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STATISTICALANDSQUEEZINGPROPERTYOFNON-DEGENERATETHREE-LEVELLASER INSIDEACAVITYDRIVENBYCOHERENTLIGHTCOUPLEDTOTWOMODETHERMALRESERVOIR

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Abstract

Inthisthesis,westudythestatisticalandsqueezingpropertiesofcoherentlydrivennon-degenerate three level lasers in a cavity coupled to two mode thermal reservoir.In the case ofinteracting driving coherent light with a cavity mode and non degenerate three level atom, weobtain the Hamiltonian equation.After that,we develop the master equation and quantumlangiven equations for the system under consideration and then, we derive the equations of evo-lutionoftheexpectationvaluesofthecavitymodeandatomicoperators.wethen,determinethesteady-statesolutionsoftheresultingequationsintheadiabatic(largetime)approxima-tion scheme.By the help of this equation and solutions, we obtained the photon statistics andquadrature squeezing.We find that the global mean photon number for mode**a(b)** increaseswiththeamplitudeofthedrivingcoherentlight(Ω)andwiththerateofstimulatedemission(γ_c).Wealsoobtainedthatthephotonnumberstatisticsofsinglelightmodeissuper-Poiso-nianphotonstatisticswhichshowsthateachofthetworadiationmodesareinachaoticstateoflight.Weverifythatthetheglo

balmeanphotonnumberfortwomodelightincreaseswiththeamplitudeofthedrivingcoherentlight(Ω)andwiththerat eofstimulatedemission(γ_c).Then, we show that the light generated by coherently driven non-degenerate three level lasercoupledtotwomodethermalreservoirownsa super-poisonianstatisticsanditisa chaoticlight.Finally,wefindthatthelightmodegeneratedbysuchlightisinasqueezedstatewithamaximumdegreeofsq ueezing(96.22%)belowthecoherentlevel.

Introduction

Quantum optics is a combination of optics with quantum mechanics. Quantum physics devel-oped through the first half of the twentieth century more by understanding how photons andmatter interacted and interrelated. And also in 1953, the maser was developed which emitscoherentmicrowave.Afteratimeofbeing,laser(lightamplificationbystimulatedemissionofradiation)was developedbydifferentquantumopticalsystems[1,2].Thequantumpropertiesof light are largely determined by the state of the light mode and the well-known quantumstates of light are the number state,the chaotic state, the coherent state and the squeezed state[3].

Squeezinglikephotonantibunchingandsub-poissonianphotonstatisticsisoneofthenon-classicalfeaturesoflightthathasbeenpredictedtheoreticallyandexperimentallyobserved[4].In a squeezed light, the noise in one quadrature is below the coherent-state level at theexpense of enhanced fluctuations in the other quadrature,with the product of the uncertaintiesin the two quadratures satisfying the uncertainty relation [5].Squeezed light has potential ap-plications in the detection of week signals, in low-noise communications and gravitational wavedetection[1,2,6].Squeezedlightcanbegeneratedbyvariousquantumopticalsystemssuchas four-wave mixing [7],parametric oscillator [8] , resonance fluorescence [9],second harmonicgeneration[10],subharmonicgenerations[11]andThree-levellaser[1,2,12].

Athree-levellaserisaquantumopticalsysteminwhichlightisgeneratedbythreelevelatomsinsideacavityusuallycoupled toavacuumreservoir.Themainclassesofthethree-levellasers arelambda(Λ),vee(*V*)andcascade(Ξ)configurations[13].Whenathree-levelatomincas-cadeconfigurationmakesatransitionfromtheuppertotheintermediatelevelandthenfromthe intermediate to the bottom level, two photons are emitted[1, 7, 14]. If the two photons havethe same frequency, the three-level atom is said to be a degenerate three-level atoms; otherwiseitiscalledanon-degeneratethree-levelatoms[2,14,15].Generallythethree-levellaserisauniquesourceofbrightlightandtocombinetheexistingoncewithittogeneratehighlysqueezed light[1, 16].The statistical and squeezing properties of the light generated by three-levelatomhavebeenstudiedbydifferentauthors[1-27].FessehaKassahun[17]hasstudiedthe squeezing and statistical properties of the light produced by a degenerate three-level laserwiththeatomsinaclosedcavityandpumpedbyelectronbombardment.Hehasshownthatthe maximum quadrature squeezing of the light generated by the laser,operating far belowthreshold, is 50% below the vacuum-state level.Tamirat Abebe [18] has analyzed the quantumproperties of cavity light produced by a coherently driven non degenerate three-level laser in aclosedcavityandcoupledtoatwo-modevacuumreservoir,ispresented.Normalorderingofthenoiseoperatorsassociatedwiththevacuumreservoirisconsidered.Applyingthesolutionsof the equations of evolution for the expectation values of the atomic operators and the quan-tum Langavin equations for the cavity mode operators, the quadrature squeezing,entanglementamplification, and the normalized second-order correlation function of the cavity radiation areobtained.Thethree-levellasergeneratessqueezedlightundercertainconditionswithmax-imumintra-cavitysqueezingbeing43%belowthevacuum-statelevel.Moreover, itisfoundthat the photon numbers of a two-mode light beams are correlated.Misrak Getahun [19] ana-lyzed the squeezing and statistical properties of the light produced by a three-level laser whosecavity contains a parametric amplifier and with the cavity mode driven by coherent light andcoupled to a squeezed vacuum reservoir.He also obtained that stochastic differential equationsassociatedwiththenormalorderingusingthepertinentmasterequationandMakinguseofthe solutions of the resulting differential equations, he has calculated the quadrature variancesand squeezing spectrum,mean and variance of the photon number and the photon numberdistributionforthecavitymodeemployingtheQfunction.

