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TWO NEW COMPLEX VALUED MONOPOLE SOLUTIONS OF SU(2) YANG-MILLS-HIGG'S FIELDS BY LIE'S SIMILARITY TRANSFORMATION METHOD

B.V.BABY,

3/88,JADKAL POST,UDUPI DISTRICT,

KARNATAKA STATE,INDIA. 576 233.

Abstract : Four new exact monopole solutions of SU(2) Yang-Mills –Higg's field is reported ,in which two solutions are real valued functions and other two solutions are complex valued functions. Lie's Similarity Transformation method is adopted. Two different similarity variables are found for any number of a class of coupled nonlinear partial differential equations that can be transformed to coupled nonlinear ordinary differential equations, also reported.

IndexTerms - Monoploe, Similarity transformation, Parameter, Nonlinear, Infinitesimal, Gauge field, Meson, Non Abelian, Symmetry.

1.Introduction

For the 5-dimensional Kaluza-Klein theory in 1938 Oscar Klein added broken symmetry of SU(2) gauge group (Actor, 1979) In 1963 Wolfgang Pauli introduced six dimensional model for interaction between meson and nucleons with SU(2) gauge group theory (Actor, 1979). In 1954 ,Yang and Mills introduced non commutative gauge group SU(2) for massless photon electrodynamics, for which in 1960 Higg's mechanism (Actor,1979)added as spontaneous symmetry breaking massive field.

Non abelian gauge theories of classical field equations are nonlinear coupled partial differential equations. Finding solutions of such classical gauge field equations are not easy and still no general solution is known.

In the historical development of differential equations ,the first part of studies were concentrated on search of exact particular solutions without any systematic methods of solving the equation. Then different systematic methods of solving linear differential equations were developed for simple cases. Lie developed a systematic method for solving different types of linear differential equations, for finding particular solutions (Lie,1874).Lie's group theoretical methods of solving ordinary differential equations by means of one parameter infinitesimal continuous similarity transformation(ICST) method developed on the basis of contact transformation of surfaces of differential geometry(Lie,1874). Based on same concept, A.V.Backlund developed Backlund transformations method (Backlund,1876) for finding particular solutions of a nonlinear partial differentia equations . ICST method developed on the basis of group theoretical aspects and initially applied to ordinary differential equations for findina systematic method of developing integrating factors. Recently ,ICST method further developed for partial differential equations (Bluman,Cole,1974), (Bluman,Anco,2002) for finding suitable transformations by which the number of independent variables reduced by one and that group theoretical aspects very extensively studied [Bluman, Cole, 1974). This method commonly known as Similarity Transformations and the resultant differential equations is called Similarity Reduced equations and its solutions are called Similarity solutions. By ICST method two independent variables partial differential equation can be reduced to an ordinary differential equation by finding suitable transformation. Then all exact solutions of resultant similarity reduced ordinary differential equation (ODE) are also exact solutions of original Partial differential equations , but converse is need not true.

Means, ICST method produced a class of exact particular solutions that are subset of all solutions of given partial differential equations and so method cannot be used for generating general solution of a partial differential equation (PDE). Moreover, in some cases ICST method fails to find suitable transformation for reducing to ODE. Even then method can be used for both linear as well as nonlinear partial differential equations .

Brief outline of the ICST method of solving partial differential equation (PDE) by reducing the number of independent variables by one is the following (Bluman, Cole, 1974) . Let the given PDE with n number of independent variables and m dependent variables be,

$\phi(x_{1,2},\ldots.x_n,\omega^1,\omega^2,\ldots..\ \omega^m,\ \omega^i_k\ ) = 0$, (1)

Where x_j are independent variables and ω^i_k are dependent variables, i=1,2,…n and j=1,2,…m. and $\omega^i_{,k}\ \omega^i_{,k}$ is the partial differential of dependent variable ω^i with respect independent variable x_k . For an infinitesimal parameter ϵ>0, get a family of infinitesimal transformations

$x_j = x_j + \epsilon X_j\ (x_i\ ,\omega^i\ ) + O(\epsilon^2)$, (2)

$\omega^i = \omega^i + \epsilon w^i\ (x_j\ ,\omega^i) + O(\epsilon^2)$, (3)

where i=1,2,……n ,the number of dependent variables and j=1,2,...n are the number of independent variables. Corresponding to the above transformations ,the first order partial derivatives of dependent variables ω^i are transformed as,

