# OPTIMAL CYCLE POLICIES FOR PERISHABLE PRODUCTS WITH PERIODIC DEMAND AND TIME VARYING DETERIORATION RATE

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***Abstract***

In the study of inventory models of deteriorating items, demand and deterioration rate of perishable product plays an important role. Demand of some seasonal products such as fruits, vegetables and herbs, flowers, sweets and cakes during festivals, cosmetic items like lotion, cream in winter and sunscreen in summer, vaccines during specific season follows the repeating pattern over time. It rises and then falls at regular time interval rather than staying constant. In this article, optimal policies are obtained for the items having periodic demand and time varying deterioration rate, does not allows shortages. The objective is to minimize the total relevant cost for an inventory system. Formulation is illustrated with the numerical example and the solution is obtained by analytical as well as AI approach.

**Keywords:** Inventory system, Periodic demand, Deterioration, Optimal policy

## I. INTRODUCTION

Most of the inventory models in the literature are based on assumption of imperishability of products with infinite useful life. However, in our real life, some products that can damaged, expired with respect to time or we can say the products that can deteriorate and become unusable after some finite time are called perishable products. Fresh food, blood products, meat, chemicals, composite materials and pharmaceuticals are all examples of perishable products. The usage of perishable products is huge such as food industry and healthcare services. Perishable goods can be broadly classified into two main categories based on deterioration and Obsolescence. Deterioration refers to damage, spoilage, depletion, decay of goods. Obsolescence is loss of value of a product because arrival of new and better product. Perishable goods are an important part of stocks carried in practice. The study of perishable inventory systems has been the theme of many articles due to its applications in sectors like food, chemicals, pharmaceuticals, photography and blood bank management. Most inventory control models assume that the demand rate to be either constant or time-dependent but independent of the stock status.

In the last few years several researchers have discussed inventory problem involving time variable demand pattern. Ghare and Schrader [15] were the first proponents for developing a model for an exponentially decaying inventory. Covert and Philip [16] extended this model by considering weibull distribution deterioration rate. Donaldson [5] solved the classical inventory problem without shortages with linear demand over finite time horizon. Shah and Jaiswal [19] developed an order level inventory model with constant rate of deterioration. Dave and Patel [3] considered an inventory model without shortages for perishable products with time dependent demand. Sachan [17] then extended this model by allowing shortages. Dave [4] developed models with shortages taking time dependent demand. But it was computationally complicated. Hariga [14] developed optimum EOQ models for perishable products with time varying demand. Yang [9] developed deterministic lot size inventory models with shortages and deterioration for fluctuating demand. K.L.Hou [11] derived an EOQ inventory model with allowable shortages for deteriorating items with stock dependent selling rates. Abdul et al. [1] derived an optimal EOQ model for perishable items having varying demand pattern which generates replenishment policies. Chung and Huang [10] generalized Goyal's [6] model by developing a new inventory model with allowable shortages. Teng and Goyal [2] obtained a method for finding the optimal solution without taking complicated differential calculus. Hu and Liu [8] extended Chung and Huang [10] to the EPQ model with shortages and unequal selling and purchasing prices. Mishra, Singh and Kumar [13] derived an inventory model with time dependent demand and time varying holding cost where deterioration is time proportional. The model allowed for shortages and the demand was partially backlogged. Shah N.H., Bhavin and Arpan [18] presented an inventory model for deteriorating items with finite production rate and stochastic demand rate. Hari Kishan, Megha Rani and Vipin kumar [7] discussed an EOQ model with shortages for deteriorating items with constant deterioration rate and inventory dependent demand rate which starts from zero in the beginning and ends to zero at the end of the cycle. Molamohamadi et al. [20] formulated an EPQ model of an exponentially deteriorating item with price sensitive demand under trade credit, where shortages are considered. Manish Pande, S.S. Gautam and N.P. Katyar [12] presented an inventory model with periodic demand and constant deterioration by allowing shortages. They assumed that the periodic demand rate increases by a constant percentage during each time interval.

