SUPER MEAN GRAPH LABELING FOR SOME SPECIAL GRAPHS

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Abstract *:* There exist many types of Graph labeling such as Graceful labeling, Edge Graceful labeling, Mean labeling, Super Mean labeling, Harmonious labeling etc., In this article, graphs such as the Pine tree graph, Tadpole graph and Combined tadpole graph are proved that it satisfies super mean graph labeling.

MSC Classification: 05C76, 05C78

Key words: Graphs, Labeling, Graceful, Mean, Super Mean, Harmonious etc.

1. Introduction

Graph theory is the branch of mathematics concerned with networks of points connected by lines. The subject of graph theory had its beginnings in recreational math problems, but it has grown into a significant area of mathematical research, with applications in chemistry, operations research, social sciences, and computer science.

2. Basic Definitions

Definition 2.1

A graph is a pair of sets **(V, E),** where **V** is the set of vertices and **E** is the set of edges, connecting the pairs of vertices.

Definition 2.2

A **connected acyclic graph** is called a tree. In other words, a connected graph with no cycles is called a tree. The edges of a tree are known as **branches**. Elements of trees are called their **nodes**. A tree with 'n' vertices has 'n-1' edges. If it has one more edge extra than 'n-1', then the extra edge must pair up with two vertices which leads to form a cycle. Then, it becomes a cyclic graph which is a contradiction for the tree graph.

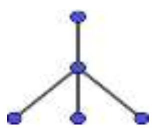
The graph shown here is a tree because it has no cycles and it is connected. It has four vertices and three edges, i.e., for 'n' vertices 'n-1' edges as mentioned in the definition.

Definition 2.3

Pine Tree Graph

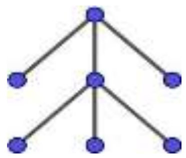
Pine tree graph satisfies all the conditions of trees. The simplest form of pine tree graph is as

follows.

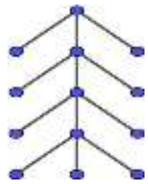


It has 5 vertices and 4 edges.

This simplest structure can be extended by adding 4 more vertices and 3 edges to the top most vertex in the above figure



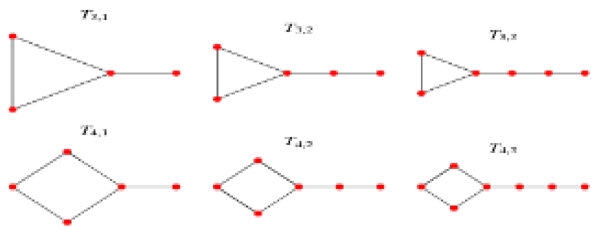
By adding such a set of edges and vertices to the top most vertex the pine tree can be extended to n vertices and n-1 edges.



Definition 2.4

Tadpole Graph

The tadpole graph, also called a dragon graph or kite graph is the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge.



Definition 2.5

Graph Labeling

A labeled graph *G* = (*V*, *E*) is a finite series of graph vertices V with a set of graph edges E of 2- subsets of V. If the vertices are assigned values subject to certain condition(s) then it is known as graph labeling. Any graph labeling will have the following three common characteristics:

(i) A set of numbers from which vertex labels are chosen;

(ii) A rule that assigns a value to each edge;

(iii) A condition that these values must satisfy.

Definition 2.6

Super Mean Labeling

Let G = (V, E) be a graph and f : V(G) →{1,2,…,p+q} be an injection for every edge e= uv, f^* in E(G) is

$$f^*(e)=\frac{f(u)+f(v)}{2} \text{ if } f(u)+f(v) \text{ is even}$$

$$f^*(e)=\frac{f(u)+f(v)+1}{2} \text{ if } f(u)+f(v) \text{ is odd}$$

f is called Super Mean Labeling if [V(G)] ∪ [E(G)] = {1,2,…….,p+q} and all the labels of V(G) and E(G) are distinct. A graph that allows Super Mean Labeling is called a Super Mean Graph.