SintayehuTesfa[20]hasadetailedanalysisforSqueezingandstatisticalpropertiesofthecavity as well as the output radiation of a non-degenerate three-level cascade laser coupled to two-modesqueezedvacuumreservoir.Thegeneratedradiationexhibitsahighdegreeofsqueezingin the minus quadrature either for a weak or strong driving radiation. In general, the degreeof squeezing and mean number of photon pairs increase with the linear gain coefficient andsqueezeparameter.Thedrivingradiationleadstosqueezingofthecavityradiationevenwhen$\gamma = 0$ and $\gamma = 1$. Moreover, he found that there was no distinct difference between the prob-ability for finding odd and even number of photon pairs in the cavity. In addition, the meannumberofphotonsinmodeaturnsouttobegreaterthanthatinmodeb.

In this thesis,we seek to study the statistical and squeezing properties of coherently drivennon-degenerate three level lasers in a cavity coupled to two mode thermal reservoir.We carryout our calculation by putting the noise operators associated with the thermal reservoir in nor-malorder.Wefirstdeterminethemasterequationforacoherentlydriventhree-levelatominaclosedcavitycoupledtoatwo-modethermalreservoirandthequantumLangevinequationsforthecavitymodeoperators.Inaddition,employingthe masterequationandthelarge-time approximation scheme, we obtain equations of evolution of the expectation values of theatomic operators.Moreover, we determine the solutions of the equations of evolution of theexpectation values of the atomic operators and the quantum Langevin equations for the cavitymode operators.Then applying the resulting solutions, we calculate the mean photon number,variance of the photon number, and the quadrature variances of the single-mode cavity lightbeams.Furthermore,applying the same solutions,we obtain the mean photon number andvariance the photon number of the two-mode cavity light.Finally, we determine quadraturesqueezingofcoherentlydrivennon-degeneratethreelevellasersinacavitycoupledtotwomodethermalreservoir.

Chapter2

DynamicsofOperators

Inthischapterwefirstobtainthemasterequationforanon-degeneratethree-levelatominacavitydrivenbytwomodecoherentlightcoupledtothermalreservoirandthequantumLangevin equations for the cavity mode operators.In addition, employing the master equationand the large-time approximation scheme, we derive the equations of evolution of the expec-tation values of the atomic operators.Finally, we determine the steady-state solutions of theresultingequations.

2.1TheModelandQuantumLangevinEquations

2.1.1 CascadeThreeLevelAtom

We consider three-level atoms with the non-degenerate states $|a\rangle$,$|b\rangle$ and $|c\rangle$ in the three possibleconfigurationsaslambda(Λ),*vee*(*V*)andladder(Ξ).Inthisstudy,weusetheladder(cascade)typemodelwhereelectronswilloccupyanyofthethreeouterenergylevelsarrangedtoptodown as shells in Bohr model of atomic configurations[1, 21].These atoms will be placed in acavity driven by two mode coherent light and coupled to two mode thermal reservoir via singleportmirror.Thetop,intermediate,andbottomlevelsofthethree-levelatomaredenotedby

$|a\rangle$,$|b\rangle$and$|c\rangle$,respectively(seeFigure2.1). Whentheatommakesatransitionfromlevel

$|a\rangle \rightarrow |b\rangle$ and from levels $|b\rangle \rightarrow |c\rangle$, two photons with different frequencies are emitted. suchlasingprocessiscallednon-degenerateortwomodethree-levellaser.Itisassumedthatthe

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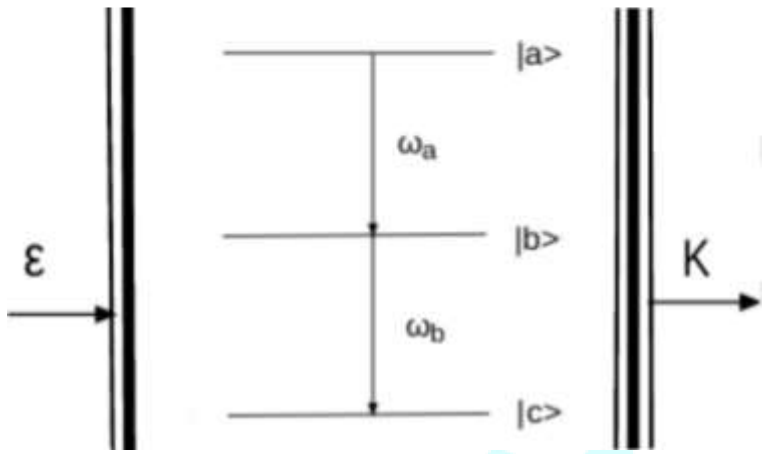

Figure2.1:Coherentlydrivennon-degeneratethree-levelatominacascadeconfiguration

cavity mode a is at resonance with transition $|a\rangle \rightarrow |b\rangle$ and the cavity mode **b** is at resonance with the transition from levels $|b\rangle \rightarrow |c\rangle$,with direct transition between $|a\rangle$ and $|c\rangle$ is to be electric-dipole forbidden.But,the top and bottom levels of the three-level atom coupled by coherent light. The interaction of a non-degenerate three-level atom with the coherent light and with the light modes a and b can be described by different Hamiltonians. The interaction of the driving two-mode coherent light with the two-mode cavity light can be described by the Hamiltonian [2,22].

