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Optimizing Inventory Replenishment: A Mathematical Approach to Cost-Effective Coffee Distribution

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Abstract : Efficient inventory management is essential for businesses dealing with physical goods, as it directly influences profitability and customer satisfaction. While inventory storage incurs costs, stockouts can severely hinder business growth. This article presents a mathematical model aimed at optimizing inventory replenishment strategies for a distribution company, specifically focusing on minimizing total inventory-related costs. Through an analysis of coffee bean demand in a coffee distribution company, the model identifies the optimal quantity and timing of orders, ignoring external environmental interactions to maintain a closed-system approach. The proposed solution leverages foundational inventory theories, case studies, and a novel mathematical concept to support improved inventory decisions.

***IndexTerms* - Inventory Management, Replenishment Strategy, Cost Optimization, Coffee Distribution, Demand Forecasting, Supply Chain Efficiency.**

INTRODUCTION

Inventories are stocks of goods held for future use or sale, essential for businesses including manufacturers, wholesalers, and retailers. While carrying inventory incurs storage and maintenance costs, the risk of running out of stock can lead to missed sales, dissatisfied customers, and long-term business losses. Therefore, companies strive to minimize storage costs while simultaneously avoiding shortage-related issues. To achieve this, firms often rely on logistics and mathematical inventory strategies to maximize efficiency and profitability. A core challenge in inventory management is determining when to reorder stock and how much to reorder. These two questions form the foundation of any effective inventory policy. In this project, we develop a mathematical model that proposes an optimal inventory replenishment strategy. The objective is to minimize total inventory costs by balancing the trade-off between holding and shortage costs, using a closed-system model for simplicity and focus.

LITERATURE SURVEY

Designing and Managing the Supply Chain: Concepts, Strategies, and Case Studies by David Simchi-Levi, Philip Kaminsky, and Edith Simchi-Levi This comprehensive text emphasizes the importance of supply chain management, covering essential models and strategies that reduce manufacturing costs and improve efficiency across supply chains. Modelling Inventory Management System at Distribution Company: A Case Study by Oksana Soshko, Vilmars Vjakse, and Yuri Merkuryev This paper offers a practical case study focused on inventory optimization in a Latvian distribution company. It explores simulation and optimization models under conditions of uncertain demand and highlights the effectiveness of each approach. Inventory Theory by Roberto Andreani This chapter discusses deterministic continuous review models and includes several case studies. It provides a mathematical basis for solving common inventory management challenges. Our model draws significant influence from this chapter while extending it through a unique and original mathematical approach.

RESEARCH GAP

The primary aim of this article is to design a mathematical inventory management model that minimizes inventory costs and enhances profitability for a distribution company. Using the example of a coffee distribution business, the study concentrates on managing the inventory of coffee beans the key determinant of other material requirements such as sugar, cups, and stirrers. By focusing solely on coffee beans, the model simplifies the system while still providing effective cost minimization strategies.

OBJECT OF STUDY

The central variable in our model is the quantity QQQ required to replenish inventory. In the context of the coffee distribution company, coffee beans represent the primary inventory object, as the demand for other materials is directly proportional to the quantity of coffee beans.