3. Main Theorems

Theorem 3.1
Statement:
All pine tree graphs are super mean graphs.
Proof:

$V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, p+q\}$
$V(G) = \{u_i, v_i, w_i; 0 \le i \le n\}$ be the vertices.
Define V(G) by,

$f(u_0) = 2$
$f(u_i) = 6i + 3 \quad ; 1 \le i \le n$
$eg: f(u_1) = 6 + 3 = 9$
$f(u_2) = 12 + 3 = 15$
$f(u_3) = 18 + 3 = 21, \dots, f(u_n) = 6n + 3$

$f(v_0) = 1$
$f(v_i) = 6i - 1 \quad ; 1 \le i \le n$
$eg: f(v_1) = 6 - 1 = 5$
$f(v_2) = 12 - 1 = 11$
$f(v_3) = 18 - 1 = 19, \dots, f(v_n) = 6n - 1$

$f(w_i) = 6i + 7 \quad ; 0 \le i \le n$
$eg: f(w_0) = 0 + 7 = 7$
$f(w_1) = 6 + 7 = 13$
$f(w_2) = 12 + 7 = 19, \dots, f(w_n) = 6n + 7$

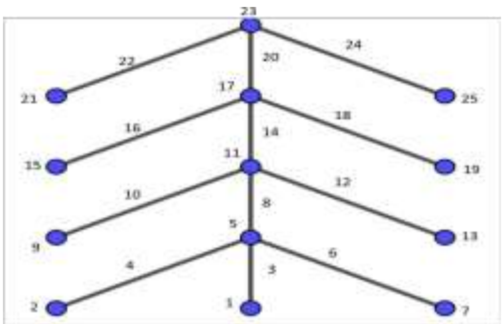
E(G) = $\{x_i, y_i, z_i; 0 \le i \le n\}$ be the vertices.

Define E(G) by,

$f(x_i) = 4 + 6i \quad ; 0 \le i \le n$
$eg: f(x_0) = 4 + 0 = 4$
$f(x_1) = 4 + 6 = 10$
$f(x_2) = 4 + 12 = 16, \dots, f(x_n) = 4 + 6n$

$f(y_0) = 3$
$f(y_i) = 8 + 6i \quad ; 1 \le i \le n$
$eg: f(y_1) = 8 + 6 = 14$
$f(y_2) = 8 + 12 = 20$
$f(y_3) = 8 + 18 = 26, \dots, f(y_n) = 8 + 6n$

$f(z_i) = 6(i + 1) \quad ; 0 \le i \le n$
$eg: f(z_0) = 6(1) = 6$
$f(z_1) = 6(2) = 12$
$f(z_2) = 6(3) = 18, \dots, f(z_n) = 6(n + 1)$



Hence by induction method all the pine trees are super mean graphs.

Theorem 3.2

Statement:
Tm ʘ Pn (tadpole graph) is a super mean graph where is n is any positive integer and m is a positive odd integer greater than or equal to 3.

Proof:
$V(G) = \{u_i ; 0 \le i \le (m+n) - 1\}$ be the vertices.
$E(G) = \{e_i \; 0 \le i \le (m+n) - 1\}$ be the edges.
$\{f(u_i) : u_i \in V(G) \cup \{g(e_i) : e_i \in E(G)\} = \{1,2,\ldots, p+q\}$

$f(u) = \lceil 2i + 1 \quad i \; \{ \quad ;i \quad \le m \quad -$
$\lfloor 2(i+1) \quad ;i > m - j$
$g(e) = | \lceil m+1 \quad ;i \quad = \quad ($
$;0 < i \le m - j$
$;i > m - j$

$i \quad \{ 2i$

$| 2(i+1) \; \lfloor$

where j is a positive integer greater than 1
i.e. j = 2, 3, 4, … respectively for m=3,5,7,...
f(ui) and g(ei) should be distinct
TmʘPn can be proved as a super mean graph by the method of induction.

Case 1:

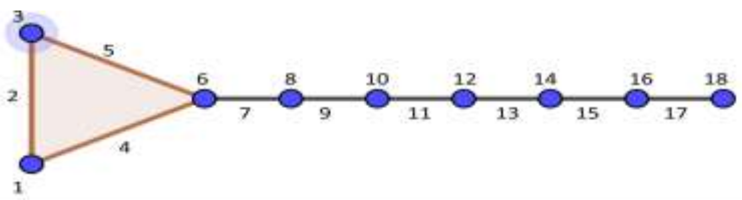
To prove T3ʘ Pn is a super mean graph.
$V(G) = \{u_i ; 0 \le i \le (3+n) - 1\}$ be the vertices.
$E(G) = \{e_i \; 0 \le i \le (3+n) - 1\}$ be the edges.
$V(G) \cup E(G) = \{1,2,3,\ldots p+q\}$

$f(u) = \lceil 2i + 1 \quad ;i \le 1$
$\{ \; \lfloor 2(i+1) \; ;i > 1$

$f(u_0)=2(0)+1=1$
$f(u_1)=2(1)+1=3$
$f(u_2)=2(2+1)=6$
$f(u_3) = 2(3+1) = 8, \ldots, f(u_{(3+n)-1}) = 2(3+n)$

$g(e) = | \lceil m+1 \quad ;i \quad = \quad (;i > 1$
$;0 < i \le 1$
$i \quad \{ 2i$
$2(i+1)$

$g(e_0) = 3 + 1 = 4$
$g(e_1) = 2(1) = 2$
$g(e_2) = 2(2) + 1 = 5$
$g(e_3) = 2(3) + 1 = 7$
,…,
$g(e_{(3+n)-1}) = 2((3+n) - 1) + 1$



Here f(ui) and g (ei) are distinct. Hence T3ʘ Pn is a super mean graph.

