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REVAN TOPOLOGICAL INDICES AND REVAN TOPOLOGICAL POLYNOMIALS FOR CROWN GRAPH, GEAR GRAPH AND FRIENDSHIP GRAPH

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Abstract : Topological indices and polynomials are valuable tools in mathematical chemistry and graph theory, used to describe the structure and properties of molecular graphs and complex networks. In this paper, we focus on the computation of Revan topological indices and Revan topological polynomials for three notable graph families: Crown graphs, Gear graphs, and Friendship graphs.

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Key words: Revan Indices, Revan topological polynomial, Crown graphs, Gear graphs, and Friendship graphs.

1.Introduction

Graph theory has emerged as a fundamental tool in various branches of science, particularly in chemistry, physics, biology, and network theory. Graphs provide an intuitive and mathematical way to model relationships and interactions between objects. In particular, molecular graphs, where atoms are represented by vertices and chemical bonds by edges, offer a valuable framework for studying molecular structure, stability, and reactivity. A core aspect of such studies involves topological indices, which are numerical descriptors derived from graph structures. These indices play a critical role in correlating the physical, chemical, and biological properties of compounds with their molecular structure.

Over the years, several topological indices have been proposed, such as the Wiener index, Zagreb indices, and the Randic index, among others. These indices capture various aspects of molecular structure, including bond connectivity, molecular branching, and distance properties. More recently, Revan topological indices have gained attention as a novel class of indices aimed at providing a more refined characterization of graph structures. Similarly, topological polynomials, such as the characteristic and chromatic polynomials, offer algebraic representations of graph properties, enabling deeper insights into their combinatorial structure. The Revan topological polynomials, in particular, extend this concept by encoding graph properties in polynomial form, facilitating the study of complex graphs and networks.

The topological index was evolved by Wiener, in 1945,while researching the alkane's boiling point [2, 3]. 1st degree-based topological index was represented by Milan Randi´c, Revan 1st and 2nd topological indices were introduced by Kulli in [1]; for study, see [4]

This paper focuses on the computation of Revan topological indices and Revan topological polynomials for three important and well-known families of graphs: Crown graphs**,** Gear graphs**,** and Friendship graphs**.** These graph families have significant applications in various scientific domains:

1. **Crown graphs** (or corona graphs) are a subclass of bipartite graphs often used in studying symmetries and molecular interactions.
2. **Gear graphs** represent a generalization of cycle graphs and have been widely studied in the context of modeling mechanical systems, molecular networks, and cyclic compounds.
3. **Friendship graphs** consist of multiple triangles sharing a common vertex and are frequently used to model social networks where mutual friendships between individuals form triangular relationships.

While these graphs have been studied extensively in the literature, the computation of Revan topological indices and Revan topological polynomials for these graph classes remains an open and unexplored area. In this paper, we aim to bridge this gap by presenting explicit formulas for these indices and polynomials for Crown, Gear, and Friendship graphs. The Revan indices and polynomials offer a more comprehensive understanding of the structural properties of these graphs, providing a new perspective on their topological characteristics.

1. Preliminaries

1.1 REVAN Indices

$$R_1(G) = \sum_{u,v\in E(G)} (r_u + r_v)$$

i. $$R_2(G) = \sum_{u,v\in E(G)} (r_u * r_v)$$

$$R_3(G) = \sum_{u,v\in E(G)} |r_u - r_v|$$

ii. $$R_2(G) = \sum_{u,v\in E(G)} (r_u * r_v)$$

iii. $$R_3(G) = \sum_{u,v\in E(G)} |r_u - r_v|.$$

1.2 REVAN Topological polynomial

$$R_1(G,x) = \sum_{u,v\in E(G)} x^{(r_u + r_v)}$$

i. $$R_2(G,x) = \sum_{u,v\in E(G)} x^{(r_u * r_v)}$$

$$R_2(G,x) = \sum_{u,v\in E(G)} x^{(r_u * r_v)}$$

ii. $$R_2(G,x) = \sum_{u,v\in E(G)} x^{(r_u * r_v)}$$

iii. $$R_2(G,x) = \sum_{u,v\in E(G)} x^{(r_u * r_v)}$$

2. Main Results

2.1 Crown graph

Let $C_n K_1$ be the crown graph with $|V(C_n K_1)| = 2n$ and $|E(C_n K_1)| = 2n$