$\hat{H}_1 = i\mu(\hat{c}_1\hat{a}^\dagger - \hat{a}\hat{c}_1^\dagger + \hat{c}_2\hat{b}^\dagger - \hat{b}\hat{c}_2^\dagger)$ where $(\hat{c}_1\hat{c}_2)$ are annihilation operators of the coherent light,μ is coupling constant between cavity modes and the coherent light and $\hat{a}$and$\hat{b}$ are annihilation operators of the cavity modes.Upon treating the coherently light classically, we see$\hat{c}_1 = \hat{c}_1^\dagger = \beta_1, \hat{c}_2 = \hat{c}_2^\dagger = \beta_2$

$$\hat{H}_1 = i\epsilon(\hat{a}^\dagger - \hat{a} + \hat{b}^\dagger - \hat{b}) \qquad (2.1)$$

with$\epsilon = \mu\beta_1 = \mu\beta_2$isapositiveandrealconstantthesameforthetwo-modesandproportionalto the amplitude of driving coherent light. The interaction of a non-degenerate three-level atom with two mode cavity light in the rotating wave approximation and in the interaction picture can be described by[23]

$\hat{H}_2 = ig[(|a\rangle\langle b| + |b\rangle\langle c|)\hat{a} - \hat{a}^{\dagger}(||b\rangle\langle a| + |c\rangle\langle b|)]$

$$\hat{H}_2 = ig(\sigma_a^{\dagger} a - a^{\dagger}\sigma_a + \sigma_b^{\dagger} b - b^{\dagger}\sigma_b) \quad (2.2)$$

Where g is the coupling constant, $\sigma_a$and$\sigma_b$areloweringatomicoperators,$\hat{a},\hat{b}$ are the annihilationoperatorsforthecavitymodes.andalso$\sigma_a^{\dagger}$and$\sigma_b^{\dagger}$areupperingatomicoperators,$\hat{a}^{\dagger}$and $\hat{b}^{\dagger}$arecreationoperatorsforthecavitymodes.Finally,thequestionofpopulationinversion,the centralpointinatomiclasing,isresolvedbythedrivingcoherentlywhichisexpecttopushor pump atoms from the bottom energy level($|c\rangle$) back to the higher energy level($|a\rangle$). This isachievedthroughcouplingbetweenthetopandbottomlevelsbythetwomodecoherentlight

Given as[2,7,24]

$$\hat{H} = i\frac{\Omega}{2}(|a\rangle\langle c| - |c\rangle\langle a|) = i\frac{\Omega}{2}(\sigma_c^{\dagger} - \sigma_c) \quad (2.3)$$

whereΩ=2$\lambda\epsilon$,consideredtoberealandconstant,istheamplitudeofthedrivingcoherentlight,andλisthecouplingconstantbetweenthedrivingcoherentlightandthethree-levelatom.TheHamiltonianfortheoverallinteractioninthecavity,thetwo-modecavitylightwithdrivingcoherentlightandthenon-degeneratethreelevelatom,twomodecoherentlightand non-degenerate three level atom (coupling), can be written as a sum to total of the threeHamiltonianstheabove

$\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_3$

Usingtherespectivevaluesabove,theoverallinteractionHamiltonianbecomes

$$\hat{H} = i\epsilon(\hat{a}^{\dagger} - \hat{a} + \hat{b}^{\dagger} - \hat{b}) + ig(\sigma_a^{\dagger} a - a^{\dagger}\sigma_a + \sigma_b^{\dagger} b - b^{\dagger}\sigma_b) + i\frac{\Omega}{2}(\sigma_c^{\dagger} - \sigma_c) \quad (2.4)$$

ThisisthetotalHamiltonianequationdescribingtheoverallprocessinanon-degeneratethree-level atom placed inside a cavity driven by two mode coherent light and coupled to twomodethermalreservoirviaasingleportmirror.

2.1.2 QuantumLangevinEquations

The dynamics of cavity mode coupled to thermal reservoir can be described using the quantumLangevinequation.HereweseektofindthequantumLangevinequationemployingthe

Hamiltonian together with the fact that the quantumLangevinequationsfortheoperators$\hat{a}$ and $b$aregivenby[1,2,25]

$$\frac{d}{dt}\hat{a}(t)=-\frac{\kappa}{2}\hat{a}-i[\hat{a},\hat{H}]+\hat{f}_a(t) \tag{2.5}$$

$$\frac{d}{dt}\hat{b}(t)=-\frac{\kappa}{2}\hat{b}-i[\hat{b},\hat{H}]+\hat{f}_b(t) \tag{2.6}$$

where κ is the cavity damping constant considered to be the same for the two cavity modesand$\hat{f}_a(t)$and$\hat{f}_b(t)$arenoiseoperatorsassociatedwiththeoperatorsaandb.Inviewofthe Hamiltonian(2.4),equations(2.5)and(2.6)reducetousingcommutationrelation,therefore theequationyields

$$\frac{d}{dt}\hat{a}(t)=-\frac{\kappa}{2}\hat{a}-g\sigma_a+\epsilon+\hat{f}_a(t) \tag{2.7}$$

$$\frac{d}{dt}\hat{b}(t)=-\frac{\kappa}{2}\hat{b}-g\sigma_b+\epsilon+\hat{f}_b(t) \tag{2.8}$$

2.2TheMasterEquation

Themasterequationfornon-degeneratethreelevellaserinacavitydrivenbytwomodecoherentlightandcoupledtotwomodethermalreservoirisexpressibleas[2,7,26].

$$\frac{d\hat{\rho}}{dt}=-i\left[\hat{H},\hat{\rho}\right]+\frac{\kappa}{2}(\bar{n}+1)\left(2\hat{a}\hat{\rho}\hat{a}^{\dagger}-\hat{a}^{\dagger}\hat{a}\hat{\rho}-\hat{\rho}\hat{a}^{\dagger}\hat{a}\right)+\frac{\kappa\bar{n}}{2}\left(2\hat{a}^{\dagger}\hat{\rho}\hat{a}-\hat{a}\hat{a}^{\dagger}\hat{\rho}-\hat{\rho}\hat{a}\hat{a}^{\dagger}\right)$$
$$+\frac{\kappa}{2}(\bar{n}+1)\left(2\hat{b}\hat{\rho}\hat{b}^{\dagger}-\hat{b}^{\dagger}\hat{b}\hat{\rho}-\hat{\rho}\hat{b}^{\dagger}\hat{b}\right)+\frac{\kappa\bar{n}}{2}\left(2\hat{b}^{\dagger}\hat{\rho}\hat{b}-\hat{b}\hat{b}^{\dagger}\hat{\rho}-\hat{\rho}\hat{b}\hat{b}^{\dagger}\right) \tag{2.9}$$

