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Modelling Techniques for Measurement of High-Speed Train-Induced Soil Vibrations

Rao Pooja S[1], Desai Atul K[2], Solanki Chandresh H[2]

[1] Department of Civil engineering, A. P. Shah Institute of Technology, Thane, Maharashtra, INDIA

[2] Civil Engineering Department, Sardar Vallabhbhai National Institute of Technology, Surat, INDIA

ABSTRACT

The propagation of ground vibrations caused by a moving train is a complex phenomenon influenced by multiple factors. A primary contributor to vibration generation is the train itself, as its interaction with the track components—including rails, sleepers, embankments, and ground support—produces dynamic forces at the wheel-rail interface. These forces result in motion within both the track structure and the train, leading to the transmission of vibrations into the surrounding environment. Accurately analyzing the induced vibrations, vibratory behavior of rail locomotives and their interaction with track foundations is crucial for minimizing the impact of railway-induced vibrations on infrastructure and nearby communities. Advances in technology have significantly enhanced the ability to study ground vibrations, offering deeper insights into their causes and effects. A comprehensive understanding of these vibrations enables more precise predictions of their intensity and facilitates the development of effective mitigation strategies. The following section provides a summary of key studies that contribute valuable knowledge to this field.

Keywords: Dynamic analysis, High speed train, Vibrations, modelling techniques, quarter car model

INTRODUCTION

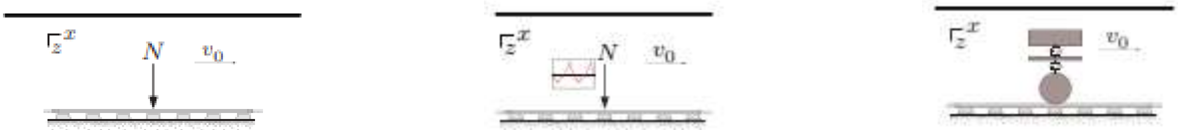
The introduction of high-speed rail systems in India, particularly the Mumbai–Ahmedabad High-Speed Rail implemented by the National High Speed Rail Corporation Limited, represents a major advancement in the country's transportation infrastructure. While these systems significantly enhance connectivity, reduce travel time, and support economic growth, they also introduce critical engineering challenges related to dynamic loading and ground vibrations. Understanding and accurately modeling the vibratory characteristics of rail locomotives and their interaction with track foundations are essential for managing and mitigating the potential impact of railway vibrations on infrastructure and nearby communities. Engineers and researchers use advanced simulation techniques to address these challenges effectively. The primary cause of vibrations in rail systems is the transfer of forces from the locomotive's wheels into the track. These forces are influenced by two main factors: the weight of the vehicle and irregularities or discontinuities at the wheel-rail interface. These forces then propagate into the supporting track foundation, leading to vibrations. Once generated, vibrations propagate through the track, the underlying soil, and nearby structures. The extent and severity of vibrations in each component depend on their natural frequencies. When the excitation frequency matches or is close to the natural frequency of a component, resonance can occur, amplifying the vibrations. To effectively address railway vibration issues, it is crucial to simulate the behavior of the vehicles and the resulting forces accurately. This simulation should closely approximate the physical problem to assess the potential impact of vibrations on track, soil, and nearby structures. Proper modelling helps engineers and planners make informed decisions regarding design and mitigation measures.

MODELLING APPROACHES FOR VEHICLE

To accurately predict ground vibrations caused by trains and understand their structural impact, two key components must be considered: (1) Correct Vehicle Modelling: The features of the train vehicles must be accurately modelled, as the vibration levels depend on the dynamic forces produced by the train. This involves modelling not only the train's mass and geometry but also factors like wheel-rail interactions and suspension systems (Figure 1). (2) Accurate Soil Modelling: The soil in the vicinity of the track must be modelled accurately. This is crucial to understanding how the induced vibration waves propagate through the soil, as different soil types and conditions can affect the transmission of vibrations.

Modelling Vehicle using Constant Axle Load: Modelling vehicles using constant axle loads is a simplified approach often used in the analysis of ground vibrations and their effects on tracks and soil. This method involves representing a vehicle as a series of discrete point loads (axles) moving along a track. While it may not capture all the complexities of real-world vehicle dynamics, it serves as a valuable tool for studying specific aspects of ground vibrations, track deflection, and critical velocities. More advanced models, such as multi-body dynamic simulations, can capture the intricate dynamics of vehicles with greater accuracy. However, these advanced models often require more computational resources and data. Thus, a constant axle load representation offers a practical way to assess the impact of rolling stock on tracks and the environment, especially in situations where a simplified representation is sufficient for the analysis.

Modelling Vehicle using Sequence of Randomly Varying Axle Load: In practice, track deflection due to axle loads $w_{st}(t)$ and track displacement induced by track irregularities $w_{dyn}(t)$ play an important role in the development of wheel and rail interaction forces. This is important for examining dynamic excitation in the evaluation of railway generated ground vibrations.



Constant Axle Load　Random Axle Load　Quarter Car Model

Fig. 1 Classification of vehicle modeling technique, [1]

The total dynamic response of the track can be expressed as the sum of the static track deflection ($w_{st}(t)$) and the dynamic response induced by irregularities ($w_{dyn}(t)$) [1]. w(t) represents the combined track displacement due to both static and dynamic effects.

$$w(t) = w_{st}(t) + w_{dyn}(t) \qquad 1$$

Track deflection refers to the vertical displacement of the railway track caused by the weight and axle loads of passing trains. This component can be determined through various analytical and numerical methods, such as finite element analysis, which consider the properties of the track, the train, and the loads applied. The track displacement induced by irregularities on the other side represents the additional track displacement caused by irregularities or imperfections in the track or wheel surfaces. Irregularities may include track geometry defects, such as misalignments, dips, or unevenness, as well as wheel surface imperfections. These irregularities lead to dynamic interactions between the wheels and track, resulting in additional displacements and forces. Thus, modeling the dynamic behavior of axle loads and their interaction with rail and surface imperfections is crucial for ensuring the safe and efficient operation of railway systems and minimizing the impact of ground vibrations on the surrounding environment.

Modelling Vehicle using Quarter Car Model: This is a multi-body or simplified model used in vehicle dynamics simulations. It divides the train or vehicle into four quarters: one-quarter representing the car body and one-quarter representing the bogie (a set of wheels and their suspension) for each end of the car. This simplification allows for easier analysis of the vehicle's behavior [[2]. Figure 2 illustrates a schematic diagram of a quarter car model. In this model, the entire carriage is divided into four separate quarter-car models. Each of these quarter-car models consists of one-fourth of the total car mass, and the bogie mass is considered to be half of the original carriage mass. To represent this multibody system, the body of car and axles are considered as rigid components, while linear springs and dampers are used to represent the primary and secondary suspensions respectively. The load transmitted to the track depends on the mass of the car, $\bar{M}$c, the mass of the bogie, $\bar{M}$b, the mass of the wheels represented as $\bar{M}$w, the primary suspension represented as $\bar{k}_1$ and $\bar{c}_1$, the secondary suspension given as $\bar{k}_2$ and $\bar{c}_2$, and train configuration $\ddot{x}c$, $\ddot{x}b$, $\ddot{x}w$ and uc represent vertical displacement of carriage, bogie wheel and track respectively. During vehicle/track interaction the forces are transmitted via the wheel/rail contact area. The coupling of vehicle and track is accomplished by the wheel-rail contact relationship assumed by Hertzian contact theory. Thus, a 3-DOF dynamic system developed to represent the vehicle model is given as

$$[\bar{M}]\{\ddot{x}\} + [\bar{c}]\{\dot{x}\} + [\bar{k}]\{x\} = \{F\} \qquad 2$$

