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Analyzing the effectiveness of different methods in creating magic squares

By Aarini Roy and Ashwath Mahesh

Abstract:

Research Objective: The objective of this research is to examine various methods of constructing magic squares and understand which method is the best. A magic square is a unique set of numbers that is arranged in a grid pattern and has each row, column, and diagonal equal the same number. Magic squares have been used in puzzles, paintings, and math for centuries. We will investigate two popular methods: the Siamese method and the Strachey method. The study will compare them and identify which method is faster, easier to use, and most effective at producing magic squares. Through experimentation with the two methods, we are hoping to identify which method is best.

Personal Interest: We have chosen the topic of Magic squares and the different methods to construct them as we were curious to understand which method is the most efficient. Magic squares have been studied for centuries in number theory, combinatorics, and algebra. Comparing construction methods would help determine which algorithm is faster and more efficient allowing us to optimize the construction of magic squares and learn about algorithm designing.

I. Research Methodology

This study is comparative analysis between Strachey's and Siamese method. The research aims to find whether odd order squares or even order squares are easier to make using the 2 methods. To compare the two methods, a set of magic squares will be created by different mathematics teachers. The steps taken to construct these squares by everyone will be documented carefully, noting the time required and any challenges faced. The efficiency of each method will be assessed based on speed, accuracy, simplicity. The results will then be recorded in tables, comparing the time taken, accuracy, and difficulty level of each method.

II. Need for the study

This study allows us to analyze the different methods that exist to create the mathematical concept of magic squares. In doing so we are able to understand which method is suitable for certain different circumstances (such as odd or even squares) by understanding each of the selected methods through their advantages, disadvantages and unique methods for creating magic squares.

III. Research question

How do different construction algorithms for magic squares (e.g., Siamese method, Strachey method) compare in terms of efficiency?

I. Introduction

About magic squares

A magic square in math is a square containing different integers in which each row, column, and diagonal of numbers have the same total sum. The magic constant is the constant sum of the integers along one side (n), which is the order of the magic square. When the grid is n x n and contains each of the integers from 1-n square, it is a normal magic square. A magic square can be of any order except n=2(IJCM_Magic_Square_Construction_Algorithms34.pdf)

Magic squares do not have repeated entries, the ones that do are called trivial. An example of a trivial is the parker square, where the square of 41 has been repeated twice.

Note: To calculate the magic sum, add all numbers from 1 to n square and then divide by the number of rows.

History of magic squares

Magic squares have been around for a long time. Third order magic squares (3x3 square) date back to at least 190 BCE in China. Lo Shu Square is one of the first recorded instances of a magic square. Legend has it that those who lived became increasingly desperate during the Lo River floods and begged with the river gods to save them. They were rescued from the water by an unusual turtle that had a pattern engraved on its back. It consisted of nine squares on a 3x3 grid containing one of the digits 1–9 in each square. Regardless of the method used to add the three numbers in each row, column, and square diagonal, the total was always 15.

An encyclopedia from Baghdad about 983 CE called the Encyclopedia of the brethren of Purity (Rasa 'il Ikhwan al-Safa) on of the earliest Islamic texts, contains magic squares of order 3 to 9. Rasa 'il Ikhwan al-Safa was written by a secretive group of scholars known as the Ikhwan al-Safa and covered a wide range of subjects.

Magic squares have not only been used in a mathematical context but also in art. The first occurrence in Western art of a magic square is in an etching by Albrecht Dürer, named Melencolia I. Artists like Man Ray also incorporated magic squares into modern abstract art.

II. Different methods of solving

Magic squares have been researched for centuries, and with that, different ways of constructing them have been found. Some of the most popular ones are the Siamese method, Strachey's method, and the LUX method. All these other methods offer a different way of constructing these magic squares of different sizes.