where,κisacavitydampingconstant,$\hat{\rho}$isdensityoperatorand$\bar{n}$isthemeanphotonnumber ofthermalreservoir.CombiningEqs(2.4)and(2.9)theequationofevaluationofthedensityoperatorforthesystemunderconsiderationis

$$\frac{d\hat{\rho}}{dt}=\epsilon\left(\hat{a}^{\dagger}\hat{\rho}-\hat{a}\hat{\rho}+\hat{b}^{\dagger}\hat{\rho}-\hat{b}\hat{\rho}-\hat{\rho}\hat{a}^{\dagger}+\hat{\rho}\hat{a}-\hat{\rho}\hat{b}^{\dagger}+\hat{\rho}\hat{b}\right)+\frac{\Omega}{2}\left(\sigma_c^{\dagger}\hat{\rho}-\sigma_c\hat{\rho}-\hat{\rho}\sigma_c^{\dagger}+\hat{\rho}\sigma_c\right)$$
$$+g\left(\sigma_a^{\dagger}\hat{a}\hat{\rho}-\hat{a}^{\dagger}\sigma_a\hat{\rho}+\sigma_b^{\dagger}\hat{b}\hat{\rho}-\hat{b}^{\dagger}\sigma_b\hat{\rho}-\hat{\rho}\sigma_a^{\dagger}\hat{a}+\hat{\rho}\hat{a}^{\dagger}\sigma_a-\hat{\rho}\sigma_b^{\dagger}\hat{b}+\hat{\rho}\hat{b}^{\dagger}\sigma_b\right)$$
$$+\frac{\kappa}{2}(\bar{n}+1)\left(2\hat{a}\hat{\rho}\hat{a}^{\dagger}-\hat{a}^{\dagger}\hat{a}\hat{\rho}-\hat{\rho}\hat{a}^{\dagger}\hat{a}\right)+\frac{\kappa\bar{n}}{2}\left(2\hat{a}^{\dagger}\hat{\rho}\hat{a}-\hat{a}\hat{a}^{\dagger}\hat{\rho}-\hat{\rho}\hat{a}\hat{a}^{\dagger}\right)$$
$$+\frac{\kappa}{2}(\bar{n}+1)\left(2\hat{b}\hat{\rho}\hat{b}^{\dagger}-\hat{b}^{\dagger}\hat{b}\hat{\rho}-\hat{\rho}\hat{b}^{\dagger}\hat{b}\right)+\frac{\kappa\bar{n}}{2}\left(2\hat{b}^{\dagger}\hat{\rho}\hat{b}-\hat{b}\hat{b}^{\dagger}\hat{\rho}-\hat{\rho}\hat{b}\hat{b}^{\dagger}\right) \tag{2.10}$$

Thisisthemasterequationforanon-degeneratethreelevellaserinacavitydrivenbytwo mode coherent light and coupled to two mode thermal reservoir where κ is cavity dampingconstant,gistheatomcouplingconstant,Ωistheamplitudeofthedrivingcoherentlight,ϵisapositiveandrealconstantproportionaltotheamplitude(Ω)ofthedrivingcoherentlightand $\bar{n}$isthemeanphotonnumberofthermalreservoir.

2.3EquationsofEvolutionofCavityModeOperators

The dynamics of a cavity mode of a three-level laser coupled to thermal reservoir can be de- scribedbythequantumLangevinequationinwhichthetimeevolutionofthethecavitymodeis carried by the operators. To this end, the expectation value of any arbitrary operator A inSchrödingerpicturecanbedeterminedusingitsdensityoperatoras[27]

$$\frac{d}{dt}\langle\hat{A}\rangle = Tr\left(\frac{d\hat{\rho}}{dt}\hat{A}\right)$$

andequationofevolutionforitsexpectationvalueintimeisgivenby

$$\frac{d}{dt}\hat{a}(t) = Tr\frac{d\hat{\rho}}{dt}\hat{a}(t) \qquad (2.11)$$

InviewofEqs.(2.10)and(2.11),wefind

$$\frac{d}{dt}\langle\hat{a}(t)\rangle = Tr\,\epsilon\big(a\hat{\rho}\hat{a} - \hat{a}\hat{\rho}\hat{a} + b\hat{\rho}\hat{a}^{\dagger} - b\hat{\rho}\hat{a} - \hat{\rho}a\hat{a} + \hat{\rho}\hat{a}^{\dagger} - \hat{\rho}b\hat{a} + \hat{\rho}b\hat{a}\big)$$

$$+\frac{\Omega}{2}\big(\sigma_c^{\dagger}\hat{\rho}\hat{a} - \sigma_c\hat{\rho}\hat{a} - \hat{\rho}\sigma_c^{\dagger}\hat{a} + \hat{\rho}\sigma_c\hat{a}\big)$$

$$+g\big(\sigma_{a\dagger}\hat{a}\hat{\rho}\hat{a} - \hat{a}^{\dagger}\sigma_a\hat{\rho}\hat{a} + \sigma_b^{\dagger}\hat{b}\hat{\rho}\hat{a} - b^{\dagger}\sigma_b\hat{\rho}\hat{a} - \hat{\rho}\sigma_{a\dagger}\hat{a}^2 + \hat{\rho}\hat{a}^{\dagger}\sigma_a\hat{a} - \hat{\rho}\sigma_b^{\dagger}\hat{b}\hat{a} + \hat{\rho}b^{\dagger}\sigma_b\hat{a}\big)$$

$$+\frac{\kappa}{2}(\bar{n}+1)\big(2\hat{a}\hat{\rho}\hat{a}^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{a}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^{\dagger}\hat{a}^2\big) + \frac{\kappa\bar{n}}{2}\big(2\hat{a}^{\dagger}\hat{\rho}\hat{a}^2 - \hat{a}\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^{\dagger}\hat{a}\big)$$

$$+\frac{\kappa}{2}(\bar{n}+1)\big(2\hat{b}\hat{\rho}\hat{b}^{\dagger}\hat{a} - \hat{b}^{\dagger}\hat{b}\hat{\rho}\hat{a} - \hat{\rho}\hat{b}^{\dagger}\hat{b}\hat{a}\big) + \frac{\kappa\bar{n}}{2}\big(2\hat{b}^{\dagger}\hat{\rho}\hat{b}\hat{a} - \hat{b}\hat{b}^{\dagger}\hat{\rho}\hat{a} - \hat{\rho}\hat{b}\hat{b}^{\dagger}\hat{a}\big)$$