All the connected bogies move at the constant speed. For simplicity it is assumed that for a given vehicle, each bogie presents the same dynamic characteristics and therefore the axle load throughout the vehicle is assumed to be same. The axle equations of motion can be written as follows:

$$\begin{bmatrix} \bar{\mathrm{M}}\mathrm{c} & 0 & 0 & 0 \\ 0 & \bar{M}b & 0 & 0 \\ 0 & 0 & \bar{M}w & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{z}c \\ \ddot{z}b \\ \ddot{z}w \\ \ddot{u}c \end{Bmatrix} + \begin{bmatrix} \bar{c}2 & -\bar{c}2 & 0 & 0 \\ -\bar{c}2 & \bar{c}1 + \bar{c}2 & -\bar{c}1 & 0 \\ 0 & -\bar{c}1 & \bar{c}1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{z}c \\ \dot{z}b \\ \dot{z}w \\ \dot{u}c \end{Bmatrix} +$$

$$\begin{bmatrix} \bar{k}2 & -\bar{k}2 & 0 & 0 \\ -\bar{k}2 & \bar{k}1 + \bar{k}2 & \bar{k}1 & 0 \\ 0 & -\bar{k}1 & \bar{k}1 + \bar{k}H & \bar{k}H \\ 0 & 0 & -\bar{k}H & \bar{k}H \end{bmatrix} \begin{Bmatrix} zc \\ zb \\ zw \\ uc \end{Bmatrix} = \begin{bmatrix} \bar{\mathrm{M}}\mathrm{c}\ \mathrm{g} \\ \bar{M}b\ g \\ \bar{M}w\ g + \bar{M}w sin\omega(t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ F_c(x,t) \end{bmatrix} \qquad 3$$

where, F_c (x, t) defines the total vertical force in the vicinity of wheelset and rail when train is in motion at any given time, while the total vertical contact force acting at the wheel and rail interface which includes both effect of vehicle load when not in motion and the contact force between wheel and rail is given as P(t) as shown below:

$$\text{where, } P(t) = Fc\ (x, t) + [\ 0.5(Mc + Mb) + Mw]\ g \qquad 4$$

Hertzian contact model devised to derive Fc as described in Equation (5) can be used to compute hertzian spring constant for known value of *Fc* (x, t) based on Equation (3).

$$Fc\ (x, t) = \bar{k}H\Delta z(t)^{3/2} \qquad 5$$

where, Δz is defined as overlap of wheel rail in the vertical direction. The overlap is defined by the relative motion of the wheel with respect to the rail in the absence of a wheel defect and is given by

$$\Delta z(t) = zw - uc\ (t) - r \qquad 6$$

where, zw, uc(t) and r, are the wheel deflection, rail deflection and track irregularities respectively in the vertical plane. Hence, according to the nonlinear Hertzian theory, the interaction force between the rail and wheel is given by,

$$\begin{aligned} Fc\,(\mathrm{x}, \mathrm{t}) &= \bar{k}H\,(\mathrm{zw} - uc\,(\mathrm{t}) - \mathrm{r})^{\frac{3}{2}},\ if\ \mathrm{zw} > uc\,(\mathrm{t}) + \mathrm{r} \\ &= 0 \qquad ,\ \mathrm{else} \end{aligned} \qquad 7$$

In the absence of track irregularities r = 0.

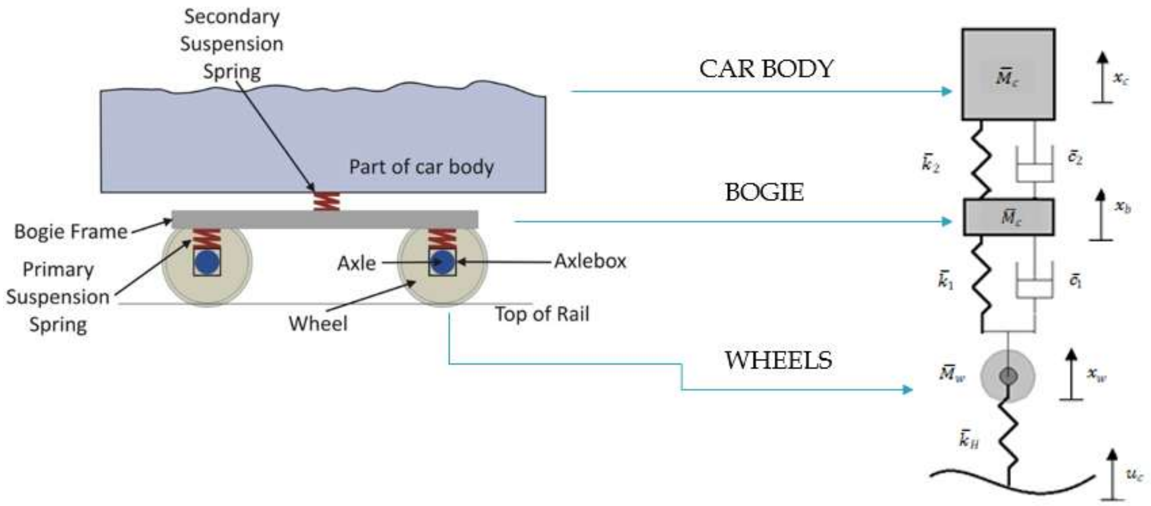


Fig.2 Schematic representation of various parts of locomotive and Quarter Car Model [1]

NUMERICAL APPROACH FOR WHEEL-RAIL CONTACT

During vehicle/track interaction the forces are transmitted via the wheel/rail contact area. On account of the geometry of the contact area between the round wheel and the rail, the relationship between force and compression, represented by the Hertzian contact spring, is not linear. Since a description of the wheel/rail relationship using transfer functions requires that all components are linear the Hertzian spring must also be linearized. The relationship between force F and indentation y of the contact surface is:

$$F = C_H y^{\frac{3}{2}} \qquad 8$$

where, C_H = hertzian spring stiffness, N/m$^{-3/2}$

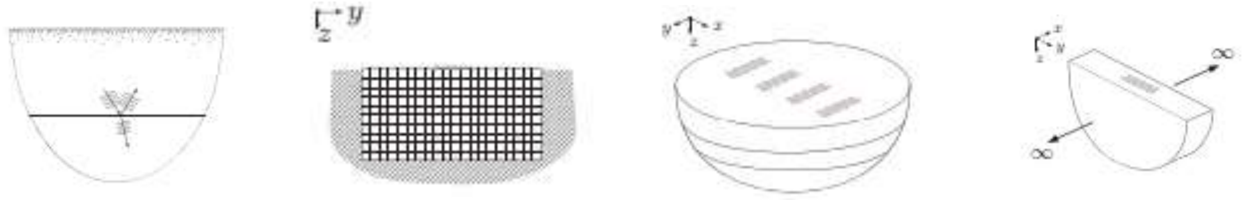
The linearized value of the stiffness can be found by considering the relationship between the force and displacement increments around the static wheel load. The linearized Hertzian spring stiffness k_H is then:

$$k_H = \frac{dF}{dy} = \frac{3}{2} C_H^{\frac{2}{3}} F^{\frac{1}{3}} \qquad 9$$

Jenkins et al, [3], determined the C_H value for old and new wheels as a function of the wheel diameter. For a wheel diameter of 1 m and a static wheel load of 7.5 t at k_H value of 1.4x10^9 N/m is found for new wheels and 1.6x10^9 N/m for old wheels (Figure 2).

PREDICTION MODELS FOR TRAIN INDUCED SOIL-GROUND VIBRATION

The vibration propagates from the track into the subsoil causing the ground vibration. This further arouse secondary vibration and noise of nearby structures and buildings, which seriously influences the living and working environment of the people. Vibration prediction plays a vital role in the environmental assessment of new railway lines and the vibration analysis of new buildings near railway lines. Theoretical analysis, numerical simulation, and field experiment predict vibration with high accuracy. These methods have their own advantages, but are time-consuming and costlier. Recently, high-accuracy prediction method takes artificial neural network as one of the supervised machine learnings together with numerical simulation and experiment method to predict train induced vibrations. Thus, recent technological advances have opened up new possibilities for analyzing ground vibrations, thereby providing a deeper insight into the subject. With a better understanding of the problem, it is possible to predict vibration levels and reduce the problems associated with these types of vibrations.