Let us begin with the Siamese method, also referred to as the de la Loubére method. This method is extremely popular for constructing odd-order magic squares, which are magic squares where the number of rows and columns is an odd number. To perform this method, put the number 1 in the center of the top row of the square. Then, keep putting each successive number in a diagonal line, moving up and to the right. If these movements lead you out of the square, go around to the other side or if the space is already occupied, put the number directly below the last number. This method makes all rows, columns, and diagonals add up to the same magic constant number.

Strachey's method, however, is a variation which is better suited towards building a series of magic squares of any order, but specifically for even-numbered squares, when both rows and columns are even. The method is to divide the square into smaller 4x4 blocks and fill them systematically into each block to form the magic square. This method has the benefit of being more systematic and geometric compared to the rest, focusing on interchanging smaller square segments to ensure that the magic properties are duly maintained throughout the entire grid. It can be finicky, but it makes up for this by allowing the building of magic squares when other methods may not be so readily applied.

The birthday magic square is a unique magic square which can be created using anyone's birthday. Ramanujan created the first ever magic square as a birthday magic square from his date of birth (in DD MM YYYY format). The magic constant of this square is 139. However, this method is not always applicable for making magic squares, depending on the person birthdate it could also form a trivial.

Lastly let us look at the LUX method. This method is a modern algorithm that is used to create magic squares. It does so by employing a sequence of matrix transformations to ensure that the sum of each row, column, and diagonal is the same. In general, this method can be used to both odd and even-order squares. The Lux method uses a mathematical technique by using pre-determined patterns and formulae. By calculating these values through matrix manipulation, the method offers a systematic formula for building much larger squares efficiently. This method is less intuitive than the Siamese or Strachey's methods, but it makes up for this by being very effective at building squares with larger sizes.

III. Construction of magic squares using different methods:

Siamese method: (Constructing a 3x3 magic square)

Start by filling the middle cell in the first row as 1.

	1	

Write the consecutive integer (2) diagonally on top of 1.

2

	1	

Place the integer 2, 180 degrees clockwise

	1	
		2

Place 3 diagonal to 2

	1		
			3
		2	

Rotate the 3, 180 degrees clockwise

	1	
3		
		2

Since 1 is diagonal to 3, we write the next integer (4) down below

	1	
3		
4		2

Write the next consecutive integers, diagonally to 4

	1	6
3	5	
4		2

Next, we are supposed to write 7 diagonally to 6 and then rotate 180 degrees clockwise. But we have 2 already written in that space, hence we will write 7 directly below 6.

	1	6
3	5	7
4		2

Next, we will write 8 diagonally to 7, but since we can't flip it 180 degrees or write it below, we will write 8, 180 degrees anti clockwise

8	1	6
3	5	7
4		2

Next, we write 9 diagonally to 8 and then flip it 180 degrees.

8	1	6
3	5	7
4	9	2

All rows and columns, rows and diagonals add to 15

Strachey's method:

We will take the 4x4 grid as a sample to demonstrate this approach. It is to place numbers from 1 to 16 in the 4x4 grid so that the sum of numbers in any row, column, and diagonal is equal (magic constant).

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Step 1: first split the table into smaller 2x2 blocks

Block A:

1	2
5	6

Block B:

3	4
7	8

Block C:

9	10
13	14

Block D:

11	12
15	16

Step 2: Now what we do is take the groups of 2x2 cubes and rearrange them to create a magic square with a magic constant. We do this by some of these common exchanges:

- Exchanging the top-left and bottom-right blocks (A and D).
- Exchanging the top-right and bottom-left blocks (B and C).
- Rotating single blocks to adjust the fit.

When doing these rotations and exchanges make sure that the magic constant is maintained, when we finish, we would've made a 4x4 magic cube like this one:

1	2	15	16
12	13	4	5
8	9	6	7
14	11	10	3

And you're done.