Applyingthecyclicpropertyofthetraceoperationtogetherwiththecommutationrelation

$$= Tr\,\epsilon\big(\hat{\rho}\hat{a}\hat{a}^{\dagger} - \hat{\rho}\hat{a}^2 + \hat{\rho}\hat{a}\hat{b}^{\dagger} - \hat{\rho}\hat{a}\hat{b} - \hat{\rho}\hat{a}^{\dagger}\hat{a} + \hat{\rho}\hat{a}^2 - \hat{\rho}\hat{b}^{\dagger}\hat{a} + \hat{\rho}\hat{b}\hat{a}\big)$$

$$+\frac{\Omega}{2}\big(\hat{\rho}\hat{a}\sigma_c^{\dagger} - \hat{\rho}\hat{a}\sigma_c - \hat{\rho}\sigma_c^{\dagger}\hat{a} + \hat{\rho}\sigma_c\hat{a}\big)$$

$$+g\big(\hat{\rho}\hat{a}\sigma_{a\dagger}\hat{a} - \hat{\rho}\hat{a}\hat{a}^{\dagger}\sigma_a + \hat{\rho}\hat{a}\sigma_b^{\dagger}\hat{b} - \hat{\rho}\hat{a}b^{\dagger}\sigma_b - \hat{\rho}\sigma_{a\dagger}\hat{a}^2 + \hat{\rho}\hat{a}^{\dagger}\sigma_a\hat{a} - \hat{\rho}\sigma_b^{\dagger}\hat{b}\hat{a} + \hat{\rho}b^{\dagger}\sigma_b\hat{a}\big)$$

$$+\frac{\kappa}{2}(\bar{n}+1)2\hat{\rho}\hat{a}^{\dagger}\hat{a}^2-\hat{\rho}\hat{a}\hat{a}^{\dagger}\hat{a}-\hat{\rho}\hat{a}^{\dagger}\hat{a}^2+\frac{\kappa\bar{n}}{2}2\hat{\rho}\hat{a}^2\hat{a}^{\dagger}-\hat{\rho}\hat{a}^2\hat{a}^{\dagger}-\hat{\rho}\hat{a}\hat{a}^{\dagger}\hat{a}$$

$$+\frac{\kappa}{2}(\bar{n}+1)2\hat{\rho}\hat{b}^{\dagger}\hat{b}\hat{a}-\hat{\rho}\hat{a}\hat{b}^{\dagger}\hat{b}-\hat{\rho}\hat{b}^{\dagger}\hat{b}\hat{a}+\frac{\kappa\bar{n}}{2}2\hat{\rho}\hat{b}\hat{a}\hat{b}^{\dagger}-\hat{\rho}\hat{a}\hat{b}\hat{b}^{\dagger}-\hat{\rho}\hat{b}\hat{b}^{\dagger}\hat{a}$$

$$\frac{d}{dt}\langle\hat{a}\rangle=-\frac{\kappa}{2}\langle\hat{a}\rangle-g\langle\sigma_a\rangle+\epsilon \quad (2.12)$$

$$\frac{d}{dt}\langle\hat{b}\rangle=-\frac{\kappa}{2}\langle\hat{b}\rangle-g\langle\sigma_b\rangle+\epsilon \quad (2.13)$$

Taking the expectation value of Eqs.(2.7) and (2.8) on both sides and comparing the results with Eqs.(2.12) and (2.13), one can see

$$\langle\hat{f}_a(t)\rangle=\langle\hat{f}_b(t)\rangle=0 \quad (2.14)$$

$$\frac{d}{dt}\langle\hat{a}^2\rangle=-\kappa\langle\hat{a}^2\rangle-g\left[\langle\hat{a}\sigma\rangle+\langle\sigma\hat{a}\rangle\right]+2\epsilon_a\langle\hat{a}\rangle \quad (2.15)$$

$$\frac{d}{dt}\langle\hat{a}^{\dagger 2}(t)\rangle=-\kappa\langle\hat{a}^{\dagger 2}\rangle-g\left[\langle\hat{a}^{\dagger}\sigma^{\dagger}\rangle+\langle\sigma^{\dagger}\hat{a}^{\dagger}\rangle\right]+2\epsilon_a\langle\hat{a}^{\dagger}\rangle \quad (2.16)$$

$$\frac{d}{dt}\langle\hat{b}^2\rangle=-\kappa\langle\hat{b}^2\rangle-g\left[\langle\hat{b}\sigma\rangle+\langle\sigma\hat{b}\rangle\right]+2\epsilon_b\langle\hat{b}\rangle \quad (2.17)$$

$$\frac{d}{dt}\langle\hat{b}^{\dagger 2}(t)\rangle=-\kappa\langle\hat{b}^{\dagger 2}\rangle-g\left[\langle\hat{b}^{\dagger}\sigma^{\dagger}\rangle+\langle\sigma^{\dagger}\hat{b}^{\dagger}\rangle\right]+2\epsilon_b\langle\hat{b}^{\dagger}\rangle \quad (2.18)$$