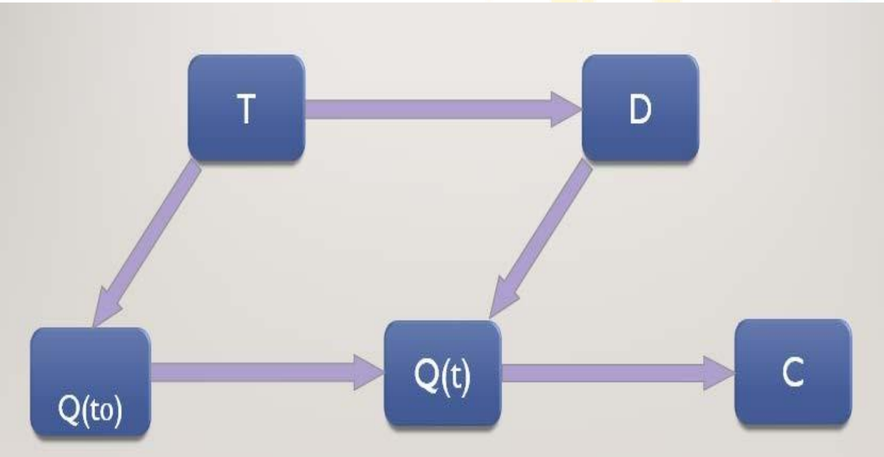
ENVIRONMENTAL FACTORS

Several environmental factors can affect inventory systems, including supply chain disruptions, natural disasters, maintenance and energy costs, seasonal fluctuations, marketing efforts, population trends, and market competition. However, for simplification and model clarity, we assume no external interactions in our case study. Thus, the system is modeled as a closed system, with a black-box approach, focusing on input-output relationships without delving into internal complexities.

VARIABLES AND PARAMETERS

To construct an effective inventory replenishment model, we have identified three primary variables that influence the system:

1.Quantity (Q): The amount of inventory replenished in each order cycle.
2.Time (t): The interval after which inventory needs to be replenished again.
3.Total Cost (TC): The overall cost associated with managing inventory, which is further broken down into:
setup Cost (K): The fixed cost incurred every time an order is placed.
holding Cost (h): The per-unit cost of storing inventory over time.
cost of Coffee (c): The per-unit purchase cost of the coffee beans.



FORMULATION

Seasons or weather has a significant impact on demand of coffee, so it would also affect the inventory. So we are dividing a year into four seasons. As mentioned before, we are assuming that demand is known from the previous year data and would be constant for an entire seasons but different for different seasons.A simple model representing this situation is the following economic order quantity model or, for short,the EOQ model.(It is some times also referred to as the economic lot-size model.) EOQ is the order quantity that minimizes the total holding costs and ordering costs. In this model it is assumed that there is constant demand (a kg/day) for the product.This model is valid for constant demand for infinite time.

Cost Optimization EOQ model-
Ordering cost per-cycle= c∗Q
Holding cost per cycle=h∗Q avg∗t;
Qavg=Q/2;t=Q/a
TotalCostpercycle(TC)=K+c∗Q+(h∗Q2)/(2∗a)
Totalcost perunit time(T)=TC/((Q/a));ForOptimizationDT/DQ=0
Optimal Quantity (Q*)=sq rt(2aK/h)
Corresponding cycle time (t*)=Q*/a=sqrt(2K/ah)

We are using EOQ model in each season and after at the end of each season (excluding last season),there are more than one paths or strategies which are possible, we analyzed each and every path and estimated the cost associated with that path. Finding the optimal path which has the minimum cost associated with it,would be our solution for the problem. Our algorithm shows how we pursue in this model-

ALGORITHM

In this model we have used dynamic programming approach.Through this approach we compared cost of all paths and chose minimum cost path.We are assuming that in the last season, inventory should become zero (termination point).

There are twenty-nine sub-paths and eleven nodes for four seasons [see Figure 1& 2]. These can vary according to number of seasons. These sub-paths shows possible way/cases that how initial inventory level finish or remains at the end of any season.Where nodes stand for total minimum cost of possible sub-paths those have same finishing point at end of season . Which means Ci node include minimum cost of

