on J.

Proof:

Let A_b be fermatean neutrosophic b-closed sets in K. So $CA_b=K\backslash A_b$ fermatean neutrosophic b-open sets in this space.

Using the mapping $\eth_b$ as fermatean neutrosophic strongly b-irresolute ,

$\eth_b^{-1}(CA_b)= \eth_b^{-1}(K\backslash A_b)$ is fermatean neutrosophic open in J.

4.3 Definition:

- Let A_b be a Fermatean neutrosophic set on (J,τ_b). In this case, the Fermatean neutrosophic b-interior and b-closure of A are defined by

  $int_b(A)=$□{O : O□A, O is a b-open fermatean neutrosophic sets}

  $cl_b(A)=\cap${C :A□C, C is a b-closed fermatean neutrosophic sets}

- Let (J,τ_b) and (K,τ_b) be Fermatean neutrosophic topological spaces. If the inverse image of all Fermatean neutrosophic pre-open sets on K is a Fermatean neutrosophic pre-open set on J, then $\eth_b : J\rightarrow K$ is called a Fermatean neutrosophic pre-continuous mapping.
- Let (J,τ_b) and (K,τ_b) be Fermatean neutrosophic topological spaces. If the image of all Fermatean neutrosophic open sets in J is a Fermatean neutrosophic pre-open set on K, then $\eth_b : J\rightarrow K$ is called a Fermatean neutrosophic pre-open.

Remark:

Let (J,τ_b) and (K,τ_b) be Fermatean neutrosophic topological spaces and $ð_b$: J□K is called a Fermatean neutrosophic b-open set, then $ð_b$ is a fermatean neutrosophic pre-open set.

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