Analytical Solution 2D FEM 3D FEM 2.5D BEM and/or FEM

Fig. 3 Classification of soil modeling technique [1]

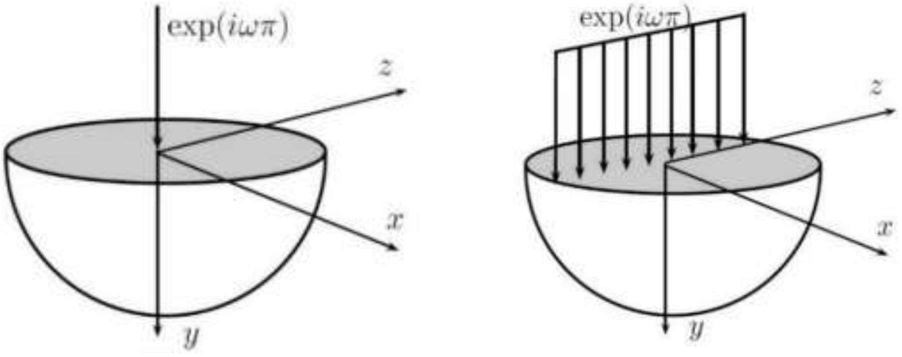
2.5.1 Empirical Method:

This method relies on real-world data collected from field measurements and observations. In the context of railway tracks, it involves monitoring and recording various parameters such as track geometry, train speeds, types of trains, track-ground conditions, and environmental factors. The data is then analysed to make predictions about the performance and response of the railway track system. Empirical methods are valuable when dealing with specific, well-defined conditions for which sufficient data is available. One of the applications of the empirical method is assessing the environmental impacts of railway tracks. This includes studying how factors like noise, vibration, and ground settlement affect the surrounding environment and communities [4], [5]. The empirical method has its limitations. It is primarily useful when dealing with specific scenarios and conditions for which there is ample data. In this context, mathematical and numerical models offer greater advantages due to their inherent flexibility, as they are not constrained by specific conditions.

2.5.2 Analytical Method:

The study of how railway tracks respond to moving loads on a flexible surface has been a subject of theoretical exploration since the early 19th century. In 1904, Lamb conducted pioneering research into the dynamic behaviour of elastic half-spaces and elastic bodies subject to impulsive loads applied at specific points or along lines on their surfaces, considering infinite boundary conditions (Figure 4). Subsequently, additional research endeavours led to the creation of diverse analytical models designed to forecast the dynamic reaction of both half and full spaces in response to harmonic point and line loads, carried out by various investigators [6], [7]. These studies reveal that when a harmonic load is applied to a surface, it leads to the generation of two distinct waves propagating away from the point of loading.

Researchers [8], [9], [10], [11]studied the dynamic response of a track structure to moving loads after investigating the dynamic response of an elastic medium when subjected to harmonic loading. They found that the dynamic response of an elastic medium increases with the increase in the speed of moving loads. This dynamic response was categorised as follows: (1) when the speed of the moving load is less than the speed of S-waves of the medium (i.e., V < Vs), it is called as subsonic condition, (2) when the speed of a moving load is lower than the speed of P- waves but higher than the speed of S-waves (i.e. Vs < V < Vp) then it called as transonic condition and (3) The condition is supersonic if the speed of a moving load is higher than the P-waves in the medium (i.e. V> Vp). The beam on Winkler foundation is the simplest and most



(a) Point load **(b) Line load**

Fig. 4 Classical Lamb's model with harmonic [12]

suitable model for simulating railway track with embankment and surrounding soil among all the analytical models. The Winkler model simulates the track as an infinite beam, discretely supported by springs and dashpots, as shown in Figure 5 to study the dynamic response of a moving load.

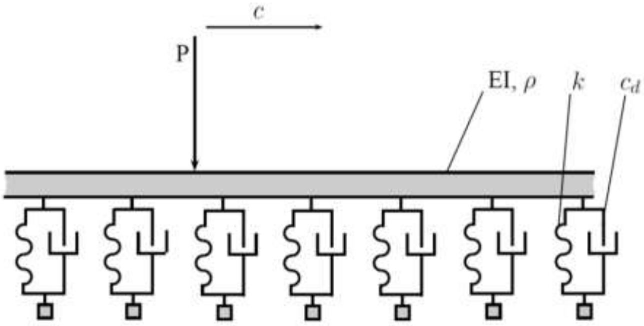


Fig. 5 Beam on Winkler foundation [12]

The general differential equation of the track response due to moving loads is given by:

$$EI\,\frac{d^4 u_v(x,t)}{\partial x^4} + \rho l\,\frac{d^4 u_v(x,t)}{\partial x^2} + 2c_d\,\frac{d^4 u_v(x,t)}{\partial x} + \mathrm{kd}u_v = \mathrm{P}\delta(\mathrm{x} - \mathrm{Ct}) \qquad 10$$

where: EI is the flexural stiffness of the Winkler beam (embankment); uv is the vertical displacement; ρl is the mass per unit length of the beam; k is the stiffness of the Winkler foundation; cd is the viscous damping of foundation; P is the vertical load and δ (x - vt) is the Dirac delta function of the moving load at speed v. The parameters of an analytical model are determined based on the characteristics of the track-ground system. This system can be represented in two different ways. In the first approach, the model employs a beam to characterize the rails, while the Winkler foundation accounts for all other components of the track structure. In the second approach, the beam represents the rails, rail pads, and sleepers, while the Winkler foundation represents the subgrade. While the beam on Winkler foundation model is valuable for predicting track deflection and serves as a suitable benchmark for validating numerical tools, it has a notable limitation—it cannot transmit shear stresses. Furthermore, due to certain assumptions and divisions in this analytical approach, the solutions it provides are often insufficient for addressing real-world practical problems.

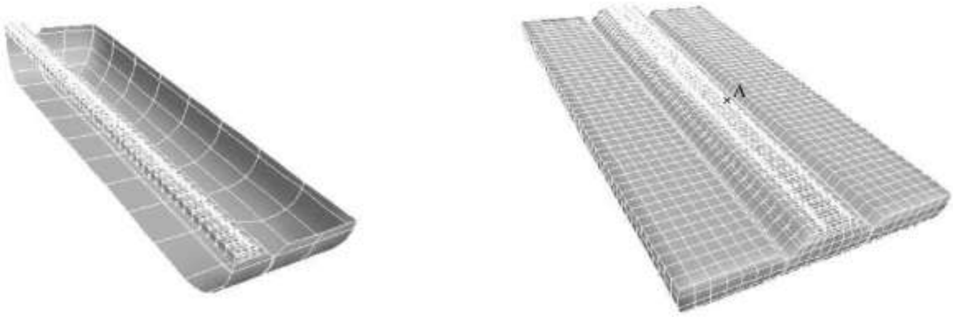
2.5.3 Numerical Method:

The need to address the limitations of empirical and analytical techniques drove the evolution of numerical methods, strengthened by the challenging computational power of modern computers. Among the numerical methods frequently employed to model the dynamic behaviour of railway tracks, the finite element method (FEM), boundary element method (BEM), finite difference method (FDM), and discrete element method (DEM) stand out. These methods employ distinct strategies to tackle the boundary value problem linked to the track-ground system. In practice, the choice of numerical method depends on the specific problem at hand, including factors such as the geometry of the track, the materials involved, and the computational resources available. Often, a combination of these methods or advanced hybrid techniques may be employed to capture different aspects of railway track dynamics efficiently. Advances in computer hardware and software continue to expand the capabilities of numerical simulations in railway track analysis, allowing engineers to make more informed decisions in track design and maintenance. The most common numerical approaches to simulate the railway track responses and their relative advantages and disadvantages are briefly discussed below.

2.5.3.1 Finite Element Method (FEM):

FEM is favored in various science and engineering fields because it offers a detailed way to define track geometry and allows for the incorporation of complex constitutive models for track materials. This makes it possible to accurately simulate the response of railway tracks to dynamic train loads within the surrounding soil.