Table1: Revan edge set partition of crown graph

Edge partition	$RE_{1,1}$	$RE_{1,3}$
Frequency	n	n

2.2 Line graph of a crown graph

Let $(L(C_nK_1))$ be the line graph of crown graph with $|V(L(C_nK_1))|=2n$ and $|E(L(C_nK_1))|=3n$.

Table 2: Revan edge set partition of line graph of crown graph

Edge partition	$RE_{2,2}$	$RE_{4,2}$
Frequency	n	$2n$

Theorem 3.1 Let G be the crown graph C_nK_1 then

1. $R_1\ (G)=6n$
2. $R_2(G)=4n$
3. $R_3(G)=2n$

Proof: Let G be the crown graph C_nK_1. The total number of vertices of G are $2n$ and total numbers of edges are $2n$ respectively.

1. To compute $R_1(G)$ using table 1, we see that

$$R_1(G)=\sum_{E_{1,3}}(r_u+r_v)+\sum_{E_{1,1}}(r_u+r_v)$$
$$R_1(G)=n(1+3)+n(1+1)$$
$$R_1\ (G)=6n$$

2. To compute $R_2(G)$ using table 1, we see that

$$R_2(G)=\sum_{E_{1,3}}(r_u*r_v)+\sum_{E_{1,1}}(r_u*r_v)$$
$$R_2(G)=n(1*3)+n(1*1)$$
$$R_2\ (G)=4n$$

3. To compute $R_3(G)$ using table 1, we see that

$$R_3(G)=\sum_{E_{1,3}}|r_u-r_v|+\sum_{E_{1,1}}|r_u-r_v|$$
$$R_3(G)=n|1-3|+n|1-1|$$
$$R_3\ (G)=2n$$

Theorem 3.2 Let G be a crown graph C_nK_1 then

1. $R_1(G,x)=nx^2(x^2+1)$
2. $R_2(G,x)=nx(x^2+1)$
3. $R_3(G,x)=n(x^2+1)$

Proof: Let G be the crown graph C_nK_1. The total number of vertices of G is $2n$ and total numbers of edges are $2n$ respectively.

1. To compute $R_1(G,x)$ using table 1, we see that

$$R_1(G,x)=\sum_{E_{1,3}}x^{(r_u+r_v)}+\sum_{E_{1,1}}x^{(r_u+r_v)}$$
$$R_1(G,x)=n(x^{(1+3)})+n(x^{(1+1)})$$
$$R_1(G,x)=nx^2(x^2+1)$$

2. To compute $R_2(G,x)$ using table 1, we see that

$$R_2(G,x)=\sum_{E_{1,3}} x^{(r_u*r_v)}+\sum_{E_{1,1}} x^{(r_u*r_v)}$$

$$R_2(G_1,x)=n(x^{(1*3)})+n(x^{(1*1)})$$

$$R_2(G,x)=nx(x^2+1)$$

3. To compute $R_3(G,x)$ using table 1, we see that

$$R_3(G,x)=\sum_{E_{1,3}} x^{|r_u-r_v|}+\sum_{E_{1,1}} x^{|r_u-r_v|}$$

$$R_3(G,x)=n(x^{|1-3|})+n(x^{|1-1|})$$

$$R_3\ (G,x)=n(x^2+1)$$

Theorem 3.3 Let G be a line graph of crown graph $L(C_nK_1)$ then

1. $R_1(G)=16n$
2. $R_2(G)=20n$
3. $R_3(G)=4n$

Proof: Let G be the line graph of crown graph $L(C_nK_1)$. The total number of vertices of G is $2n$ and total numbers of edges are $3n$ respectively.