Case 2:

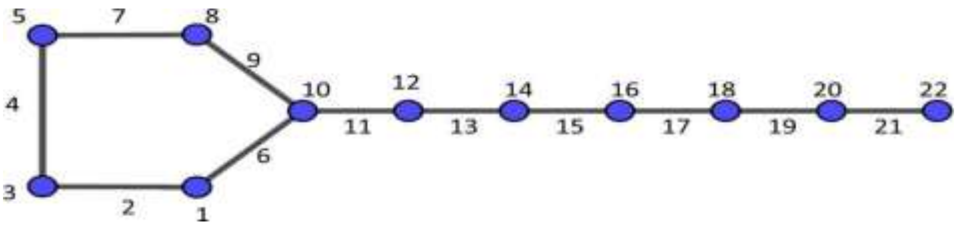
To prove T5Ꙩ Pn is a super mean graph.
$V(G) = \{u_i ; 0 \leq i \leq (5+n) - 1\}$ be the vertices.

$E(G) = \{e_i ; 0 \leq i \leq (5+n) - 1\}$ be the edges.
$V(G) \cup E(G) = \{1,2,3,\ldots p + q\}$
$f(u_i) = \begin{cases} 2i+1 & ; i \leq 2 \\ 2(i+1) & ; i > 2 \end{cases}$

$f(u_0) = 2(0) + 1 = 1$
$f(u_1) = 2(1) + 1 = 3$
$f(u_2) = 2(2) + 1 = 5$
$f(u_3) = 2(3 + 1) = 8$
$f(u_4) = 2(4+1) = 10, \ldots, f(u_{(3+n)-1}) = 2(3+n)$

$g(e_i) = \begin{cases} m+1 & ; i = (\\ 2i & ; 0 < i \leq 2 \\ 2(i+1) & ; i > 2 \end{cases}$

$g(e_0) = 5 + 1 = 6$
$g(e_1) = 2(1) = 2$
$g(e_2) = 2(2) = 4$
$g(e_3) = 2(3) + 1 = 7$
$g(e_4) = 2(4) + 1 = 9, \ldots, g(e_{(3+n)-1}) = 2((3+n) - 1)\ 1$



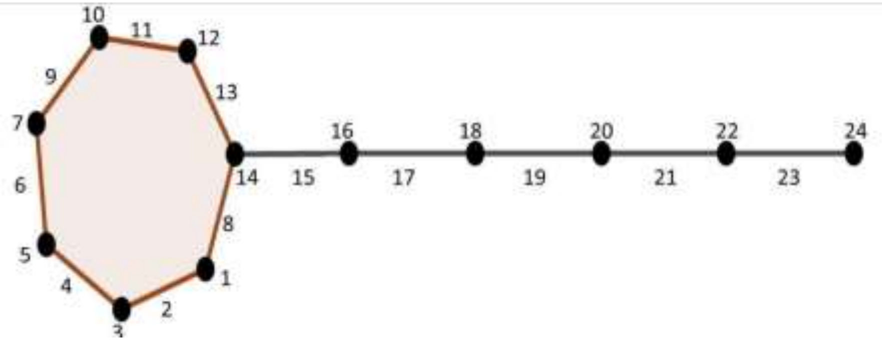
Here f(ui) and g (ei) are distinct. Hence T5Ꙩ Pn is a super mean graph.