The accuracy of FEM simulations heavily depends on how the model boundaries are treated. If not handled properly, soil modeling in finite elements can produce inaccurate results. Specifically, when the boundaries of a finite element mesh are constrained, waves generated by dynamic wheel loads may reflect at these boundaries instead of propagating outward, causing disturbances in the numerical simulation [13]. This phenomenon introduces disturbance in the numerical simulation. To address the issue of wave reflections, researchers have proposed various techniques. One method introduced by John Lysmer and Kuhlemeyer [14], involves using non-reflecting viscous boundaries that absorb incoming waves perfectly if aligned correctly with the incident wave direction. Another approach, proposed by Bettess [15] uses infinite elements to represent decay behavior as dimensions approach infinity, thus preventing reflections. John P. Wolf and Chongmin Song [16] introduced the Scaled Boundary Finite Element Method (SBFEM) in 1996. This method is particularly suited for simulating unbounded domains, such as the track substructure in railway modeling. SBFEM helps avoid wave reflections, even when support restraints are present at soil-structure interfaces. It has been shown to provide accurate results that align well with field measurements. In their 2002 study, Ekevid and Wiberg [17] employed a method to replicate the dynamic behavior of a standard railway track (Figure 6). Their findings highlighted that utilizing the SBFEM (Stabilized Boundary Finite Element Method) yielded minimal or non-existent wave reflections, even in cases where support restraints were applied at the soil-structure interface nodes. Moreover, the time-history vertical displacements obtained at a specific location on the track closely matched the real-world field measurements.



(a) discretisation of the structure-unbounded media interface **(b) the finite element model**

Fig. 6 Coupling of FEM and SBFEM [17]

For dynamic analysis, it is still essential to develop a larger model than that used for static analysis. Moreover, the mesh size should be based on the maximum frequency of loading. Subsequently, the number of elements becomes much larger than that of the static FE model, as a result of which the FE models for dynamic analysis are computationally too costly. To streamline computational efficiency, [18] initially crafted a 3D Finite Element (FE) model. This model applied static loads to ascertain equivalent beam parameters. These parameters were then transferred to a second model, employed to calculate the ballast surface force under the sleeper over time. Subsequently, this force was incorporated into a third model—a 2D plane strain representation of the structural cross-section. However, this model falls short in accurately predicting stress under the dynamic loads imposed by moving trains due to its neglect of principal stress rotation. Despite these limitations, several researchers, such as Gomes Correia et al. [19], persisted in utilizing two-dimensional (2D) plane strain FE modeling to replicate the dynamic behavior of railway tracks. Figure 7 illustrates a typical instance of a 2D plane strain model depicting a ballasted railway track.

2.5.3.2 FE-BE Method:

The Boundary Element Method (BEM) proves highly effective for simulating wave propagation in soil, but faces limitations when dealing with material nonlinearity and complex geometries. In the context of simulating railway tracks, a common practice is to employ BEM for modeling the track foundation solely. For the track structure itself, different numerical methods are often preferred to accurately capture the complex responses due to material nonlinearity and geometric intricacies. In this context, 2.5D is an efficient tool to imply the track-soil interaction, where the soil is modelled using the BEM approach. In the 2.5D model, the transversal geometry of the track structure is only discretized in 2D, and the transversal section of the model is kept invariant in the longitudinal direction [20][21], [22].

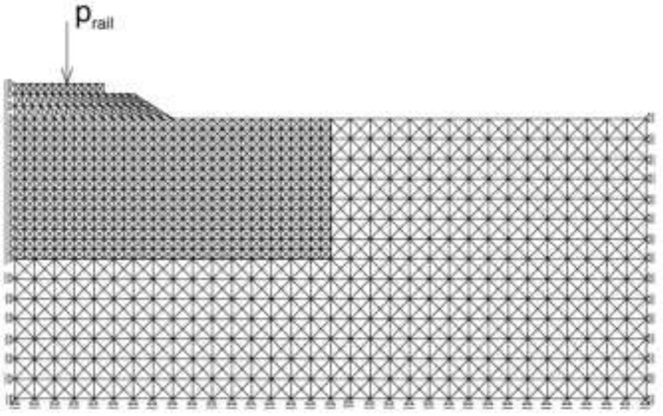


Fig. 7 Example of a 2D plane strain FE modelling of the track and soil [20]

In 2006, Sheng, Jones, and Thompson [23], introduced a predictive model that utilized a 2.5D Finite Element-Boundary Element Method (FE-BEM) to anticipate ground vibrations resulting from the passage of trains. This model had the capability to account for the irregular shape of the track's transverse geometry. However, it assumed that both the constructed structures and the ground remained uniform in the longitudinal direction. A similar approach was adopted by other researchers [22][24], [25]to simulate the vibrational response within the track and the propagation of waves through the surrounding soil, particularly in response to high-speed train movements (as depicted in Figure 7). Furthermore, [26], developed a periodic FE-BE coupling method to predict the dynamic behavior of exceptionally long structures. This method found widespread application among researchers [24], [27], [28] when simulating ground vibrations emanating from underground railway tracks.

2.5.3.3 Discrete Element Method (DEM):

The DEM (Discrete Element Method) serves as a powerful computational tool for gaining a detailed understanding of how granular materials, such as ballast, behave on a microscale. Within the DEM framework, the system consists of just two fundamental components: particles and walls. The computational process follows a time-stepping algorithm that involves several key steps.

Firstly, it necessitates repeatedly applying the laws of motion to each individual particle, continuously updating the positions of walls, and implementing a force-displacement relationship for every contact point. At the outset of each time step, the set of contact points is refreshed based on the known positions of particles and walls. Subsequently, the force-displacement relationship is employed at each contact point to modify the contact forces, considering the relative motion between the two entities at the point of contact and the specific contact constitutive model. Following this, the laws of motion are applied to each particle to adjust their velocity and position. This adjustment takes into account the resultant forces and moments generated by the contact forces and body forces acting on the particle. Additionally, the positions of the walls are updated in accordance with specified wall velocities. The Discrete Element Code YADE typically executes this calculation cycle, as illustrated in Figure 8 [29].

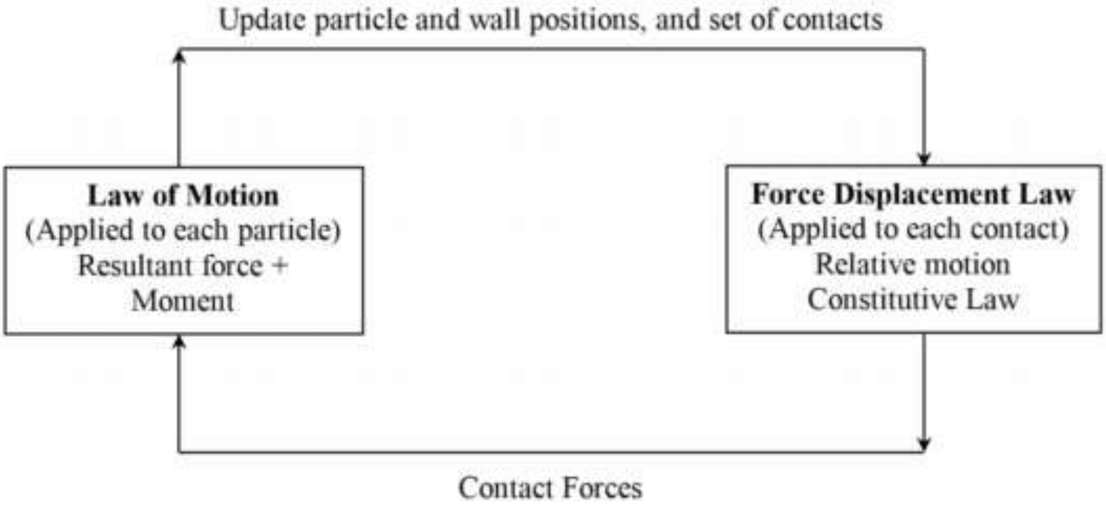


Fig. 8 The Calculation cycle of DEM in YADE[30]