$\omega^i_k = \omega^i_{,k} + \epsilon[W^i_{,k}] + O(\epsilon^2)$, (4)

Where $[W^i_{,k}\ ]$ is called 'first extension' of first order derivatives ω^i_k of dependent variables ω^i and its expansion (Bluman,Cole, 1974),

$[W^i_{,k}\ ] = \frac{\partial \omega^i}{\partial x_k} + \frac{\partial W^i}{\partial \omega^\mu}\omega^\mu_{,k} - \frac{\partial X_\nu}{\partial x_k}\omega^i_{,\nu} - \frac{\partial X_\nu}{\partial \omega^\mu}\omega^\mu_{,k}\omega^i_\upsilon$, (5) The second partial derivatives $\omega^i_{,jk}$ with respect to the variables x_j and $x_{,k}$ are transformed as,

$$\omega^i_{,jk} = \omega^i_{,jk} + \epsilon[W^i_{,jk}] + O(\epsilon^2), \quad (6)$$

where $[W^i_{,jk}]$ is called 'second extension' of the second order derivatives $\omega^i_{,jk}$ of the dependent variables ω^i with respect to the independent variables x_j and x_k , and its expansion is,

$$[W^i_{,jk}] = \frac{\partial^2 W^i}{\partial x_j\, \partial k} + \frac{\partial^2 W^i}{\partial x_j\, \partial \omega^\mu}\omega^\mu_{,k} + \frac{\partial^2 W^i}{\partial x_k \partial \omega^\mu}\omega^\mu_{,j} - \frac{\partial^2 x_\nu}{\partial x_j \partial x_k}\omega^i_\nu + \frac{\partial W^i}{\partial \omega^\mu}\omega^\mu_{,jk} - \frac{\partial X_\nu}{\partial x_j}\omega^i_{,k\nu} - \frac{\partial X_\nu}{\partial x_k}\omega^i_{,j\nu} + \frac{\partial^2 W^i}{\partial \omega^\lambda \partial \omega^\mu}\omega^\lambda_{,j}\omega^\mu_{,k} - \frac{\partial^2 X_\nu}{\partial x_j \partial \omega^\mu}\omega^\mu_{,j}\omega^i_{,\nu} - \frac{\partial^2 X_\nu}{\partial x_k \partial \omega^\mu}\omega^\mu_{,k}\omega^i_{,\nu} - \frac{\partial X_\nu}{\partial \omega^\mu}[\omega^i_{,\nu}\omega^\mu_{,jk} + \omega^\mu_{,j}\omega^i_{,\nu k} + \omega^\mu_{,k}\omega^i_{,j\nu}] - \frac{\partial^2 X_\nu}{\partial \omega^\lambda \partial \omega^\mu}\omega^\lambda_{,k}\omega^\mu_{,j}\omega^i_{,\nu}\,. \quad (7)$$

The ' invariant surface condition ' for a similarity transformation (2),(3) and (4) of the given PDE (1) for a second order PDE is

$$X_i \frac{\partial \phi}{\partial x_i} + W^i \frac{\partial \phi}{\partial \omega^i} + [W^i_{,j}]\frac{\partial \phi}{\partial \omega^i_{,j}} + \left[W^i_{,jk}\right]\frac{\partial \phi}{\partial \omega^i_{,jk}} = 0. \quad (8)$$

On solving above invariant surface condition (8) , 'infinitesimals ' $X_{,j}, W^i$ can be uniquely determined ,that give the similarity transformation under which the given PDE (1) can be transformed into another PDE with one independent variables less ..
Then for finding the similarity transformation solve,

$$X_j \frac{d\omega^i}{dx_j} + W^i \frac{d\omega^i}{dW^i} = 0\,, \quad (9)$$

using the Lagrange's conditions,

$$\frac{dx_j}{X_j} = \frac{d\omega^i}{W^i} = 0\ . \quad (10)$$

The solution of (10) gives the similarity variables η_j as function of independent variables x_j and arbitrary constants ,and the 'similarity solutions' ω^i as function of similarity variables η_j . On substituting the similarity solutions $\omega^i(\eta_j)$ in the given PDE (1) gives the similarity reduced PDE in which one independent variables less than the original PDE (1), All exact solution of similarity reduced PDE are also exact solutions of given PDE (1),but converse need not be satisfied. In the following section above method of finding solution of PDE is using to find exact solution of coupled nonlinear partial differential
equations of second order with two independent variables and two dependent variables.