Case 3:

To prove T7Ꙩ Pn is a super mean graph.
$V(G) = \{u_i ; 0 \leq i \leq (7+n) - 1\}$ be the vertices.
$E(G) = \{e_i ; 0 \leq i \leq (7+n) - 1\}$ be the edges.
$V(G) \cup E(G) = \{1,2,3,\ldots p + q\}$
$f(u_i) = \begin{cases} 2i+1 & ; i \leq 3 \\ 2(i+1) & ; i > 3 \end{cases}$
$f(u_0) = 2(0) + 1 = 1$
$f(u_1) = 2(1) + 1 = 3$
$f(u_2) = 2(2) + 1 = 5$
$f(u_3) = 2(3) + 1 = 7$
$f(u_4) = 2(4+1) = 10, \ldots, f(u_{(3+n)-1}) = 2(3+n)$

$g(e_i) = \begin{cases} m+1 & ; i = (\\ 2i & ; 0 < i \leq 3 \\ 2(i+1) & ; i > 3 \end{cases}$

$g(e_0) = 7 + 1 = 8$
$g(e_1) = 2(1) = 2$
$g(e_2) = 2(2) = 4$
$g(e_3) = 2(3) = 6$
$g(e_4) = 2(4) + 1 = 9, \ldots, g(e_{(3+n)-1}) = 2((3+n) - 1) + 1$



Here f(ui) and g (ei) are distinct. Hence T7Ꙩ Pn is a super mean graph.

Case 4:

To prove T9Ꙩ Pn is a super mean graph.

$V(G) = \{u_i ; 0 \le i \le (9+n) - 1\}$ be the vertices.

$E(G) = \{e_i ; 0 \le i \le (9+n) - 1\}$ be the edges.

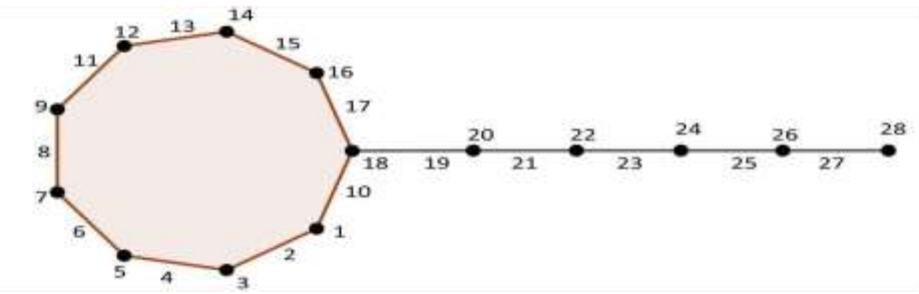
$V(G) \cup E(G) = \{1, 2, 3, \ldots p+q\}$

$$f(u_i) = \begin{cases} 2i+1 & ; i \le 4 \\ 2(i+1) & ; i > 4 \end{cases}$$

$f(u_0) = 2(0) + 1 = 1$
$f(u_1) = 2(1) + 1 = 3$
$f(u_2) = 2(2) + 1 = 5$
$f(u_3) = 2(3) + 1 = 7$
$f(u_4) = 2(4) + 1 = 9, \ldots, f(u_{(3+n)-1}) = 2(3+n)$

$$g(e_i) = \begin{cases} m+1 & ; i = 0 \\ 2i & ; 0 < i \le 4 \\ 2(i+1) & ; i > 4 \end{cases}$$

$g(e_0) = 9 + 1 = 10$
$g(e_1) = 2(1) = 2$
$g(e_2) = 2(2) = 4$
$g(e_3) = 2(3) = 6$
$g(e_4) = 2(4) = 8, \ldots, g(e_{(3+n)-1}) = 2((3+n) - 1) + 1$



Here f(ui) and g (ei) are distinct. Hence T9Ꙩ Pn is a super mean graph.

From the case1,2,3,4 it is clear that T3Θ Pn is a super mean graph T3+2Θ Pn = T5Θ Pn is a super mean graph T5+2Θ Pn = T7Θ Pn is a super mean graph T7+2Θ Pn = T9Θ Pn,…, Tk+2Θ Pn is a super mean graph , where k+2 = m , since k+2 is odd.

Hence TmΘ Pn is a super mean graph by the method of induction.

Conclusion:

The tadpole graph Tn,k is the graph created by concatenating Cn and P_k with an edge from any vertex of Cn to a pendant of P_k for integers n=3 and k=0. In this article, it is proved that the pine tree graph, tadpole graphs are super mean graphs.

References:

1. P. Jeyanthi and D. Ramya, Super mean graphs.
2. R. Vasuki and A. Nagarajan, On the Meanness of Arbitrary path super subdivision of paths, Australasian Journal of Combinatories.
3. R. Vasuki and A. Nagarajan, Meanness of the graphs Pa,b and P^b_a, International Journal of Applied Mathematics.
4. P. Jeyanthi, R. Ponraj and D. Ramya, Supermean labeling of graph, Ars Combin
5. Selvam Avadayappan, R. Vasuki, New families of mean graph
6. S. Somasundaram and R. Ponraj, Mean labelings of graphs, National Academy Science letter.
7. A. Vasuki and A. Nagarajan, Some results on super mean graphs.