Furthermore, employing Eqs.(2.10) along with Eqs.(2.11), we see that

$$\frac{d}{dt}\langle\hat{a}^{\dagger}(t)\hat{a}(t)\rangle=Tr[\epsilon(\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{a}\hat{\rho}\hat{a}^{\dagger}\hat{a}+\hat{b}^{\dagger}\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{b}\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{\rho}\hat{a}^{\dagger 2}\hat{a}+\hat{\rho}\hat{a}\hat{a}^{\dagger}\hat{a}-\hat{\rho}\hat{b}^{\dagger}\hat{a}^{\dagger}\hat{a}+\hat{\rho}\hat{b}\hat{a}^{\dagger}\hat{a})$$

$$+\frac{\Omega}{2}(\sigma_c^{\dagger}\hat{\rho}\hat{a}^{\dagger}\hat{a}-\sigma_c\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{\rho}\sigma_c^{\dagger}\hat{a}^{\dagger}\hat{a}+\hat{\rho}\sigma_c\hat{a}^{\dagger}\hat{a})$$

$$+g\sigma_a^{\dagger}\hat{a}\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{a}^{\dagger}\sigma_a\hat{\rho}\hat{a}^{\dagger}\hat{a}+\sigma_b^{\dagger}\hat{b}\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{b}^{\dagger}\sigma_b\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{\rho}\sigma_a^{\dagger}\hat{a}\hat{a}^{\dagger}\hat{a}+\hat{\rho}\hat{a}^{\dagger}\sigma_a\hat{a}^{\dagger}\hat{a}-\hat{\rho}\sigma_b^{\dagger}\hat{b}\hat{a}^{\dagger}\hat{a}+\hat{\rho}\hat{b}^{\dagger}\sigma_b\hat{a}^{\dagger}\hat{a}$$

$$+\frac{\kappa}{2}(\bar{n}+1)(2\hat{a}\hat{\rho}\hat{a}^{\dagger 2}\hat{a}-\hat{a}^{\dagger}\hat{a}\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{\rho}\hat{a}^{\dagger}\hat{a}\hat{a}^{\dagger}\hat{a})+\frac{\kappa\bar{n}}{2}(2\hat{a}^{\dagger}\hat{\rho}\hat{a}\hat{a}^{\dagger}\hat{a}-\hat{a}\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{\rho}\hat{a}\hat{a}^{\dagger 2}\hat{a})$$

$$+\frac{\kappa}{2}(\bar{n}+1)(2\hat{b}\hat{\rho}\hat{b}^{\dagger}\hat{a}^{\dagger}\hat{a}-\hat{b}^{\dagger}\hat{b}\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{\rho}\hat{b}^{\dagger}\hat{b}\hat{a}^{\dagger}\hat{a})+\frac{\kappa\bar{n}}{2}(2\hat{b}^{\dagger}\hat{\rho}\hat{b}\hat{a}^{\dagger}\hat{a}-\hat{b}\hat{b}^{\dagger}\hat{\rho}\hat{a}^{\dagger}\hat{a}-\hat{\rho}\hat{b}\hat{b}^{\dagger}\hat{a}^{\dagger}\hat{a})]$$

Applying the cyclic property of the above Eqs, we obtain

$$\frac{d}{dt}\langle\hat{a}^{\dagger}(t)\hat{a}(t)\rangle=-\kappa\langle\hat{a}^{\dagger}\hat{a}\rangle-g\langle\hat{a}\sigma^{\dagger}\rangle+\langle\sigma\hat{a}^{\dagger}\rangle+\epsilon_a\langle\hat{a}^{\dagger}\rangle+\langle\hat{a}\rangle_b \quad (2.19)$$

$$\frac{d}{dt}\langle\hat{a}(t)\hat{a}^{\dagger}(t)\rangle=-\kappa\langle\hat{a}\hat{a}^{\dagger}\rangle+\kappa-g\langle\hat{a}\sigma^{\dagger}\rangle+\langle\sigma\hat{a}^{\dagger}\rangle+\epsilon_a\langle\hat{a}^{\dagger}\rangle+\langle\hat{a}\rangle \quad (2.20)$$

$$\frac{d}{dt}\langle\hat{b}^{\dagger}(t)\hat{b}(t)\rangle=-\kappa\langle\hat{b}^{\dagger}\hat{b}\rangle-g\langle\hat{b}\sigma\rangle+\langle\sigma^{\dagger}\hat{b}\rangle+\epsilon_a\langle\hat{b}^{\dagger}\rangle+\langle\hat{b}\rangle_a \quad (2.21)$$

$$\frac{d}{dt}\langle\hat{b}(t)\hat{b}^{\dagger}(t)\rangle=-\kappa\langle\hat{b}\hat{b}^{\dagger}\rangle+\kappa-g\langle\hat{b}\sigma\rangle+\langle\sigma^{\dagger}\hat{b}\rangle+\epsilon_a\langle\hat{a}^{\dagger}\rangle+\langle\hat{a}\rangle_b \quad (2.22)$$

$$\frac{d}{dt}\langle\hat{a}\hat{b}\rangle=-\kappa\langle\hat{a}\hat{b}\rangle-g\langle\hat{a}\sigma_b\rangle+\langle\hat{b}\sigma_a\rangle+\epsilon\langle\hat{a}\rangle+\langle\hat{b}\rangle \tag{2.23}$$

$$\frac{d}{dt}\langle\hat{a}^{\dagger}\hat{b}\rangle=-\kappa\langle\hat{a}^{\dagger}\hat{b}\rangle-g\langle\hat{a}^{\dagger}\sigma_b\rangle+\langle\hat{b}\sigma_a^{\dagger}\rangle+\epsilon\langle\hat{a}^{\dagger}\rangle+\langle\hat{b}\rangle \tag{2.24}$$

$$\frac{d}{dt}\langle\hat{b}^{\dagger}\hat{a}\rangle=-\kappa\langle\hat{b}^{\dagger}\hat{a}\rangle-g\langle\hat{b}^{\dagger}\sigma_a\rangle+\langle\sigma_b^{\dagger}\hat{a}\rangle+\epsilon\langle\hat{a}\rangle+\langle\hat{b}^{\dagger}\rangle \tag{2.25}$$

$$\frac{d}{dt}\langle\hat{a}^{\dagger}\hat{b}^{\dagger}\rangle=-\kappa\langle\hat{a}^{\dagger}\hat{b}^{\dagger}\rangle-g\langle\sigma_b^{\dagger}\hat{a}^{\dagger}\rangle+\langle\hat{b}^{\dagger}\sigma_a^{\dagger}\rangle+\epsilon\langle\hat{a}^{\dagger}\rangle+\langle\hat{b}^{\dagger}\rangle \tag{2.26}$$