EX

C1 C2 Fc C3 So ca fro In

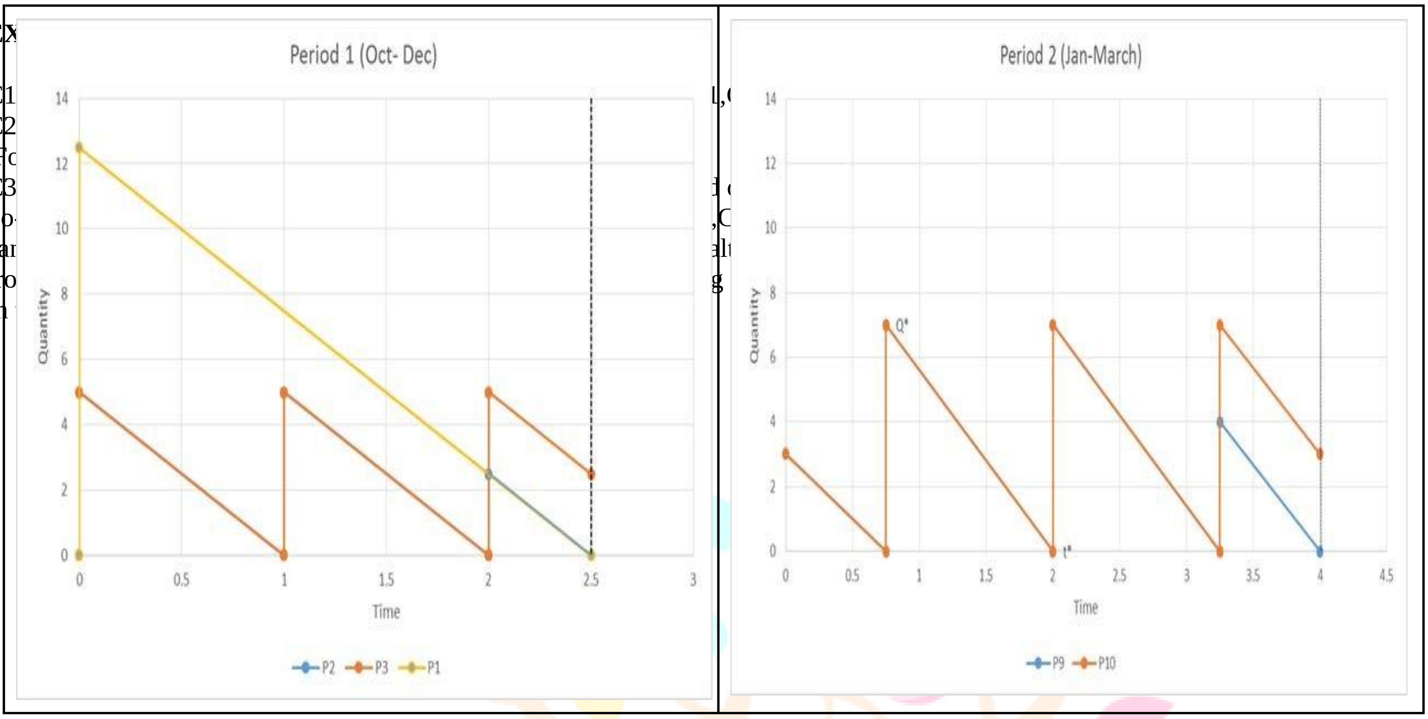


Figure 1- ALGORITHM DIAGRAM

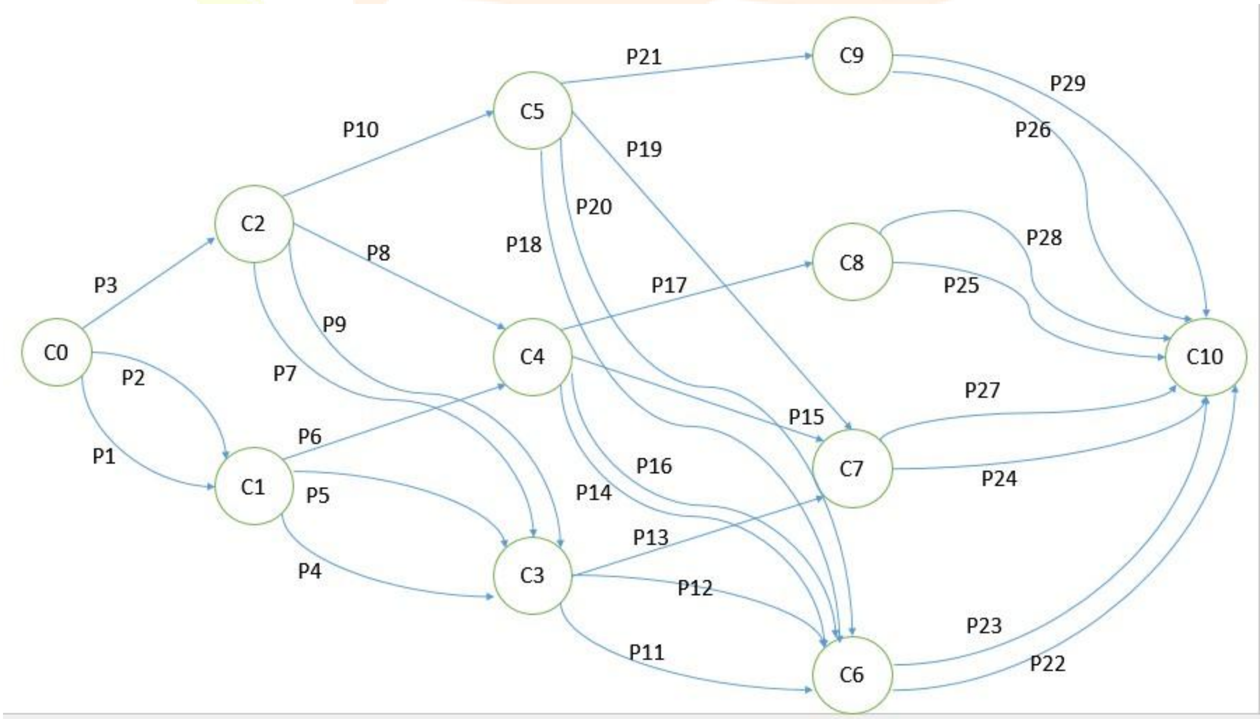


Figure 2-All the possible-paths

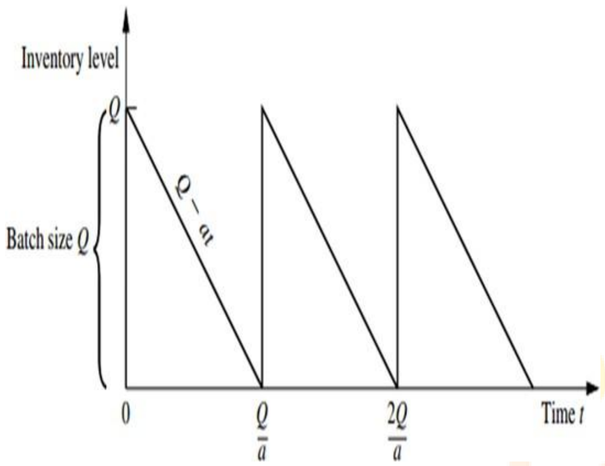
To find the optimal path,our model uses numerical values for the variables(demand) and parameters.

Parameters for given problem

Setup Cost(K) =15000Rs. per day
Holding Cost (h)= 0.85Rs. per kg
Cost (c) = 500Rs. per kg

