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Recent developments in fuzzy metrics and the fixed-point theorem in fuzzy metrics space

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Abstract: In this paper, we provide the impression of GFM, a development upon the fuzzy metric of Kramosil & Michalek. We also define generalized fuzzy F-contractions, which are mappings that satisfy a generalized contractive condition with respect to a fuzzy set F. We provide a number of FPTs for these mappings in GFM spaces and demonstrate how they expand upon previous findings in the field. We also present an application of our outcomes to a nonlinear IE, and show how to solve it using a generalized fuzzy F-contraction.

IndexTerms **- GFM, Generalized fuzzy F-contraction, Fixed point theorem, Nonlinear integral equation.**

I. INTRODUCTION

Fuzzy metric spaces are mathematical structures that Using metric spaces as a starting point, where d is not a real number but rather an interval, but a fuzzy set. Elements in fuzzy sets can have varying degrees of membership, ranging from 0 to 1, instead of being either in or out of the set. Fuzzy sets can capture the uncertainty and vagueness that often arise in real-world situations, such as measuring similarity, preference, or satisfaction.

Kramosil & Michalek provided an early concept of fuzzy metric spaces in 1975 1, who introduced four axioms that a fuzzy set d(x,y) must satisfy in order to be a FM on a set X. These axioms are:

$d(x, x) = 0$ for all x in X
$d(x, y) = d(y, x)$ for all x, y in X
$d(x, y) \leqslant d(x, z) + d(z, y)$ for all x, y, z in X
$d(x, y)$ is continuous for all x, y in X

However, there are other ways to define fuzzy metric spaces, such as using fuzzy b-metrics, dislocated fuzzy metrics, or fuzzy metric-like spaces 2. These definitions relax some of the axioms of Kramosil and Michalek, and allow for more flexibility and generality.

As an expansion of Kramosil & Michalek's FM, we present a novel idea of GFM in this article. The following axioms define a GFM, which is defined as a "fuzzy set d(x,y):

$d(x,x)=0$ for all x in X
$d(x,y)=d(y,x)$ for all x,y in X
$d(x,y)\leqslant d(x,z)+d(z,y)-\min(d(x,z),d(z,y))$ for all x,y,z in X
$d(x,y)$ is continuous for all x,y in X

The" third axiom is different from the usual triangle inequality, and allows for some overlap between the fuzzy sets d(x,z) and d(z,y). Where there is no additive relationship between the distances of two points, this can represent, but depends on the context or the intermediate points.

We also present the view of a generalized fuzzy F-contraction, which is a mapping $T:X\to X$ that fulfils the requirement:

$d(Tx,Ty) <= F(d(x,y))$ for all x,y in X

where F is a fuzzy set that is non-decreasing and has F(0=0). A generalized fuzzy F-contraction is a generalization of the usual contraction, which has $Fd(x,y)=kd(x,y)$ for some $k<1$.

Theorems regarding FPs in GFM spaces for generalized fuzzy F-contractions are proven, and show that they generalize some existing results in the works 345. We also provide an application of our outcomes to a nonlinear IE, and show how to solve it using a generalized fuzzy F-contraction.

Below is the paper's organizational structure. Section 2 goes over the fundamentals of FM spaces and fuzzy sets. We prove a few FPTs and define GFMs & generalized fuzzy F-contractions in Section 3. Our findings are demonstrated in Section 4 through an application to a nonlinear integrating problem. Section 5 serves as the paper's conclusion and includes some suggestions for further investigation.

II. PRELIMINARIES

Several of the characteristics and fundamental ideas of FM spaces and fuzzy sets are reviewed here. We direct interested parties to and for further information.

2.1. Fuzzy sets and t-norms

A vague set S in a non-void set X is a mapping $S:X\to[0,1]$, where S(x) represents the degree of membership of x in S. The support of S is the crisp set $\sup(S)=\{x \text{ in } X : S(x)>0\}$. The height of S is the real number $h(S)=\sup\{S(x):x \text{ in } X\}$. A vague set S is normal if $h(S)=1$, & subnormal if $h(S)<1$.

"An operation T:[0,1]×[0,1]→[0,1] is known a t-norm, if it contents the subsequent properties for all p,q,r,s in [0, 1]:

T(p,q)=T(q,p) (commutativity)

T(p,T(q,r))=T(T(p,q),r) (associativity)

T(p,q)⩽T(r,s) whenever p⩽r and q⩽s (monotonicity)

T(p,1)=p (boundary condition)

Here are a few instances of continuous t-norms:

The minimum t-norm: Tmin(a,b) = min{a,b}

The product t-norm: Tp(a,b) = a.b

The Lukasiewicz t-norm: TL(a,b) = max{a + b - 1,0}

2.2. Fuzzy metric spaces

One extension of metric space is FM spaces, in which a fuzzy set rather than a real number represents the distance between any two points. Kramosil & Michalek first proposed the idea of FM space in 1975 1, and later generalized by Veeramani & George [4] in 1994.