Magic sum = 34

LUX method:

Step 1: Before we start make sure your squares order is and odd number like 3x3 or 5x5, for this example let's take 3x3

Step 2: create a clear gird, in this example well make it 3x3

Step 3: Start by placing 1 in the middle of the top row of the square

	1	

Step 4: Now from where you placed the first number move diagonally up and right, if you go out of the square you:

- If you go out from the top, then write the number in the bottom row
- If you move out of the right side go to the left column

So, then the next number (2) goes in the bottom right

	1	
		2

Step 5: Now we again move diagonally up and right from 2 which leads us outside, so we wrap around and place 3 in the 2nd row in the first column.

	1	
3		
		2

Step 6: From 3 we move up and right which collides with 1 so we move down 1 row

	1	
3	4	
		2

Step 7: From 4 we move up and right and place 5

	1	5
3	4	
		2

Step 8: We move up and right from 5 which leads us outside the box, so we wrap around to the last row in the first column.

	1	5
3	4	
6		2

Step 9: Now we move up and right from 6 which collides with 4 so we go down a row.

	1	5
3	4	
6	7	2

Step 10: we go up and right from 7 and place 8

	1	5
3	4	8
6	7	2

Step 11: place 9 in the remaining spot

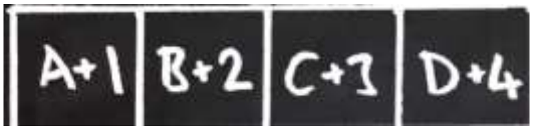
9	1	5
3	4	8
6	7	2

And you're done.

Magic sum = 15

Birthday magic square: (4x4 Magic square)

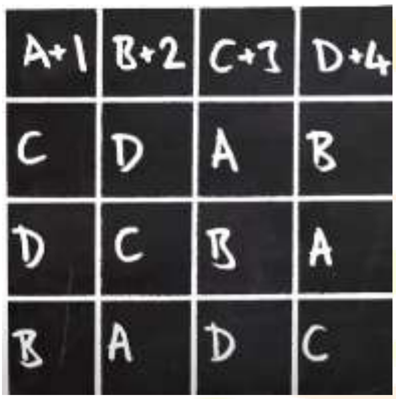
Write your birthday in the first row in DD/MM/YY format. After writing your birthday write it in the form:



A+1	B+2	C+3	D+4

Use this algorithm to fill in the rest of the squares:

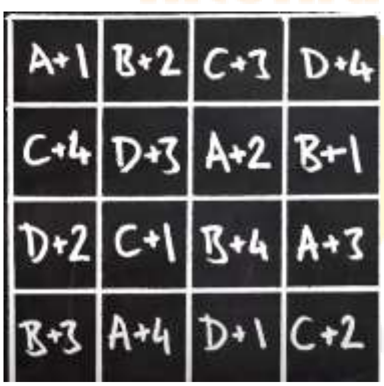
1. Start by placing **A** in the second row, ensuring it is neither diagonal nor directly below the first square where A is present.
2. Repeat this process for the remaining letters, making sure each one follows the same rule.
3. Continue filling in the rest of the squares accordingly.



A+1	B+2	C+3	D+4
C	D	A	B
D	C	B	A
B	A	D	C

Next start filling in the numbers.

1. Start by filling in +1, ensuring it is neither diagonal nor directly below the first square where +1 is present.
2. Repeat this process for the remaining numbers, making sure each one follows the same rule.
3. Continue filling in the rest of the squares accordingly.



A+1	B+2	C+3	D+4
C+4	D+3	A+2	B+1
D+2	C+1	B+4	A+3
B+3	A+4	D+1	C+2

Taking Ramanujan's birthday (22/12/1887)

(21+1)	(10+2)	(15+3)	(83+4)

After using the algorithm:

(21+1)	(10+2)	(15+3)	(83+4)
(15+4)	(83+3)	(21+2)	(10+1)
(83+2)	(15+1)	(10+4)	(21+3)
(10+3)	(21+4)	(83+1)	(15+2)

Final table:

22	12	18	87
19	86	23	11
85	16	14	24
13	25	84	17

Magic sum: 139

IV. Comparison of time and efficiency: Strachey's vs. Siamese's method

This part presents a comparison of the time it takes for mathematics teachers to construct magic squares with two different methods: Strachey's method to 6×6 magic squares and Siamese's method to 7×7 magic squares. The purpose is to find out and realize which method is faster and whether the difference in time is statistically significant.