But,onecanshowthat

$$\frac{d}{dt}\langle\hat{a}^2(t)\rangle=\langle\frac{d\hat{a}(t)}{dt}\hat{a}(t)\rangle+\langle\hat{a}(t)\frac{d\hat{a}}{dt}\rangle$$

InviewofEq.(2.7),thisreducesto

$$\frac{d}{dt}\langle\hat{a}^2(t)\rangle=\langle(-\frac{\kappa}{2}\hat{a}-g\sigma_a+\epsilon+\hat{f}_a(t))\hat{a}\rangle+\langle\hat{a}(-\frac{\kappa}{2}\hat{a}-g\sigma_a+\epsilon+\hat{f}_a(t))\rangle$$

$$=-\kappa\langle\hat{a}^2\rangle-g(\langle\sigma_a\hat{a}\rangle+\langle\hat{a}\sigma_a\rangle)+2\epsilon\langle\hat{a}\rangle+\langle\hat{f}_a\hat{a}\rangle+\langle\hat{a}\hat{f}_a\rangle \tag{2.27}$$

$$\frac{d}{dt}\langle\hat{b}^2(t)\rangle=\langle\frac{d\hat{b}(t)}{dt}\hat{b}(t)\rangle+\langle\hat{b}(t)\frac{d\hat{b}}{dt}\rangle$$

InviewofEq.(2.8),weget

$$\frac{d}{dt}\langle\hat{b}^2(t)\rangle=\langle(-\frac{\kappa}{2}\hat{b}-g\sigma_b+\epsilon+\hat{f}_b(t))\hat{b}\rangle+\langle\hat{b}(-\frac{\kappa}{2}\hat{b}-g\sigma_b+\epsilon+\hat{f}_b(t))\rangle$$

$$=-\kappa\langle\hat{b}^2\rangle-g(\langle\sigma_b\hat{b}\rangle+\langle\hat{b}\sigma_b\rangle)+2\epsilon\langle\hat{b}\rangle+\langle\hat{f}_b\hat{b}\rangle+\langle\hat{b}\hat{f}_b\rangle \tag{2.28}$$

UponcomparingEqs.(2.16)and(2.18)with(2.27)and(2.28),weseethat

$$\langle\hat{f}_a\hat{a}\rangle+\langle\hat{a}\hat{f}_a\rangle=\langle\hat{f}_b\hat{b}\rangle+\langle\hat{b}\hat{f}_b\rangle=0 \tag{2.29}$$

$$\langle\hat{a}^{\dagger}\hat{f}_a^{\dagger}\rangle+\langle\hat{f}_a^{\dagger}\hat{a}^{\dagger}\rangle=\langle\hat{b}^{\dagger}\hat{f}_b^{\dagger}\rangle+\langle\hat{f}_b^{\dagger}\hat{b}^{\dagger}\rangle=0 \tag{2.30}$$

$$\frac{d}{dt}\langle\hat{a}^{\dagger}\hat{a}\rangle=\langle\frac{d\hat{a}^{\dagger}}{dt}\hat{a}\rangle+\langle\hat{a}^{\dagger}\frac{d\hat{a}}{dt}\rangle$$

$$\frac{d}{dt}\langle\hat{a}^{\dagger}\hat{a}\rangle=\epsilon(\langle\hat{a}\rangle+\langle\hat{a}^{\dagger}\rangle)-g(\langle\sigma_a\hat{a}^{\dagger}\rangle+\langle\hat{a}\sigma_a^{\dagger}\rangle)-\kappa\langle\hat{a}^{\dagger}\hat{a}\rangle+\langle\hat{a}^{\dagger}\hat{f}_a\rangle+\langle\hat{f}_a^{\dagger}\hat{a}\rangle \tag{2.31}$$

$$\frac{d}{dt}\langle \hat{a}\hat{a}^{\dagger}\rangle = \langle \frac{d\hat{a}}{dt}\hat{a}^{\dagger}\rangle + \langle \hat{a}\frac{d\hat{a}^{\dagger}}{dt}\rangle$$

$$\frac{d}{dt}\langle \hat{a}\hat{a}^{\dagger}\rangle = \epsilon\langle \hat{a}\rangle + \langle \hat{a}^{\dagger}\rangle - g\langle \hat{a}\sigma\rangle + \langle \sigma^{\dagger}\hat{a}\rangle - \kappa\langle \hat{a}\hat{a}^{\dagger}\rangle + \langle \hat{a}\hat{f}_a^{\dagger}\rangle + \langle \hat{f}_a\hat{a}^{\dagger}\rangle \qquad (2.32)$$

$$\frac{d}{dt}\langle \hat{b}^{\dagger}\hat{b}\rangle = \langle \frac{d\hat{b}^{\dagger}}{dt}\hat{b}\rangle + \langle \hat{b}^{\dagger}\frac{d\hat{b}}{dt}\rangle$$

$$\frac{d}{dt}\langle \hat{b}^{\dagger}\hat{b}\rangle = \epsilon\langle \hat{b}\rangle + \langle \hat{b}^{\dagger}\rangle - g\langle \sigma\hat{b}^{\dagger}\rangle + \langle \hat{b}\sigma^{\dagger}\rangle - \kappa\langle \hat{b}^{\dagger}\hat{b}\rangle + \langle \hat{b}^{\dagger}\hat{f}_b\rangle + \langle \hat{f}_b^{\dagger}\hat{b}\rangle \qquad (2.33)$$

$$\frac{d}{dt}\langle \hat{b}\hat{b}^{\dagger}\rangle = \langle \frac{d\hat{b}}{dt}\hat{b}^{\dagger}\rangle + \langle \hat{b}\frac{d\hat{b}^{\dagger}}{dt}\rangle$$

$$\frac{d}{dt}\langle \hat{b}\hat{b}^{\dagger}\rangle = \epsilon\langle \hat{b}\rangle + \langle \hat{b}^{\dagger}\rangle - g\langle \hat{b}\sigma\rangle + \langle \sigma^{\dagger}\hat{b}\rangle - \kappa\langle \hat{b}\hat{b}^{\dagger}\rangle + \langle \hat{b}\hat{f}_b^{\dagger}\rangle + \langle \hat{f}_b\hat{b}^{\dagger}\rangle \qquad (2.34)$$