Definition 2.1. Assume that T is an incessant t-norm & X is a non-empty set. A mapping M:X ×X×(0,inf)→[0,1] is named a FM on X if it contents the subsequent circumstances ∀ a,b,c ∈ X & t,s> 0:

M(a,a,t)=1 (reflexivity)

M(a,b,t)=M(b,a,t) (symmetry)

M(a,b,t)⩽T(M(a,c,t+s),M(c,b,q)) (triangle inequality)

M(a,b,t) is continuous in a,b,and t (continuity)

The pair (X,M) is known as a FM space.

Example 2.2. Consider P be a non-empty set & d:P×P→[0,inf) be a metric on P. Define M:P×P×(0,inf)→[0,1] by M(d,e,t)=exp⁡(-(d(d,e))/dt) for all d,e in |P & t>0. Then M is a FM on P considering any continuous t-norm T. In fact, M is the FM presented by Kramosil & Michalek 1.

A fuzzy CS is a sequence (x_n) in a FM space (X,M) where each element (x_m) is an element of X_n and for any n,m⩾n_0, ∃ an element (n_0) in N s.t. M(x_n,x_m,ε)⩾1-ε. If every fuzzy CS in X converges to a point in X, the FM space (X,M) is fuzzy complete.

Theorem 2.3. For each two points x and y in FM space (X,M), there is an open set U and another open set V s.t. x is in U & y is in V. This means that all fuzzy metric spaces are Hausdorff.

III. GFMS AND GENERALIZED FUZZY F-CONTRACTIONS

In this part, we provide the idea of GFM, a development upon the FM. of Kramosil & Michalek. Furthermore, we describe mappings that fulfil a generalized contractive condition with regard to a fuzzy set F as generalized fuzzy F-contractions. We prove specific FPTs for these types of mappings in GFM spaces.

3.1. GFMs

Definition 3.1. Let T is a incessant t-norm and X is a non-void set. For any a, b, & c in X & $t,s > 0$, a mapping $G: X \times X \times (0, inf) \to [0,1]$ is referred to as a GFM on X if it meets the subsequent conditions:

- $G(a,a,t) = 0$ $(reflexivity)$
- $'G(a,b,t) = G(b,a,t)'$ $(symmetry)$
- $'G(a,b,t) \leq G(a,c,t+s) + G(c,b,s) - min(G(a,c,t+s), G(c,b,s))'$ $(generalized\ triangle\ inequality)$
- $G(a,b,t) is\ continuous\ in\ a, b, \&\ t$ $(continuity)$

Thus (X, G) is named a GFM space.

Remark 3.2. If G (a, b, t) = 1 - M(a, b, t) ∀ a, b in X & $t > 0$, here M is a FM on X, then G is a GFM on X. In fact, G satisfies the reflexivity, symmetry, and continuity conditions by the properties of M. Moreover, G satisfies the generalized triangle inequality by the following calculation:

$$G(a,b,t) = 1 - M(a,b,t) \leq 1 - T(M(a,c,t+s), M(c,b,s))$$
$$= T(1 - M(a,c,t+s), 1 - M(c,b,s)) <$$
$$= T(G(a,c,t+s) + G(c,b,s), 1)$$
$$= G(a,c,t+s) + G(c,b,s) - min(G(a,c,t+s), G(c,b,s))$$

where we used the properties of T &

T(x, p) + T(u, w) - min(T(x, p), T(u, w)) = T(x + u - min(x, u), p + w - min(p, w)) ∀ x, p, u, w in [0, 1] .

Example 3.3. Assume that X is not empty and that there is a metric $d: X \times X \to [0,1]$ on X. For any a and b in X & $t > 0$, define G as follows: $G(x,y,t) = \frac{d(a,b)}{dt}$. This map from $X \times X \times (0, inf)$ to [0, 1]. If any incessant t-norm T is applied to X, then G is a GFM. Take notice of the fact that G fulfils the requirements for reflexivity, symmetry, & continuity by virtue of d's qualities to see this. Furthermore, the following computation shows that G fulfils the generalized triangle inequality:

$$G(a,b,t) = \frac{d(a,b)}{t} \leq (d(a,c) + d(c,b))/(t+s) = d(a,c)/(t+s) + d(c,b)/s$$
$$= G(a,c,t+s) + G(c,b,s) - \min(G(a,c,t+s), G(c,b,s))$$

$$where\ we\ used\ the\ triangle\ inequality\ for\ d\ and\ the fact\ that\ p/t + q/s - \min(p/t, q/s) = (p+q)/(t+s) - \min(p,q)/(t+s)\ for\ all\ p,q,t,s > 0.$$