Demand rate-

Period	DemandRate
Oct.-Dec.	50 Kg/Day
Jan.-March	70 Kg/Day
April-June	45 Kg/Day
July-Sept.	60 Kg/Day



RESULTS

After using the dynamic programming approach with numerical values of all the variables and parameters,we find out that Optimum Path would be-P3-P10-P20-P23, and our findings from the model are-

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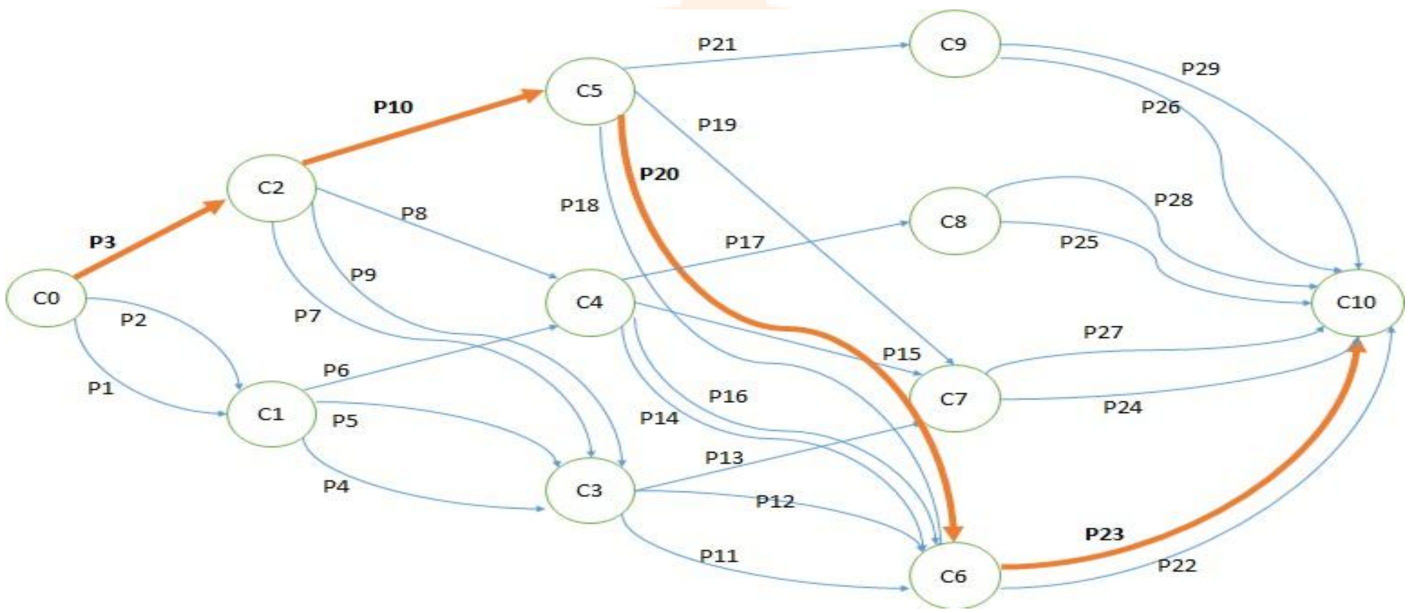