This paper deals with the inventory model of perishable products having periodic demand which starts from zero in the beginning and ends to zero at the end of the cycle and time varying deterioration rate. This model attempts to obtain the average total cost per unit time of the inventory system. Here, the model considers the case, without shortages.

For modelling the inventory system, we will use the following notations and assumptions.

**ASSUMPTIONS**

(1) The demand is periodic function of time and is given by $asint$.
(2) The deterioration rate $\theta$ is time varying, where $0 < \theta < 1$.
(3) The lead time is zero.
(4) The replenishment time is infinite.
(5) Inventory holding cost is charged only on the amount of undecayed stock.
(6) Shortages are not allowed.
(7) There is no repair or replacement of the deteriorated items.

**NOTATIONS**

(1) $\theta_0 t$ = the variable rate of deterioration
(2) $q(t)$ = the inventory level at time $t, 0 \leq t \leq T$
(3) $C_r$ = the replenishment cost
(4) $C_0$ = the ordering cost per order
(5) $C_d$ = the deterioration cost
(6) $C_h$ = the holding cost
(7) $C$ = the purchasing cost per unit
(8) $h$ = unit holding cost per unit time
(9) $S$ = the maximum inventory level
(10) $T$ = the replenishment cycle time
(11) $P$ = the production rate
(12) $t_1$ = time during which production runs and decreases due to demand and deterioration
(13) $Z(T)$ = the total relevant cost per year which consists of replenishment cost, deterioration cost, holding cost and shortage cost.

## II. MATHEMATICAL MODELLING

Let $q(t)$ is the inventory level at any time$t$. We consider $S$ is the highest inventory. The demand is periodic function of time and is given by $asint$. The variable rate of deterioration is $\theta_0 t$, where $0 < \theta_0 < 1$. The inventory increases with the production rate $P$ and decreases due to the market demand and deterioration during$(0, t_1)$. During$(t_1, T)$, the inventory depletes to zero due to demand and deterioration.
So, the model formulated by the differential equations,

$$\frac{dq}{dt} = P - asint - \theta_0 tq, 0 \leq t \leq t_1 \quad (1)$$

with the boundary condition $q(0) = 0$ (2)

$$\frac{dq}{dt} = -asint - \theta_0 tq, t_1 \leq t \leq T \quad (3)$$

with the boundary condition $q(T) = 0$ (4)

## III. ANALYSIS

Solving (1), we get

$$q_1(t)e^{\frac{\theta_0 t^2}{2}} = \int (P - asint)e^{\frac{\theta_0 t^2}{2}}dt + c_1$$

Neglecting higher order terms in exponential series expansion,

$$q_1(t) = \int (P - asint)\left[1 + \frac{\theta_0 t^2}{2}\right]dt + c_1 \tag{5}$$

Now using boundary condition (2), we have $c_1 = 0$ , so we get

$$q_1(t) = \left(1 - \frac{\theta_0 t^2}{2}\right)\left(Pt - a(1 - cost)\right); 0 \le t \le t_1 \tag{6}$$

Solving (3), we get

$$q_2(t)e^{\frac{\theta_0 t^2}{2}} = \int (-asint)e^{\frac{\theta_0 t^2}{2}}dt + c_2$$

Neglecting higher order terms in exponential series expansion and using boundary condition,

$$q_2(t) = -a\int (sint)\left[1 + \frac{\theta_0 t^2}{2}\right]dt + c_2$$

$$q_2(t) = a(cost - cost_1) - \frac{(\theta_0 t^2)}{2}a(cost - cost_1); t_1 \le t \le T \tag{7}$$

Total replenishment cost per unit is given by,

$$C_r = \frac{C_0}{T} \tag{8}$$

Total inventory holding cost per unit time is given by,

$$C_h = \frac{h}{T}\int_0^T q(t)dt$$

$$C_h = \frac{h}{T}\left[\int_0^{t_1} q_1(t)dt + \int_{t_1}^T q_2(t)dt\right] \tag{9}$$

$$I_1 = \int_0^{t_1} q_1(t)dt$$

$$\therefore I_1 = \frac{Pt_1{}^2}{2} - \frac{P\theta_0 t_1{}^4}{8} - \frac{\theta_0 a}{2}\left(\frac{t_1^3}{3} - t_1{}^2 sint_1 - 2t_1 cost_1 + 2sint_1\right) - a(t_1 - sint_1) \tag{10}$$