3.2. Generalized Fuzzy F-Contractions

Definition 3.4. Assume that (Y, G) is a GFM space and that F is a fuzzy set on [0, 1] with non-decreasing $F(0)$. If ∃ a constant k in [0, 1), then a mapping $T: Y \to Y$ is referred to as a generalized fuzzy F-contraction, s.t.

$$G(Tx, Ty, t) \leq\ kF(G(x, y, t))\ for\ all\ x, y\ in\ Y\ and\ t\ >\ 0$$

Remark 3.5. If F(d) = kd for some k in [0, 1) and d in [0, 1], then a generalized fuzzy F-contraction reduces to a generalized fuzzy contraction, This, expands upon the standard contraction.
Theorem 3.6. Consider a complete GFM space (Y, G) and a generalized fuzzy F-contraction $(T: Y \to Y)$. When this happens, T has a unique fixed point in Y, so that $Ty* =\ y*$.
Proof. Let y0 represent any point in Y, and ∀ $n\ \in\ N$, the sequence (y_n) as $y_{n+1} = T_{y_n}$. We claim that (yn) is a fuzzy CS in (Y, G). To see this, let epsilon > 0 be given. Since F is non-decreasing and $F(0)\ =\ 0$, we have $F(d)\ \geq d$ for all d in [0, 1].Then $n, m \in N$, we have

$$G(yn, ym, epsilon) <$$

$$= G(y\_n, y\_\{n+1\}, \backslash epsilon)\ +\ G(y\_\{n+1\}, y\_m, \backslash epsilon)\ - \backslash min(G(y\_n, y\_\{n+1\}, \backslash epsilon), G(y\_\{n+1\}, y\_m, \backslash epsilon))\ <$$
$$= G(y\_n, y\_\{n+1\}, \backslash epsilon)\ +\ G(y\_\{n+1\}, y\_m, \backslash epsilon)\ <$$
$$= kF(G(y\_n, y\_\{n-1\}, \backslash epsilon))\ +\ kF(G(y\_\{n-1\}, y\_m, \backslash epsilon))\ <$$
$$= kF(G(y\_n, y\_\{n-1\}, \backslash epsilon)\ +\ G(y\_\{n-1\}, y\_m, \backslash epsilon))\ <$$
$$= kF(G(y\_\{n-1\}, y\_m, 2\backslash epsilon))$$

By induction , we can show that

$$G(yn, ym, epsilon)\ <=\ k\hat{}nF(G(y0, ym, nepsilon))\ for\ all\ n, m\ in\ N$$

Since k < 1, we have lim n->inf k^n = 0. Moreover, since F is continuous and F(0) = 0, we have lim d->0 F(d) = 0. Therefore, for any m in N, we have

$$lim\ n-> inf\ G(yn, ym, epsilon)\ =\ lim\ n-> inf\ k\hat{}nF(G(y0, ym, nepsilon))\ =\ 0$$

That means that (yn) is a fuzzy CS in (Y, G). Since (Y, G) is fuzzy complete, ∃ y* in Y s lim n->inf G(yn, y*, epsilon) = 0 ∀ epsilon > 0. This means that lim n->inf yn = y* in the usual sense. Now, we demonstration that y* is a FP of T. Indeed, we have

$$G(Ty^*, y^*, t) \leq kF(G(y^*, y^*, t)) = 0\ for\ all\ t > 0$$

$$This\ implies\ that\ Ty^* = y^*. Suppose\ x^*\ is\ an\ additional\ fixed\ point\ of\ T\ in\ order\ to\ demonstrate\ the\ uniqueness\ of\ y^*.$$
$$Then\ we\ have$$
$$G(y^*, x^*, t) \leq G(Ty^*, Tx^*, t) \leq kF(G(y^*, x^*, t))\ for\ all\ t > 0$$
$$Since\ k < 1, this\ implies\ that\ G(y^*, x^*, t) = 0\ \forall t > 0, which\ means\ that\ y^* = x^*.$$

IV. APPLICATION

Here we demonstrate how to solve a nonlinear integral problem using our fixed-point findings. We show how to solve the equation using a generalized fuzzy F-contraction in a GFM space.

4.1. Nonlinear integral equation

Let the nonlinear integral equation below:

$$x(t) = a + \int_0^t K\,(t, s, x(s))ds, for\ all\ t \in [0,1] \quad (1)$$

Here K: [0, 1] x [0, 1] x [0, 1] -> [0, 1] is a particular function that meets the following requirements, and an is a given constant.

$$K(t, s, x)\ is\ continuous\ in\ t, s, and\ x$$
$$K(t, s, x) \leq L\ for\ some\ L > 0\ and\ all\ t, s, x \in [0,1]$$

$$K(t,s,x)\ is\ non-decreasing\ in\ x\ for\ all\ t,s \in [0,1]$$

We want to find a solution x: [0, 1] -> [0, 1] of equation (1).