1. To compute $R_1(G)$ using table 2, we see that

$$R_1(G)=\sum_{E_{2,2}}(r_u+r_v)+\sum_{E_{4,2}}(r_u+r_v)$$

$$R_1(G)=n(2+2)+2n(4+2)$$

$$R_1\ (G)=16n$$

2. To compute $R_2(G)$ using table 2, we see that

$$R_2(G)=\sum_{E_{2,2}}(r_u*r_v)+\sum_{E_{4,2}}(r_u*r_v)$$

$$R_2(G)=n(2*2)+2n(4*2)$$

$$R_2(G)=20n$$

3. To compute $R_3(G)$ using table 2, we see that

$$R_3(G)=\sum_{E_{2,2}}|r_u-r_v|+\sum_{E_{4,2}}|r_u-r_v|$$

$$R_3(G)=n\,|\,2-2\,|+2n\,|\,4-2\,|$$

$$R_3\ (G)=4n$$

Theorem 3.4 Let G be a line graph of crown graph L (C_nK_1) then

$R_1(G,x)=nx^4(2x^2+1)$

1. $R_2(G,x)=nx^4(2x^4+1)$

$R_3(G,x)=n(2x^2+1)$

2. $R_2(G,x)=nx^4(2x^4+1)$
3. $R_3(G,x)=n(2x^2+1)$

Proof: Let G be the line graph of crown graph $L(C_nK_1)$. The total number of vertices of G is $2n$ and total numbers of edges are $3n$ respectively.

1. To compute $R_1(G,x)$ using table 2, we see that

$$R_1(G,x)=\sum_{E_{2,2}} x^{(r_u+r_v)}+\sum_{E_{4,2}} x^{(r_u+r_v)}$$

$$R_1(G,x)=n(x^{(2+2)})+2n(x^{(4+2)})$$

$$R_1(G,x)=nx^4(1+2x^2)$$

2. To compute $R_2(G,x)$ using table 2, we see that

$$R_2(G,x)=\sum_{E_{2,2}} x^{(r_u*r_v)}+\sum_{E_{4,2}} x^{(r_u*r_v)}$$

$$R_2(G,x)=n(x^{(2*2)})+2n(x^{(4*2)})$$

$$R_2(G,x)=nx^4(2x^4+1)$$

3. To compute $R_3(G,x)$ using table 2, we see that

$$R_3(G,x)=\sum_{E_{2,2}} x^{|r_u-r_v|}+\sum_{E_{4,2}} x^{|r_u-r_v|}$$

$$R_3(G,x)=n(x^{|2-2|})+2n(x^{|4-2|})$$

$$R_3(G,x)=n(2x^2+1)$$

3.2 Gear Graph

Let G_n be the gear graph with $|V(G_n)|=2n$ and $|E(G_n)|=3n$

Table 3: Revan edge partition of gear graph

Edge partition	$RE_{2,(n-1)}$	$RE_{(n-1),n}$
Frequency	n	$2n$

3.3 Line graph of a gear graph

Let $L(G_n)$ be the gear graph with $|VL(G_n)|=3n$ and $|EL(G_n)|=\frac{8n+n^2-2}{2}$

Table 4: Revan edge set partition of line graph of gear graph

Edge partition	$RE_{3,3}$	$RE_{3,(n+1)}$	$RE_{(n+1),(n+1)}$
Frequency	$\frac{n(n-1)}{2}$	$2n$	$2n$

Theorem 3.5 Let G be a gear graph then (G_n)

1. $R_1(G)=n(3n-1)$
2. $R_2(G)=2n(n^2-1)$
3. $R_3(G)=n(5-n)$

Proof: Let G be the crown graph (G_n). The total number of vertices of G is $2n$ and total numbers of edges are $3n$ respectively.