$$I_2 = \int_{t_1}^T q(t)dt$$

$$I_2 = a(sinT - sint_1 - (T - t_1)cost_1)\frac{-a\theta_0}{2}\Big[(T^2 sinT + 2TcosT - 2sinT) - (t_1^2 sint_1 + 2t_1 cost_1 - 2sint_1) - cost_1\frac{(T^3 - t_1^3)}{3}\Big] \tag{11}$$

From (9), (10) and (11), we get

$$C_h = \frac{h}{T}\Big(\frac{Pt_1{}^2}{2} - \frac{P\theta_0 t_1{}^4}{8} - \frac{\theta_0 a}{2}\left(\frac{t_1^3}{3} - t_1{}^2 sint_1 - 2t_1 cost_1 + 2sint_1\right) - a(t_1 - sint_1) + a(sinT - sint_1 - (T - t_1)cost_1)\frac{-a\theta_0}{2}\Big[(T^2 sinT + 2TcosT - 2sinT) - (t_1^2 sint_1 + 2t_1 cost_1 - 2sint_1) - cost_1\frac{(T^3 - t_1^3)}{3}\Big]\Big) \tag{12}$$

Total deterioration cost is given by,

$$C_d = \frac{C}{T}\left[\int_0^{t_1}\theta_0 t q_1(t)dt + \int_{t_1}^T \theta_0 t q_2(t)dt\right]$$

$$C_d = \frac{C\theta_0}{T}\left[\int_0^{t_1} tq_1(t)dt + \int_{t_1}^T tq_2(t)dt\right]$$

Neglecting higher-order terms and then integrating, we have

$$C_d = \frac{C\theta_0}{T}\left[\frac{Pt_1^3}{3} - a\left(\frac{t_1^2}{2} - t_1 sint_1 - cost_1 + 1\right) + a\left(TsinT + cosT - t_1 sint_1 - cost_1 - \frac{cost_1}{2}(T^2 - t_1^2)\right)\right] \tag{13}$$

The total inventory cost per unit time is given by,

$Z(T) = C_r + C_h + C_d$

$$Z(T) = \frac{C_0}{T} + \frac{h}{T}\left(\frac{Pt_1{}^2}{2} - \frac{P\theta_0 t_1{}^4}{8} - \frac{\theta_0 a}{2}\left(\frac{t_1^3}{3} - t_1{}^2 sint_1 - 2t_1 cost_1 + 2sint_1\right) - a(t_1 - sint_1) + a(sinT - sint_1 - (T - t_1)cost_1) - \frac{a\theta_0}{2}\left[(T^2 sinT + 2TcosT - 2sinT) - (t_1^2 sint_1 + 2t_1 cost_1 - 2sint_1) - cost_1 \frac{(T^3 - t_1^3)}{3}\right]\right) + \frac{C\theta_0}{T}\left[\frac{Pt_1^3}{3} - a\left(\frac{t_1^2}{2} - t_1 sint_1 - cost_1 + 1\right) + a\left(TsinT + cosT - t_1 sint_1 - cost_1 - \frac{cost_1}{2}(T^2 - t_1^2)\right)\right] \quad (14)$$

To get the optimum cycle time $T$ and to minimize total relevant cost, we differentiate $Z(T)$ with respect to $T$ and for optimal value necessary condition is $\frac{dZ}{dT} = 0$, which can be solved numerically.

## IV. NUMERICAL EXAMPLE

For the Illustration of proposed model, We consider following inventory system in which values of different parameters are $a = 20, P = 100, h = 5, \theta_0 = 0.02, C_0 = 500, C_d = 100$ which provide the solution as given below. The optimal cycle time is obtained using analytical approach by using python programming and also a genetic algorithm based optimization approach.

| Methods | optimal production time $t_1$ | optimal cycle time $T$ | minimum cost $Z(T)$ |
|---|---|---|---|
| Analytical solution | 1.0294 | 2.2024 | 466.1732 |
| Solution by Genetic algorithm | 1.3458 | 3.0598 | 496.8825 |

## V. CONCLUSION

Numerical example confirms that the parameters such as production rate, demand coefficient, deterioration rate, holding cost and deterioration cost are highly affect to the optimal policy. Also, solution obtained using a genetic algorithm is nearest or slightly higher than the analytical solution obtained by using python programming. So genetic algorithm also can be apply to manage inventory.