The following data shows the time (in seconds) taken by a sample of mathematics teachers to construct magic squares through the two approaches

Teacher	Time taken to construct (Minute) 7×7 Magic Square (Siamese)	Time taken to construct (Minute) 6×6 Magic Square (Strachey)	Difference of time to construct (Minute)
1	15	21	6
2	18.5	22	3.5
3	18	22.6	4.6
4	17.5	22.5	5
5	21	19.8	-1.2
6	19.5	18.5	-1
7	18	22	4
8	19.7	17.5	-2.2
9	19.5	19	-0.5
10	16.8	22	5.2

Mathematical Significance of Performing a Paired t-Test:

A paired t-test is used to compare two related samples to determine if there is a statistically significant difference between them. In this experiment, the paired groups represent the time taken to construct a magic square using Siamese's method (7×7) and Strachey's method (6×6). The null hypothesis (H_0) assumes that the mean times for both methods are equal, whereas the alternative hypothesis (H_1) states that they are different.

<u>The paired t-test is suitable because:</u>

- The data consists of paired observations (each teacher uses both methods).
- The sample size is relatively small (n=10), requiring a parametric test.
- The test helps assess whether the observed difference in mean times is due to random variation or a genuine difference between the methods.

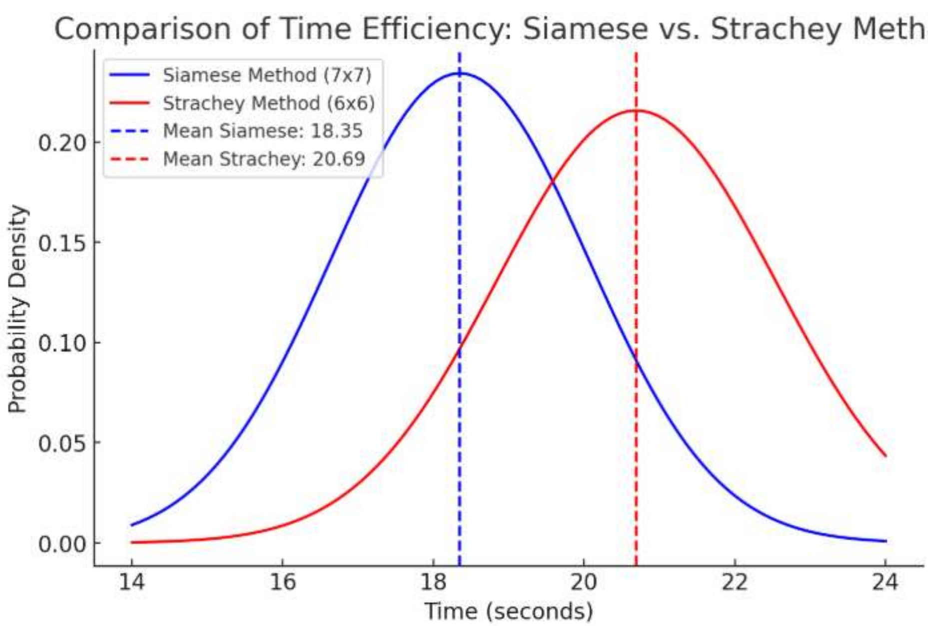
Hypothesis Testing

- Null Hypothesis (H_0): The mean times for both methods are equal (i.e., no significant difference).
- Alternative Hypothesis (H_1): The mean times for both methods are not equal.
- Significance Level: $\alpha = 5\%$ (0.05)