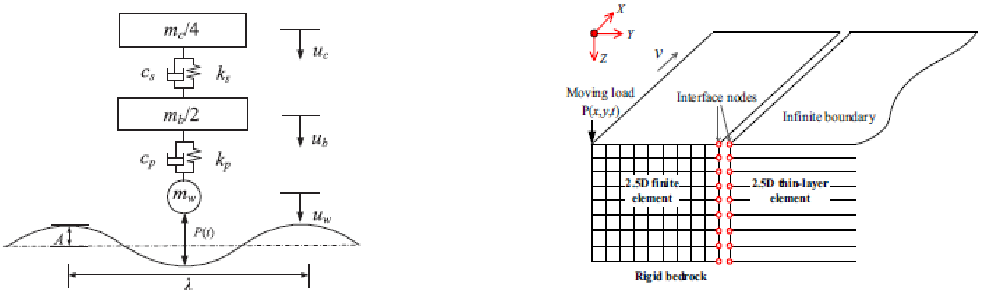
Several studies have utilized Discrete Element Method (DEM) to replicate and explore the micromechanical behavior of ballast material in both triaxial compression tests [31], [32], [33]However, the extensive computational resources required by DEM have limited its application in simulating ballast behaviour within comprehensive numerical track models. In instances where DEM was employed to model ballast, it only managed to replicate a small section of the track, neglecting interaction with the subgrade. Moreover, the length of the models was typically reduced to encompass just 1 to 5 sleepers in 2D configurations [33] [34], [35]. These limited longitudinal models were inadequate for simulating the movement of loads while considering factors such as time-dependent modulus.

The following section presents brief of the work done by various researchers using above mentioned techniques.

Rao P S et. ai., 2021 [2] presented a quarter car model (lumped mass model) to replicate CHR3-type high-speed train. The passage of train over the track structure is represented as a sinusoidal harmonic load. The study establishes that the statistical analysis of the attenuation of the waves is exponentially related to the distance from the source and follows the Bornitz equation. The frequency-independent damping ratio is computed considering near-field and far-field effect of induced vibration in the soil. Finally, the damping ratio and transmissibility are used to derive the critical velocity for the given track near the proposed site.

Bian et al. 2015[36] presented study based on the 2.5D finite element/thin- layer element coupled analysis model to examine the vibration response caused by train moving loads. The quarter-car model was used to deduce the equation for the wheel–rail contact force under the condition of track irregularity as shown in Figure 9. Here the track and the near-field region of the model (the track, foundation and part of the subsoil) were modelled using the 2.5D finite elements, whereas the wave propagation in the subsoil from the near-field region laterally outwards to infinity on the two sides was modelled by the thin-layer element as shown. The influence of track irregularities of four typical wavelengths on the vibrations of the track and the surrounding ground environment were also investigated.

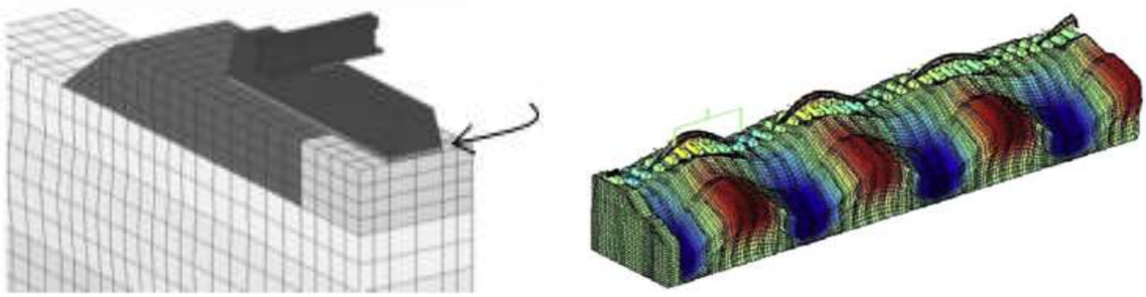
Ferreira and López-Pita 2015**,** [37] presented a numerical model Dynavoie, Figure 10, which was built to predict train/track dynamic response, not only instantaneously but also in a long-term perspective. The model developed is a FE based that considers dynamic interaction between the train and track with all its components and is able to make multiple simulations with very low computation time.



(a) **(b)**

Fig. 9 (a) Quarter car model (b) 2.5D finite element/thin layer element coupled model [37]

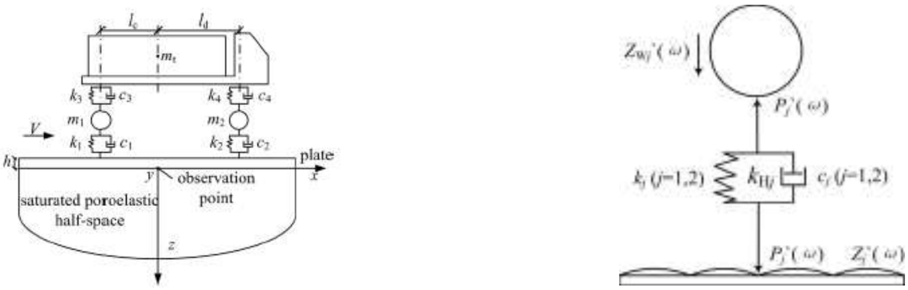
The use of modal sub-structuring approach allows the significant decrease on the total number of degrees of freedom as the track was assumed to be longitudinally split in several cross-sectional slices. They build slice model using 3D finite elements techniques and takes into account the models of the rail, sleepers, fastening and rail pads, ballast and sub-ballast layers and all other layers of soil including the subgrade soil foundations. Time simulation of a vehicle travelling on the railway track at a certain speed was achieved through Newmark time integration implicit algorithm with variable and auto-adopted step.



(a) sleeper with under sleeper pads **(b) Image of a mode shape of the complete model**

Fig. 10 Part of the model in Dynavoie [38]

Cai et al. 2015[38] established a truck–pavement–ground coupling model Figure 11 to study the dynamic responses of a saturated poroelastic half-space generated by a moving heavy truck on the uneven pavement. The ground was simulated as a fully saturated poroelastic half-space governed by Biot's theory ignoring the compressibility and the apparent mass density of soil grains. The asphalt mixture in the flexible pavement system was modeled as a Kirchhoff thin plate which extended to infinity in the horizontal directions. The heavy truck was modelled as a rigid body system connected with spring sand dampers. For the solution of the pavement-ground system the tire-pavement contact area was simplified as a rectangle as given by Cai et al. 2015 [39] **.** The traffic loads were modelled as four rectangular load pressures. With the assumption of a sinusoidal pavement surface, the dynamic wheel–pavement force was obtained through a linear Hertizian contact model against the excitation frequency of two truck speed. They solved the governing equations by Fourier transform and calculated the time domain results by fast inverse Fourier transform. The stress and the excess pore water responses in the half space generated by both the axle load and dynamic load were studied at different truck speeds.



(a) Truck-pavement-ground coupling model **(b) Wheel-set and ground coupled model**

Fig. 11 Saturated poroelastic half-space model with road irregularities[39]

Connolly et al. 2014**,** [41] proposed vibration prediction tool ScopeRail based on 3D finite element model that was developed using a statistical approach as proposed by the Federal Rail Road Administration. The tool help to analyze change in vibration levels due to under the wide range of soil without performing physical investigations at large number of test sites, each with diverse soil characteristics. The soil was modelled as a stratified half space with four sides truncated by an absorbing boundary condition (infinite elements). The vehicle was modelled using a lumped mass multi-body approach Figure 12. The wheel and rail interface force were computed using non-linear Hertzian contact spring. They modelled only half of all three components like track, soil and vehicle in finite element package ABAQUS using dynamic explicit central differencing scheme.