2.SU(2) Yang-Mills-Higgs Field Equations Reduced by Ansatz.

The basic dynamical variables of SU(2) Yang-Mills (YM) field (Actor,1979),(Ni Jun,2014), Yang,Mills, 1954),(Julia,Zee, 1975)] are vector potential B^α_μ and the YM field $F_{\mu\nu}$ related by

$$F^\alpha_{\mu\nu} = \partial_\mu B^\alpha_\nu - \partial_\nu B^\alpha_\nu + e\,\epsilon^{\alpha bc} B^b_\mu B^c_\nu\ , \quad (11)$$

where ϵ is the coupling constant . Then the equation of motion is

$$D_\mu F^{\mu\nu} = 0 \quad (12)$$

The Lagrangian for a YM theory with local gauge symmetry breaking potential is

$$\mathscr{L} = -F^\alpha_{\mu\nu}F^{\mu\nu}_\alpha - \frac{1}{4}\lambda\left(A^2 + \frac{\mu^2}{A^2}\right). \quad (13)$$

The Lagrangian for SU(2)YM gauge field with Higgs triplet[8] is,

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu\alpha}F^{\alpha}_{\mu\nu} - G^{\mu\alpha}G^{\alpha}_{\mu} + \frac{1}{2}\mu^2\phi^2\varphi^2 - \frac{1}{2}\lambda(\phi^{\alpha}\phi^{\alpha})^2, \quad (14)$$

where

$$G^{\alpha}_{\mu\nu} = \partial_{\mu}\phi^{\alpha} + e\epsilon^{abc}B^{b}_{\nu}\phi^{c} . \quad (15)$$

Then equation of motion becomes,

$$\partial_{\nu}G^{\mu\alpha 3} + e\epsilon^{abc}B^{b}_{\mu}G^{\mu c} - \frac{\partial P(\phi)}{\partial\phi} = 0 , \quad (16)$$

where

$$P(\emptyset) = -\frac{1}{2}\mu^2\phi^{\alpha}\phi^{\alpha} + \frac{1}{4}\lambda(\phi^{\alpha}\phi^{\alpha})^2. \quad (17)$$

Using Wu-Yang-t'Hooft-Julia Zee ansatz (Actor,1979), (Yang,Mills, 1954),('t hooft,1979),(Prasad,Sommerfield, 1975) equation of motion reduced to a pair of coupled nonlinear partial differential equation of two variables u(r, t) and v(r, t) as,

$$r^2[u_{,rr} - u_{,tt}] = u(u^2 - 1) + uv^2, \quad (18)$$

and

$$r^2 [v_{,rr} - v_{,tt}] = 2vu^2 + \frac{\lambda}{e^2}(v^3 - C^2r^2v^2). \quad (19)$$

Where r is the spherical symmetry variable . For λ=0 , obtained ,

$$r^2(u_{,rr} - u_{,tt}) = u(u^2-1) + uv^2, \quad (20)$$

and

$$r^2(v_{,rr} - v_{,tt}) = 2vu^2 . \quad (21)$$

for which, static point like finite energy monopole solutions reported (Actor,1979),(Prasad,Sommerfield,1975),(Wu,Yang,!969).

In this study ,using ISCT method two independent variables nonlinear coupled PDEs(20) and (21) are transformed to a pair of coupled nonlinear ODEs , for which two sets of new exact solutions are reporting with complex valued circular and hyperbolic trigonometric functions. Whereas ,known solutions are with different real valued hyperbolic and circular trigonometric functions (Actor,1979),(Prasad,Sommerfield,1975),(Babu Joseph,Baby, 1985), (Baby, 2021).(Baby,2023).(Baby, 2024a).

3.ICST Method of Converting Coupled PDEs with Two Independent Variables to Coupled ODEs