## REFFERENCES

[1] Abdul, Ibraheem, Atsuo Murata: An optimal EOQ model for perishable products with varying demand pattern, IEEE SMC Hiroshima Chapter, Hiroshima University, Japan, November 10, 11 & 12, 2009

[2] Chang, C.T, Teng J.T., and S. K. Goyal, Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand, International Journal of Production Economics, vol. 123, no. 1, (2010) 62–68.

[3] Dave U., Patel L. K., (T, Si) policy inventory model for deteriorating items with time proportional demand, Journal of the Operational Research Society, Vol. 32, (1981) 137–142.

[4] Dave U., On a heuristic inventory replenishment rule for items with a linearly increasing demand incorporating shortages, Journal of Operation Research Society, 40, (1989) 827-830.

[5] Donaldson W. A., Inventory replenishment policy for a linear trend in demand, an analytical solution, Operational Research Quarterly (28), (1977) 663-670.

[6] Goyal, S.K., Economic Order Quantity under Conditions of Permissible Delay in Payments, Journal of Operation Research Society, 36, (1985)335-338.

[7] Hari Kishan, Vipin Kumar, An EOQ model of deteriorating products with inventory level dependent demand rate under trade credit and time discounting, International Journal of Advanced Research In Engineering Technology and Science, 2 (2014), 11-19.

[8] Hu F. and Liu D., Optimal replenishment policy for the EPQ model with permissible delay in payments and allowable shortages, Applied Mathematical Modelling, vol. 34, no. 10, (2010) 3108–3117.

[9] H. L. Yang, J. T. Teng, M. S. Chern, Deterministic lot size inventory models with shortages and deterioration for fluctuating demand, Nav. Res. Logist. 48 (2001) 144-158.

[10] K.J. Chung and C.K. Huang, An ordering policy with allowable shortage and permissible delay in payments, Applied Mathematical Modelling, vol. 33, no. 5, (2009) 2518–2525.

[11] K.L. Hou, An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting, European Journal of Operational Research, vol. 168, no. 2, (2006) 463–474.

[12] Manish Pande, S. S. Gautam, N. P. Katyar, An inventory model with periodic demand, constant deterioration and shortages, Industrial Engineering Letters. (2015) 26-30.

[13] Mishra V.K., Singh L., Kumar R., An inventory model for deteriorating items with time dependent demand and time varying holding cost under partial backlogging. Journal of Industrial Eng. Int., (2013) 9, 4-8.

[14] M. A. Hariga, Optimal EOQ models for deteriorating items with time-varying demand, Journal of Operation Research Society, 47 (1996) 1228-1246.

[15] P. M. Ghare, G. P. Schrader, A model for an exponentially decaying inventory, Journal of Industrial Engineering, 14 (1963) 238-243.

[16] R. B. Covert, G. S. Philip, An EOQ model with Weibull distribution deterioration, AIIE Trans. 5 (1973) 323-326.

[17] Sachan R. S. On (T, Si) policy inventory model for deteriorating items with time proportional demand, Journal of Operational Research Society, (1984) 1013-1019.

[18] Shah N. H, Bhavin J. shah and Arpan D Shah, Deteriorating inventory model with finite production rate and two level of credit financing for stochastic demand, Opsearch, 50 (3), (2013) 358-371.

[19] Shah Y. K., Jaiswal M. C., An order level inventory model for a system with constant rate of deterioration, Opsearch, 14 (1977) 174-184.

[20] Zohreh Molamohamadi, Rahman Arshizadeh, Napsiah Ismail, An EPQ inventory model with allowable shortages for deteriorating items under trade credit policy, Discrete dynamics in Nature and society, (2014).