1. To compute $R_1(G)$ using table 3, we see that

$$R_1(G)=\sum_{E_{2,n-1}}(r_u+r_v)+\sum_{E_{n-1,n}}(r_u+r_v)$$

$$R_1(G)=n(2+n-1)+2n(n-1+n)$$

$$R_1(G)=n(3n-1)$$

2. To compute $R_2(G)$ using table 3, we see that

$$R_2(G)=\sum_{E_{2,n-1}}(r_u*r_v)+\sum_{E_{n-1,n}}(r_u*r_v)$$

$$R_2(G)=n(2*(n-1))+2n((n-1)*n)$$

$$R_2(G)=2n(n^2-1)$$

3. To compute $R_3(G)$ using table 3, we see that

$$R_3(G)=\sum_{E_{2,n-1}}|r_u-r_v|+\sum_{E_{n-1,n}}|r_u-r_v|$$

$$R_3(G)=n|2+n-1|+(2n(|n-1-n|))$$

$$R_3(G)=n(5-n)$$

Theorem 3.6 Let G be a gear graph (G_n) then

$$R_1(G,x)=n(x^{n+1}+2x^{2n-1})$$

1. $R_2(G,x)=n(x^{2n-2}+2(x^{n^2-n}))$

$$R_3(G,x)=n(x^{3-n}+2x)$$

2. $R_2(G,x)=n(x^{2n-2}+2(x^{n^2-n}))$
3. $R_3(G,x)=n(x^{3-n}+2x)$

Proof: Let G be the crown graph (G_n) The total number of vertices of G is $2n$ and total numbers of edges are $3n$ respectively.

1. To compute $R_1(G,x)$ using table 3, we see that

$$R_1(G,x)=\sum_{E_{2,n-1}}x^{(r_u+r_v)}+\sum_{E_{n-1,n}}x^{(r_u+r_v)}$$

$$R_1(G,x)=n(x^{2+n-1})+2n(x^{n-1+n})$$

$$R_1(G,x)=n(x^{n+1}+2x^{2n-1})$$

To compute $R_2(G,x)$ using table 3, we see that

$$R_2(G,x)=\sum_{E_{2,n-1}}x^{(r_u*r_v)}+\sum_{E_{n-1,n}}x^{(r_u*r_v)}$$

$$R_2(G,x)=n(x^{2*(n-1)})+2n(x^{n*(1+n)})$$

$$R_2(G,x)=n(x^{2n-2}+2(x^{n^2-n}))$$

To compute $R_3(G,x)$ using table 3, we see that

$$R_3(G,x)=\sum_{E_{2,n-1}}x^{|r_u-r_v|}+\sum_{E_{n-1,n}}x^{|r_u-r_v|}$$

$$R_3(G,x)=n(x^{|2-(n-1)|})+2n(x^{|n-(1+n)|})$$

$$R_3(G,x)=n(x^{3-n}+2x)$$

Theorem 3.7 Let G be a line graph of gear graph $L(G_n)$ then

1. $R_1(G)=3(3n^2+4n-1)$
2. $R_2(G)=\dfrac{4n^3+29n^2+7n}{2}$
3. $R_3(G)=2n(2-n)$

Proof: Let G be the crown graph $L(G_n)$ the total number of vertices of G is $3n$ and total numbers of edges are $\dfrac{8n+n^2-2}{2}$ respectively.

1. To compute $R_1(G)$ using table 4, we see that

$$R_1(G)=\sum_{E_{3,3}}(r_u+r_v)+\sum_{E_{3,n+1}}(r_u+r_v)+\sum_{E_{n+1,n+1}}(r_u+r_v)$$

$$R_1(G)=\frac{n(n-1)}{2}(3+3)+2n(3+n+1)+2n(n+1+n+1)$$

$$R_1(G)=3(3n^2+4n-1)$$

2. To compute $R_2(G)$ using table 4, we see that

$$R_2(G)=\sum_{E_{3,3}}(r_u*r_v)+\sum_{E_{3,n+1}}(r_u*r_v)+\sum_{E_{n+1,n+1}}(r_u*r_v)$$