Coupled nonlinear PDEs (20) and (21) have two independent variables r and t and two dependent variables u(r, t) and v(r, t), and four second order partial differential derivatives, $u_{,rr}$, $u_{,tt}$,$v_{,rr}$,$and\ v_{,tt}$ for which second extensions (7) are

$$[U_{,rr}] = U_{,rr} + 2(U_{,ru}u_{,r} + U_{,rv}v_{,r}) - R_{,rr}u_r - T_{,rr}u_{,t} + U_{,u}u_{,rr} + U_{,v}v_{,rr} - 2R_{,r}u_{,r} - 2T_{,ru}u_{rt} + U_{,uu}u^2_{,r} + 2U_{,uv}u_{,r}v_{,r} + U_{,vv}v^2_{,r} - 2(R_{,ru}u^2_{,r} + R_{,rv}u_{,r}v_{,r} + T_{,ru}u_{,r}u_{,t} + T_{,rv}v_{,r}u_{,t}) - 3R_{,u}u_{,r}u_{,rr} - R_{,v}(u_{,r}v_{,rr} + 2v_{,r}u_{rr}) - T_{,u}(u_{,t}u_{,rr} + 2u_{,r}u_{,rt}) - T_{,v}(u_{,t}v_{rr} + 2v_{,r}u_{,rt}) + R_{,vv}v^2_{,}u_{,r} - R_{,uu}u^3_{,r} - 2R_{,uv}v_{,r}u^2_{,r} - T_{,uu}u_{,t}u^2_{,r} - 2T_{,uv}u_{,r}u_{,t}v_{,r} - -T_{,vv}v^2_{,r}u_{,t} , \quad (22)$$

$$[U_{,tt}] = U_{,tt} + 2(U_{,ut}u_{,t} + U_{,vt}v_{,t}) - R_{,tt}u_{,r} - T_{,tt}u_{,t} + U_{,u}u_{,tt} + U_{,v}v_{,tt}$$

$-2(R_{,t}u_{,rt} + T_{,t}u_{,tt}) + U_{,uu}u_{,t}^2 + 2U_{,uv}u_{,t}v_{,t} + U_{,vv}v_{,t}^2$
$-2(R_{,ut}u_{,t}u_{,r} + R_{,vt}u_{,v}v_{,t} + T_{,ut}u_{,t}^2 + T_{,vt}v_{,t}u_{,t}) - R_{,u}(u_{,r}u_{,tt} + 2u_{,t}u_{,rt}) - R_{,v}(u_{,r}v_{,tt} + 2v_{,t}u_{,rt}) - 3T_{,u}u_{,t}u_{,tt} - T_{,v}(u_{,t}, v_{,tt}, t + 2v_{,t}u_{,tt}) \quad -R_{,uu}u_{,t}^2u_{,r} - 2R_{,uv}u_{,t}u_{,r}v_{,t} - R_{,vt}v_{,t}^2u_{,r} - T_{,uu}u_{,t}^3 - 2T_{,uv}u_{,t}^2v_{,t} + T_{,vv}v_{,t}^2u_{,t}.$ (23)

$[V_{,rr}] = V_{,rr} + 2(V_{,rv}v_{,r} + V_{,ru}u_{,r}) - R_{,rr}v_{,r} - T_{,rr}v_{,t} + V_{,u}u_{,rr} + V_{,r}v_{,rr} - 2(R_{,r}v_{,rr} + T_{,r}v_{,rt}) + V_{,uu}u_{,r}^2 + 2V_{,uv}u_{,r}v_{,r} + V_{,vv}v_{,r}^2$
$-2(R_{,ur}u_{,r}v_{,r} + R_{,vr}v_{,r}^2 + T_{,ur}u_{,r}v_{,t}) - R_{,u}(v_{,r}u_{,rr} + 2u_{,r}v_{,rr}) - 3R_{,v}v_{,r}v_{,rr} - T_{,u}(v_{,t}u_{,rr} + 2u_{,r}v_{,rt}) - (2R_{,uv}u_{,r}v_{,r}^2 + R_{,uu}u_{,r}^2v_{,r} + R_{,vv}v_{,r}^3) - (T_{,uu}u_{,r}^3v_{,t} + 2T_{,uv}u_{,x}v_{,r}v_{,t} + T_{,vv}v_{,r}^2v_{,t})$. (24)

