

# Mathematical expressions to solving the problems of some Chronic and epidemic diseases

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**Abstract:-** Infectious diseases such as tuberculosis chickenpox, Hiv/AIDS poses significant challenge to global health, characterized by diverse transmission methods varying severity, and substantial socioeconomic impacts while modern medicine has provided effective vaccines and treatments these diseases still cause millions of deaths annually particularly in developing regions

In 2026, mathematical modeling will remain a key tool for understanding the global impact of tuberculosis, chickenpox and HIV/AIDS. These diseases are described using compartmental models that divide the population into groups such as susceptible, infected and recovered to predict spread and evaluate control strategies.

**Introduction:-** There are many diseases that we want to analyze mathematically so that we can understand their rate of spread, containment, and recovery, Are people recovering is the disease spreading, or is it subsiding. Having mathematical concepts is crucial for solving these problems. The mathematical data obtained in this way makes it easier for us to prevent many types of dangerous diseases.

For example, the SEIR model is useful for tuberculosis, the SIR model for chickenpox. And SI and SIR model for diseases like HIV/AIDS. These models help determine the rate at which these diseases spread and also identify their causes.

**Key word:-** Diseases, Epidemic, Mathematical expression, Mathematical models, infected populations, reproduction number, recovery numbers

## Theory

Mathematical concept to solve the contagious diseases:- (1),(2),(3)

- $S(t)$  = Susceptible population
- $I(t)$  = Infected population
- $R(t)$  = Recovered population
- $N = S + I + R$  (total population)

$$\frac{ds}{dt} = -\beta SI$$

$$dI/dt = \beta SI - \gamma I$$

$$dR/dt = \gamma I$$

Where:

$\beta$  (beta) = transmission rate

$\gamma$  (gamma) = recovery rate

Rate of epidemic spread depends on  $\beta SI$

SIR MODEL IS BENEFICIAL FOR IDENTIFICATION PERMANENT IMMUNITY FOR chicken pox Therefore:

Basic Reproduction Number:

$$R_0 > 1 \rightarrow \text{epidemic spreads}$$

$$R_0 < 1 \rightarrow \text{disease dies out}$$

Mathematical solution of chickenpox spreading and control

Chickenpox is a highly contagious viral disease, so it is well described by (4),(5)

the **SIR epidemic model**.

### Population Division

Let total population = **N**

- **S(t)** = Susceptible individuals
  - **I(t)** = Infected individuals
  - **R(t)** = Recovered (immune) individuals
- $$N = S(t) + I(t) + R(t)$$

### Differential Equations (Rate of Spread)

$$\frac{ds}{dt} = -\beta SI$$

$$dI/dt = \beta SI - \gamma I$$

$$dR/dt = \gamma I$$

### Meaning of Parameters

- **β (beta)** = Transmission rate (how fast infection spreads)
- **γ (gamma)** = Recovery rate
- **1/γ** = Average infectious period

### Rate of Spread of Chickenpox (6),(7),(8)

The infection spreads when:

$$\frac{dI}{dt} > 0$$

$$\begin{aligned} \beta SI - \gamma I &> 0 \\ \beta S &> \gamma \end{aligned}$$

Basic Reproduction Number( $R_0$ )

$$R_0 = \beta S_0 / \gamma$$

Where  $S_0$  is the initial susceptible population.

- **$R_0 > 1$**  → Disease spreads (epidemic)
- **$R_0 < 1$**  → Disease dies out (controlled)

For chickenpox

$$R_0 \approx 8-12 \text{ (very high)}$$

$$R_0 < 1$$

$$\frac{\beta S}{\gamma} < 1$$

This can be achieved by:

- Reducing  $\beta$  (less contact)
  - Reducing  $S$  (vaccination)
  - Increasing  $\gamma$  (faster recovery)
- Effect of Vaccination (Control Model)

Let  $v$  = vaccination coverage

$$S=(1-v)N$$

Condition for control:-

$$R_0(1-v)<1$$

$$v>1-\frac{1}{R_0}$$

For chickenpox ( $R_0 \approx 10$ ):

$$v>0.9(90\% \text{ vaccination required})$$

### Graphical Interpretation (Conceptual)

- $S(t)$  decreases continuously
- $I(t)$  increases rapidly, reaches a peak, then declines
- $R(t)$  increases steadily

Peak infection occurs when:

$$S=\frac{\gamma}{\beta}$$

### Medical Control

- Antiviral drugs (Acyclovir)
- Fever control (avoid aspirin  $\rightarrow$  Reye's syndrome)
- Adequate hydration
- Chickenpox spread follows **SIR model**
- High  $R_0$  (**8–12**)  $\rightarrow$  rapid spread
- Control achieved by:
  - Vaccination
  - Isolation
  - Reduced contact
- Mathematical condition for control:

$$R_0<1$$

Graph:-

Tuberculosis:-

### Spreading of Tuberculosis (TB)

TB spreads **slowly** and many people remain **latent** (infected but not infectious).  
So we use the **SEIR model**.