$$R_2(G)=\frac{n(n-1)}{2}(3*3)+2n(3*(n+1))+2n((n+1)*(n+1))$$

$$R_2(G)=\frac{4n^3+29n^2+7n}{2}$$

3. To compute $R_3(G)$ using table 4, we see that

$$R_3(G)=\sum_{E_{3,3}}|r_u-r_v|+\sum_{E_{3,n+1}}|r_u-r_v|+\sum_{E_{n+1,n+1}}|r_u-r_v|$$

$$R_3(G)=\frac{n(n-1)}{2}|3-3|+2n|3-(n+1)|+2n|(n+1)-(n+1)|$$

$$R_3(G)=2n(2-n)$$

Theorem 3.8 Let G be a line graph of gear graph $L(G_n)$ then

1. $R_1(G,x)=2x^2(n^2-n)+n(x^{2n+2}+x^{n+4})$
2. $R_2(G,x)=x^9(n^2-n)+4n(x^{3(n+1)}+x^{(n+1)^2})$
3. $R_3(G,x)=n(n-1)+4n(1+x^{(n-2)})$

Proof: Let G be the crown graph $L(G_n)$ The total number of vertices of G is $3n$ and total numbers of edges are $\frac{8n+n^2-2}{2}$ respectively

$$R_1(G,x)=\sum_{E_{3,3}}x^{(r_u+r_v)}+\sum_{E_{3,n+1}}x^{(r_u+r_v)}+\sum_{E_{n+1,n+1}}x^{(r_u+r_v)}$$

1. To compute $R_1(G,x)$ using table 4, we see then $R_1(G,x)=\frac{n(n-1)}{2}(x^{3+3})+2n(x^{3+n+1})+2n(x^{n+1+n+1})$

$$R_1(G,x)=2x^2(n^2-n)+n(x^{2n+2}+x^{n+4})$$

2. To compute $R_2(G,x)$ using table 4, we see then

$$R_2(G,x)=\sum_{E_{3,3}}x^{(r_u*r_v)}+\sum_{E_{3,n+1}}x^{(r_u*r_v)}+\sum_{E_{n+1,n+1}}x^{(r_u*r_v)}$$

$$R_2(G,x)=\frac{n(n-1)}{2}(x^{3*3})+2n(x^{3*(n+1)})+2n(x^{(n+1)*(n+1)})$$

$$R_2(G,x)=x^9(n^2-n)+4n(x^{3(n+1)}+x^{(n+1)^2})$$

3. To compute $R_3(G,x)$ using table 4, we see then

$$R_3(G,x)=\sum_{E_{3,3}}x^{|r_u-r_v|}+\sum_{E_{3,n+1}}x^{|r_u*r_v|}+\sum_{E_{n+1,n+1}}x^{|r_u*r_v|}$$

$$R_3(G,x)=\frac{n(n-1)}{2}(x^{|3-3|})+2n(x^{|3-(n+1)|})+2n(x^{|(n+1)-(n+1)|})$$

$$R_3(G,x)=n(n-1)+4n(1+x^{(n-2)})$$

3.5 Friendship graph

Let C_n^m be the friendship graph with $|V(C_n^m) = m(n-1)+1|$ and $|E(C_n^m) = nm|$

Table 5: Revan edge set partition of friendship graph

Edge partition	$RE_{2,2m}$	$RE_{2m,2m}$
Frequency	$2m$	$(n-2)m$

3.6 Line graph of a friendship graph

Let $L(C_n^m)$ be the line graph of friendship graph for For $n>3$ then, $|V(C_n^m)| = nm$ and $|E(C_n^m)| = nm + 2(2m+k) \quad for\ 2 \le k \le (2m-2)$

Table 6: Revan edge set partition for line graph of friendship graph

Edge partition	$RE_{2,2m}$	RE_{2m2m}	$RE_{2,2}$
Frequency	$m(n-1)$	$m(n-1)$	$m(2m-1)$

Theorem 3.9 Let G be a friendship graph C_n^m then,

1. $R_1(G) = 4m - 4m^2(1-n)$
2. $R_2(G) = 4m^3(n-2) + 8m^2$
3. $R_3(G) = 4m(m-1)$

Proof: Let G be the friendship graph C_n^m the total number of vertices are $m(n-1)+1$ and total numbers of edges are mn respectively.