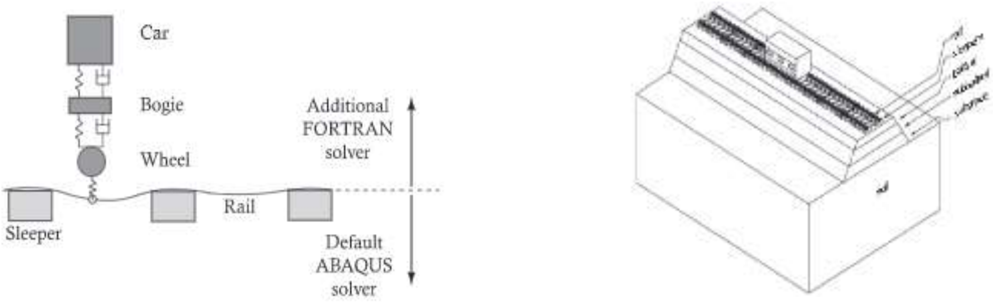


Fig. 12 (a) Vehicle model and coupling mechanism (b) Schematic representation of FE model [42]

Kouroussis and Verlinden 2013, [42] proposed a finite element based numerical model to study the dynamic behaviour of the vehicle/track/foundation subsystem and free-field response of the soil due to the loads acting on the soil surface. In order to mimic the infinite dimension of the soil both infinite elements and viscous boundaries as non-reflecting boundaries are used. To avoid excessive computational resources, a coupled lumped mass model (CLM model) of the soil has been considered in the study. The influence of ballast and soil stiffness is also considered through the CLM model. The study is followed by a comprehensive analysis to show the benefit of the finite element model with the proper radiation conditions at infinity, for analysing the structural response of a building located in the vicinity of the track Figure 13.

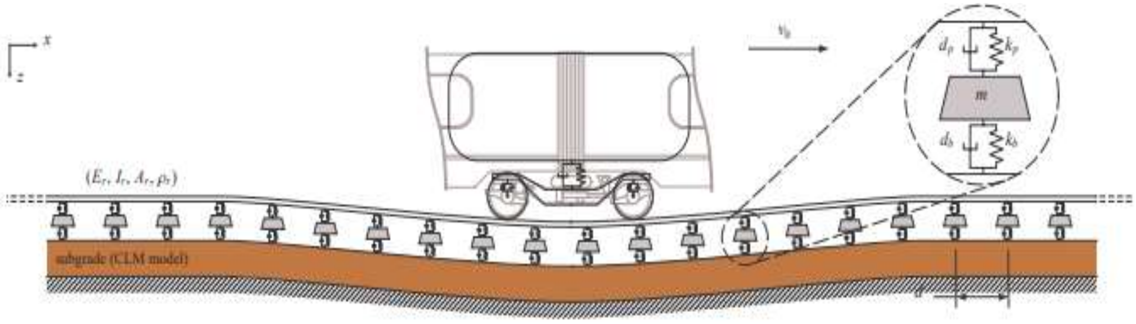


Fig. 13 The vehicle/track/foundation model [43]

Motazedian et al. 2012, [43] investigated the behaviour of railway train induced vibrations and soil amplifications during an earthquake for weak motion at soil sites in the Ottawa area. These two effects, especially in an area where railways are located on very loose soil, may cause large unusual soil amplification during an earthquake (at least for weak motion).

Xia, Cao, and De Roeck 2010**,** [44] modelled the track considering it as an infinite Euler-Bernoulli beam on elastic supports. They modelled subsoil under the track as a three-dimensional layered visco-elastic medium with the bottom modelled as half-space, by assuming each parallel layer isotropic and homogeneous. The dynamic Green function was used to characterize propagation of waves through soil. The wheel-rail interaction force is obtained through Hertz contact theory by taking into account track irregularity under quasi-static excitation induced by moving gravity axle loads. The train sub-model, the track sub-model and the subsoil sub-model are coupled through dynamic interactions of wheel–rail and sleeper-soil, respectively as shown Figure 14. The solution is based on the Fourier transforms in time and along the track.

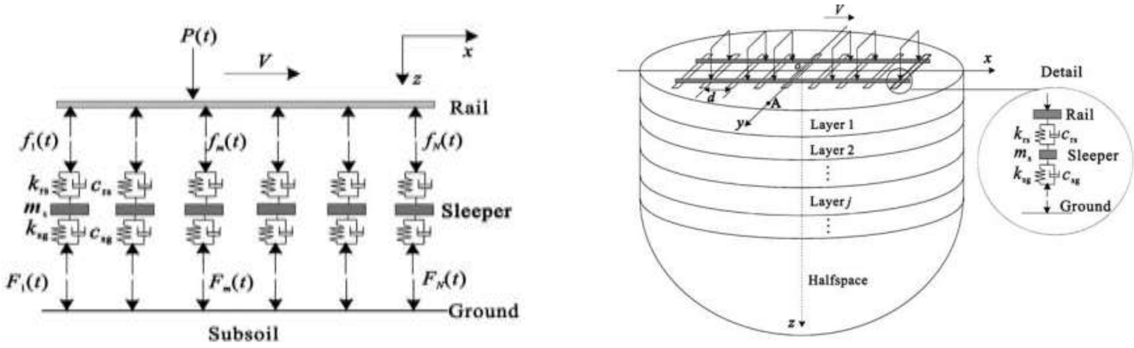


Fig. 14 (a) Interaction forces among train, track and soil (b) Track-soil dynamic interaction model [45]

Galvín, Romero, and Domínguez [46] conducted a study on vibrations generated by train movement on both ballast and non-ballast tracks. They employed a comprehensive three-dimensional multi-body finite element–boundary element model in the time domain. The boundary element method (BEM) incorporated Barkan's expression to account for soil material damping, enabling the time-dependent analysis of displacement and traction at boundaries. Meanwhile, the finite element method (FEM) utilized various elements, including spring-dampers, beams, shells, and solids, with dynamic stiffness matrices derived using Newmark's method. The train vehicle was represented as a multi-body system, as illustrated in Figure 15(a). Three key load components were considered (1) quasi-static excitation; forces that are generated by moving axle loads, (2) parametric excitation which is caused by discrete rail supports (3) excitation due to rail and wheel roughness – including track irregularities. To assess train-induced vibrations, the researchers evaluated track receptances for various train speeds and analyzed vibration levels in both the track structure and the surrounding free-field environment. Additionally, the study explored the interaction between soil and the track system. Finally, they examined the effectiveness of floating slab track systems in reducing surface vibrations, as depicted in Figure 15(b). The findings demonstrated that this method significantly minimizes vibrations in high-speed rail systems.

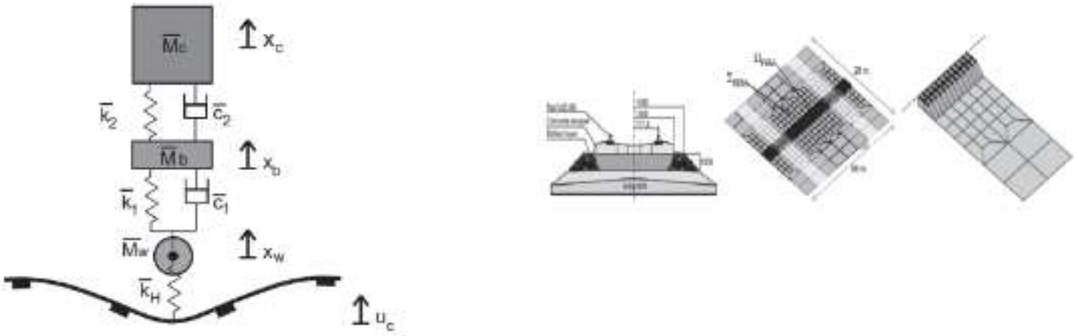
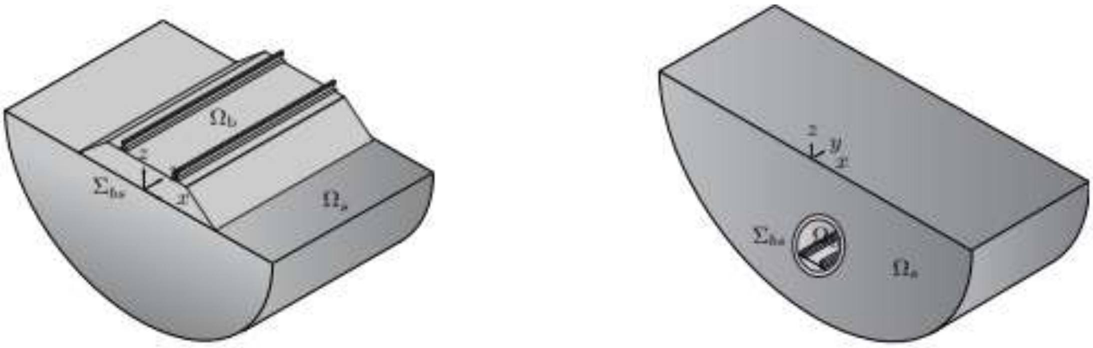