Figure 3-Optimum Path (in red lines)

RESULTS AFTER NUMERICAL CALCULATIONS-

Solution Cost = Rs10847921 (using Optimum Path:P3-P10-P20-P23)
Cost without the model = Rs 11112541
Saving (profit)=Rs 264620

LIMITATIONS - We have tried to estimate a model who is a solution for the all manufacturing and distribution companies who face inventory issues and needs to minimize their costs associated with inventory management.But still there are some limitations associated with our model-

❖This model can be used only for constant demand rate

❖If the number of periods increase,then complexity of the algorithm will also increase

❖To get the optimum solution,numeric values of parameters are required, because general solution is not possible

❖This model requires high holding cost and lowest up cost,so may not be applicable for some manufacturing companies having high setup cost

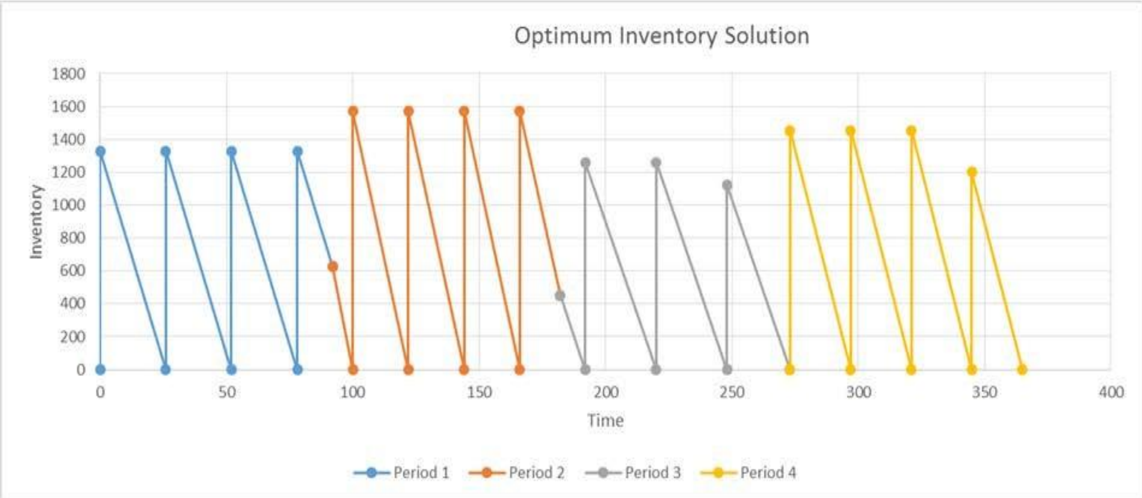

Figure4-Optimum Inventory Solution

CONCLUSIONS

As mentioned before, inventory management tries to find the best strategies and policies which provides the answers to the two most important and key questions for the inventory management- when to order and how much to order. Our model is a modest effort to find the best promising, feasible and profitable answers to these two questions. As we can see from the results that using our model provides good profit to the company. So inventory management is a key factor for the growth and success of the company. This dynamic and mathematical model and approach we are presenting in our project has a very useful attribute that it can be applied for any distribution and manufacturing company having similar inventory system and demand.

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