1. To compute $R_1(G)$ using table 5, we see then

$$R_1(G) = \sum_{E_{2,2m}} (r_u + r_v) + \sum_{E_{2m,2m}} (r_u + r_v)$$

$$R_1(G) = 2m(2+2m) + (n-2)m(2m+2m)$$

$$R_1(G) = 4m - 4m^2(1-n)$$

2. To compute $R_2(G)$ using table 5, we see then

$$R_2(G) = \sum_{E_{2,2m}} (r_u * r_v) + \sum_{E_{2m,2m}} (r_u * r_v)$$

$$R_2(G) = 2m(2*2m) + (n-2)m(2m*2m)$$

$$R_2(G) = 4m^3(n-2) + 8m^2$$

3. To compute $R_3(G)$ using table 5, we see then

$$R_3(G) = \sum_{E_{2,2m}} |r_u - r_v| + \sum_{E_{2m,2m}} |r_u - r_v|$$

$$R_3(G) = 2m|2-2m| + (n-2)m|2m-2m|$$

$$R_3(G) = 4m(m-1)$$

Theorem 3.10 Let G be a friendship graph C_n^m then,

1. $R_1(G,x) = mx^{2m}(2x^2 + x^{2m}(n-2))$
2. $R_2(G,x) = m(2x^{4m} + (n-2)x^{4m^2})$
3. $R_3(G,x) = m(\frac{2x^2}{x^{2m}} + n - 2)$

Proof: Let G be the friendship graph C_n^m then total number of vertices are $m(n-1)+1$ and total numbers of edges are mn respectively.

1. To compute $R_1(G,x)$ using table 5, we see then

$$R_1(G,x) = \sum_{E_{2,2m}} x^{(r_u+r_v)} + \sum_{E_{2m,2m}} x^{(r_u+r_v)}$$

$$R_1(G,x) = 2m(x^{2+2m}) + (n-2)m(x^{2m+2m})$$

$$R_1(G,x) = mx^{2m}(2x^2 + x^{2m}(n-2))$$

2. To compute $R_2(G,x)$ using table 5, we see then

$$R_2(G,x) = \sum_{E_{2,2m}} x^{(r_u*r_v)} + \sum_{E_{2m,2m}} x^{(r_u*r_v)}$$

$$R_2(G,x) = 2m(x^{2*2m}) + (n-2)m(x^{2m*2m})$$

$$R_2(G,x) = m(2x^{4m} + (n-2)x^{4m^2})$$

3. To compute $R_3(G,x)$ using table 5, we see then

$$R_3(G,x) = \sum_{E_{2,2m}} x^{|r_u-r_v|} + \sum_{E_{2m,2m}} x^{|r_u-r_v|}$$

$$R_3(G,x) = 2m(x^{|2-2m|}) + (n-2)m(x^{2m-2m})$$

$$R_3(G,x) = m(\frac{2x^2}{x^{2m}} + n - 2)$$

Theorem3.11 Let G be a line graph of friendship graph $L(C_n^m)$ for $n > 3$ then,

1. $R_1(G) = 2(m(4m^2 + 2m - 3) + n(m+1) - 1)$
2. $R_2(G) = 4m(2m^3 - m^2 + m + nm - 1)$
3. $R_3(G) = 2(mn - nm^2 - 1 + m)$

Proof: Let G be the line graph of friendship graph $L(C_n^m)$ then total number of vertices are nm and total numbers of edges are $nm + 2(2m+k)$ respectively.