Fig. 15 (a) The multi-body model for an axle (b) Details of discretization [46]

Galvín, François, et al. [23] applied a 2.5D methodology that integrates the finite element method (FEM) with the boundary element method (BEM) to predict railway-induced vibrations. Their approach utilized a regularized boundary integral equation, where Green's function for a layered half-space was employed in the boundary element formulation. By comparing predicted and measured free-field vibrations, they observed that modeling the ballast and embankment as a continuum provided accurate results at low frequencies (quasi-static loading). However, at higher frequencies (dynamic axle loading), the accuracy decreased. Additionally, they investigated the behavior of a tunnel embedded in a half-space using the same 2.5D methodology, as shown in Figure 16.

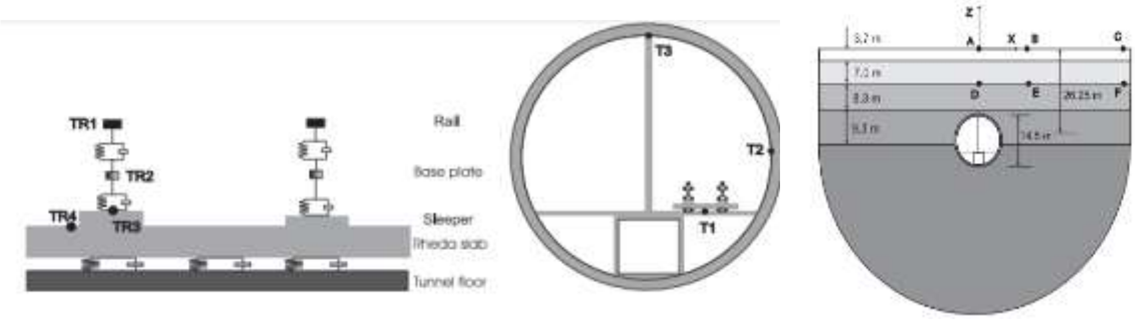


(a) ballasted track at grade **(b) a tunnel in a half space**

Fig. 16 The 2.5D coupled FE-BE models [23]

Gupta et al. 2010 [45], presented numerical model based on coupled periodic finite element-boundary element to predict vibrations from a Thalys high speed train in the Groene Hart Tunnel. The track and the tunnel were modelled using a finite element method, while the soil was modelled as a layered half space using a boundary element method. The tunnel and the soil are assumed to be invariant in the longitudinal direction, but modelled as a periodic structure using the Green-Floquet transformation. The time dependent position and the time history of the axle load was computed based on Dirac functions. A coupled FE-BE method is used to compute the transfer function in the frequency – wave number domain. A general analytical formulation to compute the response of three-dimensional periodic media excited by moving loads was adopted. The numerical model helps to understand the effect of soft layers found in the surroundings of the tunnel on vibration levels, resulting in an amplification of the horizontal response and a large contribution of the quasi-static forces at high train speeds Figure 17.

Lombaert and Degrande 2009, [46] in his work has evaluated the quasi-static excitation and the dynamic excitation due to random track unevenness. The quasi-static and dynamic contributions to the track and free-field response were analyzed for both Inter-City and HSTs, where the train speed was below the critical phase velocity of the coupled track–soil system. The dynamic axle loads are due to random rail unevenness, characterized by its power spectral density (PSD) function.



(a) **(b)**

Fig.17 Observation points (a) on the slab track and tunnel (b) in the free field [47]

A comparison was made with field measurements that have been performed during the homologation tests of the high-speed line L2 between Brussels and Koln. During these tests, vibrations of the track and in the free field have been measured for several passages of the Thalys HST and an InterCity train. A solution strategy is presented for the calculation of the non-stationary second-order statistical characteristics of the response based on the PSD of the random track unevenness, which allows for the computation of the mean square response in both the time and the frequency domain at a moderate computational cost. They also presented the influence of the train speed and the type of train on the quasi-static and dynamic excitation. Finally, they measured and predicted one-third octave band spectra and running rms values (velocity for all passages of Inter-city and Thalys train) of the track and free-field velocity, Figure 18.

Galvín and Domínguez 2009, [47] analyzed soil and structural vibration data obtained experimentally during on-field certification testing of the high-speed train line between Cordoba and Malaga (Spain) passing at speeds ranging from 151 to 298 km/h. The dynamic properties of the soil where the measurements were taken were obtained experimentally by spectral analysis of surface waves (SASW) testing procedure. The vertical response of the soil was measured away from the platform at eight different points situated at distances ranging from 1.55 to 20.56m from the point of impact, which itself was at a distance of 10.10m from the centre of the track. The soil response recorded data were used to conduct a surface wave analysis and, by employing an inversion process, the difference between the experimental and theoretical curves was minimized. They also presented the efficacy of three-dimensional boundary element method (BEM)/finite element method (FEM) numerical approach for the analysis of train induced vibrations, Figure 19. The model represented local soil conditions, discontinuities such as underpasses, as well as structures placed next to the rail track. Vibrations in those structures were computed taking into account, dynamic soil–structure interaction and local soil properties. Results for an overhead contact support structure were also evaluated.

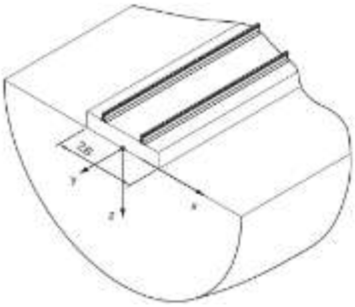


Fig.18 Schematic representation of problem geometry [48]

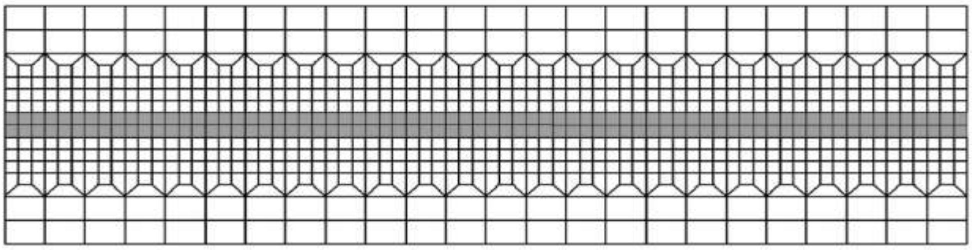


Fig. 19 Discretization of the track and soil surfaces in the proximity of the track [49]

Cai, Sun, and Xu 2008, [48] based on the theory of Biot, introduced an analytical model for the track-ground system taking into account the coupling of soil particles and pore- water of the soil medium. They described rail by introducing the Green function for an infinitely long Euler beam subjected to the action of the moving axle loads of the train and reactions of the sleeper. Sleepers were represented by a continuous mass and the effect of the ballast was considered by introducing the Cosserat model for granular medium. Using the double Fourier transform, the governing equations of motion were then solved analytically in the frequency-wave-number domain. The time domain responses were evaluated by the inverse Fourier transform computation for a particular train speed. The influence of the soil intrinsic permeability on soil responses was considered in both time domain and frequency domain, Figure 20.