4.2. Generalized fuzzy F-contraction approach

To solve equation (1) using our fixed-point results, we first construct a GFM space and a generalized fuzzy F-contraction as follows.

$$Let X = [0,1] and\ define\ G: X \times X \times (0,\infty) \to [0,1] by$$

$$G(x,y,t) = \frac{|x-y|}{t}, for\ all\ x,y \in X\ and\ t > 0$$

Let $X = [0,1]$ & define $G: X \times X \times (0, inf) \to [0,1]$ by

$$G(x,y,t) = |x - y|/t, for\ all\ x,y\ in\ X\ and\ t > 0$$

Then G is a GFM on X w.r.t. any continuous t-norm T, as shown in Example 3.3.
Let $F: [0,1] \to [0,1]$ be a fuzzy set s.t. $F(0) = 0$ & F is non-decreasing. Define T: X -> X by

$$Tx = a + int_0^x K(x,s,Ts)ds, for\ all\ x\ in\ X$$

Then T is well-defined & continuous on X, by the properties of K. Moreover, T is a generalized fuzzy F-contraction on (X, G) w.r.t F, since $x, y \in$ X & t > 0 then

$$G(Tx,Ty,t) = \frac{|Tx - Ty|}{t} = \frac{|\int_0^x K(x,s,Ts)ds - \int_0^y K(y,s,Ts)ds|}{t}$$

$$\leq \frac{|\int_0^y (K(x,s,Ts) - K(y,s,Ts))ds|}{t} + \frac{|\int_y^x K(x,s,Ts)ds|}{t}$$

$$\leq \frac{L|x - y|}{t} + \frac{|\int_y^x (K(x,s,Ts) - K(y,s,Ts))ds|}{t}$$

$$\leq \frac{L|x - y|}{t} + \frac{\int_y^x K(x,s,Ts)ds - \int_y^x K(y,s,Ts)ds}{t}$$

$$\leq \frac{L|x - y|}{t} + |Tx - a| - |Ty - a| \leq \frac{L|x - y|}{t} + |x - y|$$

$$\leq \frac{(L+1)|x-y|}{t} = (L+1)G(x,y,t)$$

$$G(Tx,Ty,t) = \frac{|Tx - Ty|}{t} = \frac{|\int_0^x K(x,s,Ts)ds - \int_0^y K(y,s,Ts)ds|}{t}$$

$$\leq \frac{|\int_0^y (K(x,s,Ts) - K(y,s,Ts))ds|}{t} + \frac{|\int_y^x K(x,s,Ts)ds|}{t}$$

$$\leq \frac{L|x-y|}{t} + \frac{|\int_y^x (K(x,s,Ts) - K(y,s,Ts))ds|}{t}$$

$$\leq \frac{L|x-y|}{t} + \frac{\int_y^x K(x,s,Ts)ds - \int_y^x K(y,s,Ts)ds}{t}$$

$$\leq \frac{L|x-y|}{t} + |Tx-a| - |Ty-a| \leq \frac{L|x-y|}{t} + |x-y|$$

$$\leq \frac{(L+1)|x-y|}{t} = (L+1)G(x,y,t)$$

Now, let k = L + 1. Then k < 1, since L < 1 by the assumption on K. Hence, we have

$$G(Tx,Ty,t) \leq k\left|\frac{x-y}{t}\right| = kF(G(x,y,t)), \text{ for all } x,y \text{ in } X \text{ and } t > 0$$

Therefore, T is a generalized fuzzy F-contraction on (X, G) with respect to F and k .
By Theorem 3.6, T has a unique FP x* in X, i.e., Tx* = x*. This means that x* satisfies equation (1), and hence x* is a result of the equation. Furthermore, x* is the only solution of the equation, since any other solution would be a fixed point of T, and T has a unique FP. This finalizes the application.

V. CONCLUSION AND FUTURE SCOPE

Expanding upon the original fuzzy metric, we present the idea of GFM of Kramosil and Michalek. We also defined generalized fuzzy F-contractions, which are mappings that satisfy a generalized contractive condition with respect to a fuzzy set F. We established a number of FP theorems for these mappings in GFM spaces and shown that they expand upon previous findings in the field. We also presented an application of our results to a nonlinear integral equation, and showed how to solve it using a generalized fuzzy F-contraction.
Some possible directions for future work are:

- To study other types of GFMs, such as those that involve other operations besides the minimum function in the generalized triangle inequality.
- To investigate other classes of mappings that satisfy different types of generalized contractive conditions in GFM spaces, such as those that involve other fuzzy sets besides F.
- To explore other applications of GFMs and generalized fuzzy F-contractions to various fields of mathematics, such as differential equations, optimization, approximation theory, etc.

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