1. To compute $R_1(G)$ using table 6, we see then

$$R_1(G) = \sum_{E_{2,2m}} (r_u + r_v) + \sum_{E_{2m,2m}} (r_u + r_v) + \sum_{E_{2,2}} (r_u + r_v)$$

$$R_1(G) = m(n-1)(2+2m) + m(n-1)(2m+2m) + m(2m-1)(2+2)$$

$$R_1(G) = 2(m(4m^2 + 2m - 3) + n(m+1) - 1)$$

2. To compute $R_2(G)$ using table 6, we see then

$$R_2(G) = \sum_{E_{2,2m}} (r_u * r_v) + \sum_{E_{2m,2m}} (r_u * r_v) + \sum_{E_{2,2}} (r_u * r_v)$$

$$R_2(G) = m(n-1)(2*2m) + m(n-1)(2m*2m) + m(2m-1)(2*2)$$

$$R_2(G) = 4m(2m^3 - m^2 + m + nm - 1)$$

3. To compute $R_3(G)$ using table 6, we see then

$$R_3(G)=\sum_{E_{2,2m}}|r_u-r_v|+\sum_{E_{2m,2m}}|r_u-r_v|+\sum_{E_{2,2}}|r_u-r_v|$$

$$R_3(G)=m(n-1)|2-2m|+m(n-1)|2m-2m|+m(2m-1)|2-2|$$

$$R_3(G)=2(mn-nm^2-1+m)$$

Theorem 3.12 Let G be a line graph of friendship graph $L(C_n^m)$ for $n>3$ then,

$$R_1(G,x)=m(x^2x^{2m}(n-1)+x^{4m}(2m-1)+x^4(2m-1))$$

1. $R_2(G,x)=m((n-1)x^{4m}+(2m-1)x^{4m^2}+(2m-1)x^4)$

$$R_3(G,x)=m((n-1)\frac{x^2}{x^{2m}}+2(2m-1))$$

2. $R_2(G,x)=m((n-1)x^{4m}+(2m-1)x^{4m^2}+(2m-1)x^4)$
3. $R_3(G,x)=m((n-1)\frac{x^2}{x^{2m}}+2(2m-1))$

Proof: Let G be the line graph of friendship graph $L(C_n^m)$ then total number of vertices are nm and total numbers of edges are $nm+2(2m+k)$ respectively.

1. To compute $R_1(G,x)$ using table 6, we see then

$$R_1(G,x)=\sum_{E_{2,2m}}x^{(r_u+r_v)}+\sum_{E_{2m,2m}}x^{(r_u+r_v)}+\sum_{E_{2,2}}x^{(r_u+r_v)}$$

$$R_1(G,x)=m(n-1)x^{(2+2m)}+m(n-1)x^{(2m+2m)}+m(2m-1)x^{(2+2)}$$

$$R_1(G,x)=m(x^2x^{2m}(n-1)+x^{4m}(2m-1)+x^4(2m-1))$$

2. To compute $R_2(G,x)$ using table 6, we see then

$$R_2(G,x)=\sum_{E_{2,2m}}x^{(r_u*r_v)}+\sum_{E_{2m,2m}}x^{(r_u*r_v)}+\sum_{E_{2,2}}x^{(r_u*r_v)}$$

$$R_2(G,x)=m(n-1)x^{(2*2m)}+m(n-1)x^{(2m*2m)}+m(2m-1)x^{(2*2)}$$

$$R_2(G,x)=m((n-1)x^{4m}+(2m-1)x^{4m^2}+(2m-1)x^4$$

3. To compute $R_3(G,x)$ using table 6, we see then

$$R_3(G,x)=\sum_{E_{2,2m}}x^{|r_u-r_v|}+\sum_{E_{2m,2m}}x^{|r_u-r_v|}+\sum_{E_{2,2}}x^{|r_u-r_v|}$$

$$R_3(G,x)=m(n-1)x^{|2-2m|}+m(n-1)x^{|2m-2m|}+m(2m-1)x^{|2-2|}$$

$$R_3(G,x)=m((n-1)\frac{x^2}{x^{2m}}+2(2m-1)$$

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