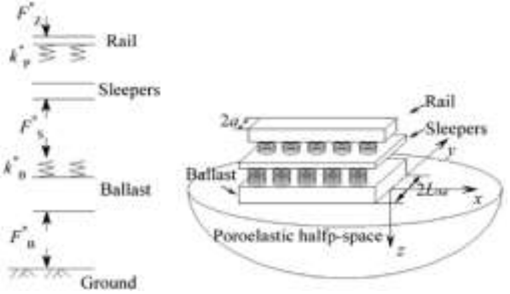
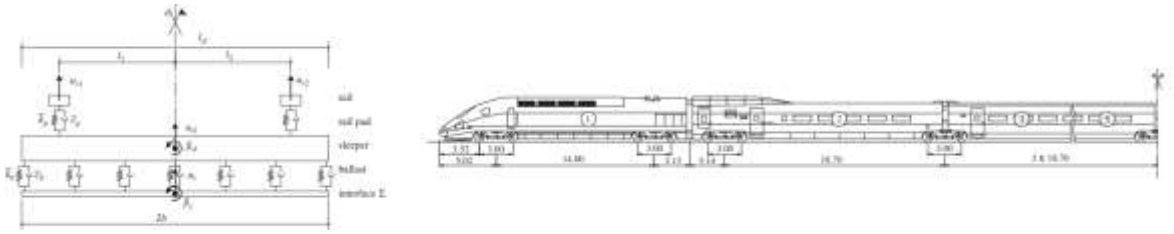


Fig. 20 Coupled track-ground model (a) the details of the model and (b) the main model [50]

Auersch 2008 [49] presents a detailed study of the moving-load effect for the tracks on soft soil. The moving-load effects have been studied for beam structures and beams on Winkler foundation. The analysis was carried out by evaluating the related integrals in the wave-number domain using fast fourier transform algorithm for several cases and parameters to find the rules for the moving load-effects.

Lombaert et al. 2006**,** [50] presents the experimental validation of a numerical model for the prediction of train induced vibrations. The model is validated by means of several experiments that have been performed at the occasion of the homologation tests of the new HST track on the line L2 between Brussels and Koln. These homologation tests have been performed with an IC train at a speed between 155.9 and 225.3 km/h and a Thalys HST at a speed between 218.1 and 326.1 km/h, Figure 21.



(a) **(b)**

Fig. 21 (a) Cross section of a ballasted track model (b) The configuration of the Thalys HST[52]

Frequency-domain analysis is a tool of utmost importance as it shows how the response of the system is distributed over a range of frequencies while time-domain analysis shows how response of the structure changes over time. Thus, frequency response gives the idea of bandwidth which the system can pass and which it cannot, which is not that evident from time domain representation. The response of the system can be converted between the time and frequency domains with a pair of mathematical operators called a transform. In order to understand the leading edge of time domain and frequency domain analysis, following literatures have been studied related to this topic, Table 1.

Table 1 Method of Analysis and Formulation

Year	Researcher	Method of Analysis	Method of Formulation
1995	Mohammadi et. al., [51]	FE-BE	Frequency Domain
2005	Anderson & Nielsen, [52]	FE-BE	Frequency Domain
2005	Brien J et. al., [53]	3D BEM-FEM	Frequency Domain
2006	Celebi et. al., [54]	3D BEM-FEM	Time Domain
2006	Sheng X Jones et. al., [23]	Wave number FE/BE	Time Domain
2009	Galvin et. al. [47]	Numerical	Time Domain
2010	Galvin et. al., [55]	3D FEM/BEM	Time Domain
2010	Galvin P et. al., [56]	2.5D	Time Domain
2010	Costa et. al., [57]	2.5D FE/BE	Time Domain

Mathematical and numerical models, on the other hand, are more flexible and versatile. They involve creating mathematical equations and computer simulations that describe the behaviour of railway tracks under various conditions. These models can be adjusted to simulate a wide range of scenarios, train speeds, and track conditions. They are especially useful when dealing with complex and varied situations that may not be adequately covered by empirical data. Often, researchers use a combination of empirical data and mathematical modelling to get the best of both worlds. Empirical data can be used to validate and refine mathematical models, ensuring that they accurately represent real-world conditions.

GROUND VIBRATION STUDY BASED ON EXPERIMENTAL WORK

Besides various techniques as discussed above, scaled model testing methods are also adopted by researchers to study the response of track structure and supporting soil against train induced vibrations. Sekine E, Ishikawa T and Kohata Y [58] built a 1:5 scale model of ballasted track and studied the outcome of moving loads on the permanent deformation of the ballast. Al Shaer et al. 2008, formed a 1:3 scale ballasted track for studying the settlement behaviour under load induced by moving bogies under various simulated train speeds through adjustment in the loading frequencies. Ishikawa T, Sekine E, and Miura S [59], used 1:5 scale model of ballasted track to study the mechanical behaviour of railway ballast owing to repeated train passages [60] conducted laboratory test to study the effect of wheel load on wheel vibration and sound radiation. They used finite element method (FEM) and boundary element method (BEM) to derive a rolling noise prediction model. Jiang et al., [63] used a 1:2 scale model of a ballastless track to gauge the dynamic performance of the subgrade under the loading of a single set of wheels. Marolt Čebašek et al., [61] developed full-scale laboratory-based testing to compare the long-term settlement performance of a precast concrete slab track section to a ballasted track (with concrete sleepers) resting on a compacted substructure. The railway track substructure is constructed from a 1.2 m deep combined subgrade and frost protection layer, according to modern high-speed rail standards specified in Germany. Long-term operation of the rail also causes wear and tear and makes the surface of the rail curvy resulting in track irregularities. Presence of such track irregularities magnify vibrations of the track and the surrounding ground environment during operation and may also lead to uneven subgrade settlement. In Germany, due to the track irregularities, the wheel–rail interaction forces of the Berlin–Hannover high-speed railway was intensified and the vibration intensity of the track structure increased nearly four-fold at train speed of 250 km/h [62].

Patent titled ground vibration proportion properties detection method and equipment thereof depicts a method of detection of ground vibration induced by actual load of traffic vehicles or load equivalent to the seismic intensity of an earthquake. The equipment for executing the above method comprises of a supporting stand, vibrator, vibration measuring unit, a driving motor, sprockets, roller chain, heavy weights, wheel for lifting heavy weight, an electro-magnetic clutch, a lifting sensor to detect the vibration level, a vibration level meter. The vibration is generated by dropping heavy weights (in tonnes) from predetermined height. The heavy weights commensurate the designed wheel loads. The entire setup is mounted on a supporting stand which is positioned on a paved surface. The method disclosed, uses electromagnetic clutch for lifting and dropping heavy weight from a specified height at a frequency of one loading per second. Each designed load for testing, simulates and is governed by the weight of number of heavy vehicles passing at a point per day on the pavement.

Another patent titled system for kinetic model test of ballastless track subgrade of high speed railway unveils a system for kinetic model test of the ballastless track subgrade of a high-speed railway. The system comprises of a model test box, a model of the ballastless track subgrade of the high-speed railway, an excitation system and a monitoring system, wherein the model of the ballastless track subgrade of the high-speed railway is formed by the various track components; the excitation system comprises a series of vibration exciters; and dynamic earth pressure sensors and layered settlement gauges are arranged in the model of the subgrade. The system can be used for carrying out study on the kinetic model test of the subgrade under the running load of the train and can evaluate and predict different foundation conditions, subgrade structures and track irregularity.

Concluding Remarks:

1. Numerical modeling is an essential tool for predicting train-induced vibrations, with 2D, 2.5D, and 3D models widely used in railway studies. However, inherent assumptions in these models can introduce deviations from real-world conditions, underscoring the need for experimental validation.
2. The study demonstrates that train passage can be idealized as a sinusoidal harmonic load, inducing negligible non-linearity in soil behavior ($<10^{-5}$). Consequently, the track superstructure and substructure can be effectively modeled using 2D isoparametric quadrilateral elements with linear elastic properties.
3. Furthermore, axle geometry significantly influences vibration propagation in the surrounding soil. Accurate modeling of train geometrical parameters enables differentiation between peak frequencies caused by track irregularities and axle distribution, thereby minimizing locomotive-induced vibrations on the subgrade. This approach enhances vibration isolation and improves the reliability of predictive models.

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