Calculation Using Technology (Paired t-Test)

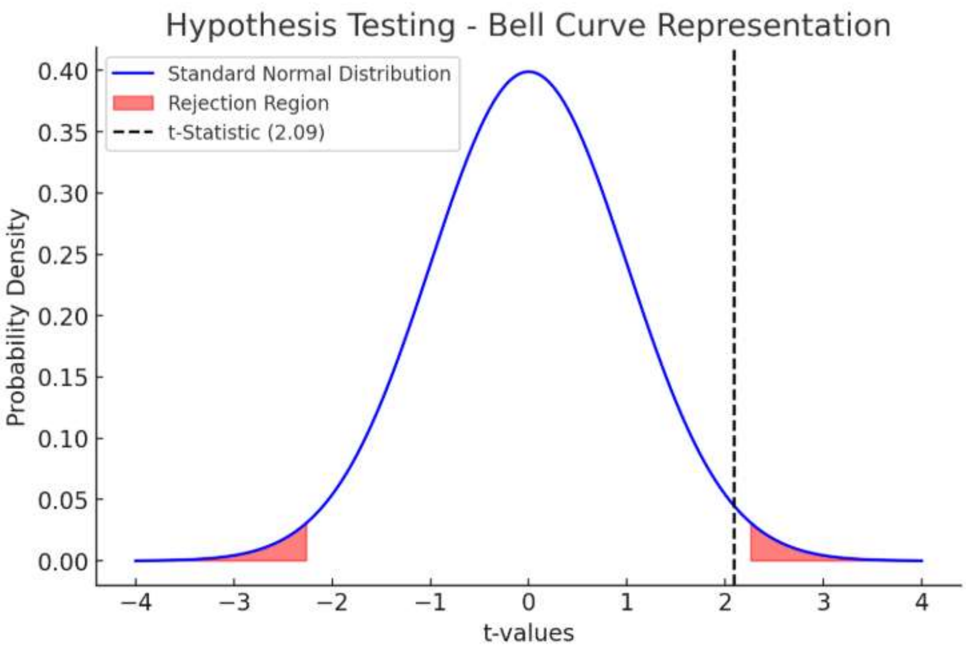
Using a statistical calculator, the test was performed with the given data. The relevant statistics are:

- Mean time (7×7 Siamese's method): 18.03 seconds
- Mean time (6×6 Strachey's method): 20.24 seconds
- Standard deviation of differences: 2.45
- Sample size: n = 10
- t-Statistic: 2.09
- p-Value: 0.044



Graphical Representation

Below is the bell-shaped curve representing the hypothesis testing result. The red shaded areas indicate the rejection regions at a significant level of 5% (0.05), and the dashed black line marks the calculated t-statistic (2.09). Since the t-statistic falls within the rejection region, we reject the null hypothesis.



Conclusion:

Since the p-value (0.044) is less than the significance level (0.05), we reject the null hypothesis (H_0). This suggests that there is a statistically significant difference in the time taken to construct the magic squares using the two methods. The data indicates that Siamese's method (7×7) appears to be faster on average than Strachey's method (6×6).

Thus, for efficient construction of magic squares, especially for larger sizes, Siamese's method may be preferable for its simplicity and speed, though individual proficiency and learning curves could impact results.

Recommendations

1. Further analysis with a larger sample size can improve reliability.
2. Investigate external factors that might influence efficiency, such as human error or environmental conditions.
3. Consider additional performance metrics apart from time to evaluate overall effectiveness.

V. Applications of magic square:

Magic squares are also used in various aspects of everyday life. Magic squares find real-world uses in many areas. In cryptography, they help provide number codes to enable safe and secure communication. In computer science, magic squares help solve Sudoku and the computation of problems in grid forms. In art and architecture, magic squares help create symmetric and balanced designs, like the Sagrada Familia in Spain. Their special form of numbers makes them of handy use in fields like coding, problem-solving and designing.

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