and

$[V_{,tt}] = V_{,tt} + 2(V_{,ut}u_{,t} + V_{,vt}v_{,t}) - R_{,tt}v_{,r} - T_{,tt}v_{,t} + V_{,u}u_{,tt} + V_{,v}v_{,tt} - 2(R_{,t}v_{,rt} + T_{,t}v_{,tt}) + V_{,uu}u_{,t}^2 + 2V_{,uv}u_{,t}v_{,t} + V_{,vv}v_{,t}^2 - 2(R_{,ut}u_{,t}v_{,r} + T_{,ut}u_{,t}v_{,t} + T_{,vt}v_{,t}^2 + R_{,vt}v_{,r}v_{,r}) - R_{,u}(v_{,r}u_{,tt} + 2u_{,t}v_{,rt}) - R_{,v}(v_{,r}v_{,tt} + 2v_{,t}v_{,rt}) - T_{,u}(v_{,t}u_{,tt} + 2u_{,t}v_{,tt}) - T_{,v}v_{,t}v_{,tt} - R_{,uu}u_{,t}^2v_{,r} - 2R_{,uv}u_{,t}v_{,t}v_{,r} - R_{,vv}v_{,t}^2v_{,r} - T_{,uu}u_{,t}^2v_{,t} - 2T_{,uv}u_{,t}v_{,t}^2 - T_{,uv}v_{,t}^3$. (25)

Then the 'invariant surface condition ' corresponds to the coupled equations (20) and (21) is

$$T\frac{\partial\phi}{\partial t} + R\frac{\partial\phi}{\partial r} + U\frac{\partial\phi}{\partial u} + V\frac{\partial\phi}{\partial v} + [U_{,rr}]\frac{\partial\phi}{\partial u_{,rr}} + [U_{,tt}]\frac{\partial\phi}{\partial u_{,tt}} + [V_{,rr}]\frac{\partial\phi}{\partial v_{,rr}} + [V_{,tt}]\frac{\partial\phi}{\partial v_{,tt}} = 0 \quad . \tag{26}$$

Substitute the second extensions given above and equate coefficients of various derivatives of dependent variables u and v as zero , we get around eighty constrained equations ,out of which the essential constrained equations are the following,

$2rR(u_{,rr} - u_{,tt}) - 3u^2U + U - v^2U - 2uvV = 0$,
$2rR(v_{,rr} - v_{,tt}) - 4uvU - 2vu^2 = 0$,
$2U_{,tu} + T_{,tt} + T_{,rr} = 0$, $R_{,tt} + 2U_{,ru} - R_{,rr} = 0$,
$2rR + r^2(U_{,u} - 2T_{,t}) = 0$, $2U_{,uu} + R_{,u} = 0$,
$2rR + r^2(U_{,u} - 2R_{,r}) = 0$, $T_{,t} + rT_{,rt} - R_{,r} = 0$
$2R_{,tu} - 2T_{,ru} = 0$, $R_{,t} - rT_{,t} = 0$, $rR_{,r} - R = 0$, $rT_{,t} - R = 0$,
$U_{,u} = U_{,v} = U_{,r} = U_{,t} = V_{,v} = V_{,u} = V_{,r} = V_{,t} = 0$,
$R_{,rr} = R_{,tt} = T_{,rr} = R_{,u} = R_{,v} = T_{,u} = T_{,v} = 0$. (27)

From above constrained equations get,

$U = V = 0,\ rR_{,r} - R = 0, rT_{,t} - R = 0,\ T_{,t} - R_{,r} = 0.$ (28)

On solving (28) get,

$R = 2\alpha rt + \beta r,$ (29)

$T = \alpha(r^2 + t^2) + \beta t + \frac{\beta^2}{4\alpha}$, (30)

and

$U = V = 0$, (31)

where α *and* β are arbitrary integration constants and $\alpha \neq 0$.

The similarity variable s(r, t) can be found out from the Lagrange's condition

$\frac{dr}{R}=\frac{dt}{T}=\frac{du}{U}=\frac{dv}{V}$. (32)

From equations (29),(30) ,(31) and (32) get the similarity variable (Babu Joseph,Baby, 1985),(Baby,2021),(Baby, 2023) as,

$s(r,t)=\frac{r}{[r^2-(t^2+\beta t+\frac{\beta^2}{4\alpha})}$. (33)

The similarity solutions of the coupled PDEs (20) and (21) are

$u(r,t)=\ u(s),$ (34)

and

$v(r,t)=\ v(s).$ (35)

Substitute the similarity solutions (34) and (35) in PDEs (20) and (21) get the following similarity reduced coupled nonlinear ordinary differential equations,

$s^2\frac{d^2u}{ds^2}=u(u^2-1)+v^2$, (36)

and

$s^2\frac{d^2v}{ds^2}=2vu^2$. (37)

All exact solutions of the above similarity reduced equations (36) and (37) are also exact solutions of the nonlinear coupled PDEs (20) and (21) by replacing the similarity variable s(r, t) by its value (33).