### Population compartments

Let total population =  $N$

- $S(t)$  = Susceptible (healthy but can get TB)
- $E(t)$  = Exposed (latent TB, not infectious)
- $I(t)$  = Infectious (active TB)
- $R(t)$  = Recovered (treated or immune)

$$N=S+E+I+R$$

### Differential Equations (TB Spread)

$$\frac{dS}{dt} = \Lambda - \beta SI - \mu S$$

$$\frac{dE}{dt} = \beta SI - (\sigma + \mu)E$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

### parameters

- $\beta$  = Transmission rate of TB
- $\sigma$  = Rate of progression from latent to active TB
- $\gamma$  = Recovery rate due to treatment
- $\delta$  = TB-induced death rate
- $\mu$  = Natural death rate
- $\Lambda$  = Recruitment (birth) rate

### Basic Reproduction Number $R_0$

$$R_0 = \frac{\beta\sigma}{(\sigma + \mu)(\gamma + \mu + \delta)}$$

- If  $R_0 > 1 \rightarrow$  TB spreads in the population
- If  $R_0 < 1 \rightarrow$  TB dies out

### Mathematical Control of Tuberculosis

Control measures reduce  $\beta$  and increase  $\gamma$ .

#### (a) Treatment (DOTS)

$$\gamma \uparrow \Rightarrow I(t) \downarrow$$

#### (b) Isolation & Mask use

$$\beta \downarrow \Rightarrow \text{Reduced transmission}$$

#### (c) Vaccination (BCG)

$$S \downarrow \Rightarrow E \downarrow$$

#### (d) Early detection

$$\sigma \downarrow \Rightarrow \text{Less progression to active TB}$$

Controlled Model (With Control Function  $u(t)$ )

$$\frac{dI}{dt} = \sigma E - (\gamma + u(t))I$$

## 6. Precautions Against Tuberculosis

Precaution	Mathematical Effect
Wearing mask	$\downarrow \beta$
Good ventilation	$\downarrow \beta$
BCG vaccination	$\downarrow S$
Early diagnosis	$\downarrow \sigma$
Complete treatment	$\uparrow \gamma$
Nutrition & immunity	$\downarrow$ progression

## Conclusion

**TB spread follows an SEIR mathematical model**

- Control aims to reduce  $R_0 < 1$
- Treatment and prevention decrease infection rate
- Mathematical modeling helps in planning TB control programs

HIV/AIDS spreading in society :-

## Mathematical Model for HIV/AIDS Spread (9),(10),(11)

HIV spreads **slowly** and mainly through **specific contacts**, so an **SI or SIR-type model with treatment** is used.

## Population division

Let total population  $N(t)$  be divided into:

- $S(t)$  – Susceptible (uninfected people)
- $I(t)$  – HIV infected (can transmit)
- $A(t)$  – AIDS patients (advanced stage)
- $T(t)$  – Treated individuals (on ART)

$$N(t) = S(t) + I(t) + A(t) + T(t)$$

## Differential Equations

### (i) Susceptible population

$$\frac{dS}{dt} = \Lambda - \beta SI - \mu S$$

$\Lambda$  = birth/entry rate

$\beta$  = HIV transmission rate

$\mu$  = natural death rate

(ii) HIV infected population

$$\frac{dI}{dt} = \beta SI - (\alpha + \mu + \tau)I$$

$\alpha$  = progression rate from HIV to AIDS

$\tau$  = treatment (ART) rate

### AIDS population

$$\frac{dA}{dt} = \alpha I - (\mu + d)A$$

$d$  = AIDS-related death rate

**Treated population** (12), (13), (14), (15)

$$\frac{dT}{dt} = \tau I - \mu T$$

### Basic Reproduction Number $R_0$

$R_0 =$

$$\frac{\beta S_0}{\alpha + \mu + \tau}$$

#### Interpretation:

- $R_0 > 1$ : HIV spreads in population
- $R_0 < 1$ : HIV infection declines

#### Reduce transmission rate $\beta$

- Safe sex practices
- Condom use
- Sterile needles
- Safe blood transfusion

#### Increase treatment rate $\tau$

- Early diagnosis
- ART (Antiretroviral Therapy)

(iii) Reduce susceptible population (16), (17), (18)

- Awareness programs
- Pre-exposure prophylaxis (PrEP)

#### ART effect:

- Reduces viral load  $\rightarrow$  transmission  $\approx 0$
- Converts HIV into **manageable chronic disease** (19), (20), (21)

In model:

$$\tau \uparrow \Rightarrow I \downarrow \Rightarrow R_0 < 1$$

## 7. Equilibrium (Long-Term Result)

### Disease-free equilibrium:

$(I, A) = (0, 0)$  if  $R_0 < 1$

### Endemic equilibrium:

$(I > 0)$  if  $R_0 > 1$

conclusion

In conclusion, this type of mathematical model can help us analyze data related to various diseases. These can be very helpful in preventing, cure them and control them. (22),(23),(24),(25)

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