4.Conclusions

ICST method of solving PDEs by reducing the number of independent variables by one is possible only if there exists suitable transformations among independent variables ,that relation is called similarity variable. There are cases such possibility may not exists such cases ICST method fails .At the same time if the exact solutions of PDEs are not simple functions then ICST method may be the only way to find that (Baby, 2021).

The above similarity reduced equations (36) and (37) are nonlinear coupled ODEs and so not easy to solve. Still found following four exact new solutions, of which two are real valued circular functions and other two are complex valued hyperbolic functions First exact real valued circular function solution of (36) and (37) is,

$u(s)=[-1+\gamma s\tan(\gamma s+\tau)],$ (38)

and

$v(s)=\ \gamma s\sec(\gamma s+\tau),$ (39)

where $\gamma\ and\ \tau$ are arbitrary integration constants. In the similarity variable (33) when arbitrary constant $\beta=0$, get another similarity variable $s_0(r,t)$, (Babu Joseph,Baby, 1985),(Baby, 2023) as,

$s_0(r,t)=\frac{r}{(r^2-t^2)}$, (40)

For which the similarity reduced equations are

$s_0^2\frac{d^2u}{ds_0^2}=u(u^2=1)+v^2,$ (41)

and

$s_0^2\frac{d^2v}{ds_0^2}=2vu^2$. (42)

For (41) and (42) get second exact circular function solution same as (38) and (39) with $\beta=0$.

For (41) and (42) complex valued hyperbolic solution is

$u(s) = [-1 + \gamma s \tanh(\gamma s + \tau)]$, (43)
and
v(s)= $i\gamma s \operatorname{sech}(\gamma s + \tau)$, (44)
where s is the similarity variable(33).

As in the last case for second similarity variable $s_0(r,t)$ the second complex valued exact solution is,
$u(s_0) = [-1 + \gamma s_0\, tanh(\gamma s_0 + \tau)]$, (45)
and
$v(s_0)$=$i\gamma s_0 \operatorname{sech}(\gamma s_0$+$\tau)$. (46)

In all above solutions when replace similarity variables by their values (33) and (40) get exact solutions of the nonlinear coupled PDEs (20) and (21). In above results it is obvious that the right hand sides of the coupled PDEs (20) and (21) are retained even after similarity reduction. That means all coupled PDEs of same form as (20) and (21) with right hand sides as exclusively polynomials have two above similarity variables s(r,t) and $s_0(r,t)$.Moreover, their similarity reduced equations are same form as above results with right hand sides retain the same polynomials of right hand sides of the coupled PDEs .So in general PDEs of the form,
$r^2\left(u^i_{j,rr} \mp u^i_{j,tt}\right) = F_j(u^i)$, (47)
Where u^i are any number of dependent variables i= 1,2,….n and j=1,2,…m are m number of coupled PDEs with polynomials $F_j\left(u^i\right)$ of dependent variables u^i will have common similarity transformation as, $u^i(s)$ where common similarity variable is,
$$s(r,t) = \frac{r}{[r^2 \mp \left(t^2 + \beta t + \frac{\beta^2}{4\alpha}\right)]} \ , \qquad (48)$$
where $\alpha\ and\ \beta$ are arbitrary integration constants and $\alpha \neq 0$. For the similarity reduced equations are
$$s^2 \frac{d^2 u^i_j}{ds^2} = F_j(u^i)\ . \qquad (49)$$
For $\beta = 0$, there exists another similarity variable $s_0(r,t)$ as,
$$s_0(r,t) = \frac{r}{[r^2 \mp t^2]}\,, \qquad (50)$$
and the respective similarity reduced ODEs are,
$$s_0^2 \frac{d^2 u^2_j}{ds^2} = F_j(u^i). \qquad (51)$$

There are some more exact solutions of above PDEs (20) and (21) are known with circular as well as hyperbolic function but unlike above complex valued functions (43),(44),(45) and (46) they are all real valued functions (Babu Joseph,Baby, 1985), (Baby, 2024a). All above four new exact solutions of (20) and (21) are monopole solutions of SU(2) Yang-Mills –Higg's field system. No general solution of YM field equations is known ,still there are many particular solutions by different methods are reported,(Actor,1979), (Julia,Zee,1975),(Babu Joseph,Baby,1985),(Baby,2021),(Baby,2023),(Carmeli, et al, 1978),(Hulehil, 1985),(Baby,2024a).(Baby,2024b),(Baby, 2024c),(Carmeli, 1982).

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