UponcomparingEqs.(2.19)and(2.20)with(2.31)and(2.32),wecouldseethat

$$\langle \hat{a}^{\dagger}\hat{f}_a\rangle + \langle \hat{f}_a^{\dagger}\hat{a}\rangle = 0 \qquad (2.35)$$

$$\langle \hat{a}\hat{f}_a^{\dagger}\rangle + \langle \hat{f}_a\hat{a}^{\dagger}\rangle = \kappa \qquad (2.36)$$

Similarly,comparingEqs.(2.21)and(2.22)with(2.33)and(2.34),onecouldget

$$\langle \hat{b}^{\dagger}\hat{f}_b\rangle + \langle \hat{f}_b^{\dagger}\hat{b}\rangle = 0 \qquad (2.37)$$

$$\langle \hat{b}\hat{f}_b^{\dagger}\rangle + \langle \hat{f}_b\hat{b}^{\dagger}\rangle = \kappa \qquad (2.38)$$

Thefollowingrelationscouldalsobeshownfollowingthesameprocess

$$\frac{d}{dt}\langle \hat{a}\hat{b}\rangle = \langle \frac{d\hat{a}}{dt}\hat{b}\rangle + \langle \hat{a}\frac{d\hat{b}}{dt}\rangle$$

$$\frac{d}{dt}\langle \hat{a}\hat{b}\rangle = \epsilon\langle \hat{a}\rangle + \langle \hat{b}\rangle - \kappa\langle \hat{a}\hat{b}\rangle - g\langle \hat{a}\sigma\rangle + \langle \hat{b}\sigma\rangle + \langle \hat{f}_a\hat{b}\rangle + \langle \hat{a}\hat{f}_b\rangle \qquad (2.39)$$

$$\frac{d}{dt}\langle \hat{a}^{\dagger}\hat{b}\rangle = \langle \frac{d\hat{a}^{\dagger}}{dt}\hat{b}\rangle + \langle \hat{a}^{\dagger}\frac{d\hat{b}}{dt}\rangle$$

$$\frac{d}{dt}\langle \hat{a}^{\dagger}\hat{b}\rangle = -\kappa\langle \hat{a}^{\dagger}\hat{b}\rangle + \epsilon\langle \hat{a}^{\dagger}\rangle + \langle \hat{b}\rangle - g\langle \hat{a}^{\dagger}\sigma\rangle + \langle \hat{b}\sigma^{\dagger}\rangle + \langle \hat{f}_a^{\dagger}\hat{b}\rangle + \langle \hat{a}^{\dagger}\hat{f}_b\rangle \qquad (2.40)$$

$$\frac{d}{dt}\langle\hat{b}^{\dagger}\hat{a}\rangle=\langle\frac{d\hat{b}^{\dagger}}{dt}\hat{a}\rangle+\langle\hat{b}^{\dagger}\frac{d\hat{a}}{dt}\rangle$$

$$\frac{d}{dt}\langle\hat{b}^{\dagger}\hat{a}\rangle=-\kappa\langle\hat{b}^{\dagger}\hat{a}\rangle+\epsilon\langle\hat{a}\rangle+\langle\hat{b}^{\dagger}\rangle-g\langle\hat{b}^{\dagger}\sigma\rangle+\langle\sigma^{\dagger}\hat{a}\rangle+\langle\hat{f}_b^{\dagger}\hat{a}\rangle+\langle\hat{b}^{\dagger}\hat{f}_a\rangle \quad (2.41)$$

$$\frac{d}{dt}\langle\hat{a}^{\dagger}\hat{b}^{\dagger}\rangle=\langle\frac{d\hat{a}^{\dagger}}{dt}\hat{b}^{\dagger}\rangle+\langle\hat{a}^{\dagger}\frac{d\hat{b}^{\dagger}}{dt}\rangle$$

$$\frac{d}{dt}\langle\hat{a}^{\dagger}\hat{b}^{\dagger}\rangle=-\kappa\langle\hat{a}^{\dagger}\hat{b}^{\dagger}\rangle+\epsilon\langle\hat{a}^{\dagger}\rangle+\langle\hat{b}^{\dagger}\rangle-g\langle\sigma^{\dagger}\hat{a}^{\dagger}\rangle+\langle\hat{b}^{\dagger}\sigma^{\dagger}\rangle+\langle\hat{f}_a^{\dagger}\hat{b}^{\dagger}\rangle+\langle\hat{a}^{\dagger}\hat{f}_b^{\dagger}\rangle \quad (2.42)$$

Comparingequations(2.23-2.26)withtherespectiveequationsin(2.39-2.42),thefollowing relationscouldbeestablished.

$$\langle\hat{f}_a\hat{b}\rangle+\langle\hat{a}\hat{f}_b\rangle=0 \quad (2.43)$$

$$\langle\hat{f}_a^{\dagger}\hat{b}\rangle+\langle\hat{a}^{\dagger}\hat{f}_b\rangle=0 \quad (2.44)$$

$$\langle\hat{f}_b^{\dagger}\hat{a}\rangle+\langle\hat{b}^{\dagger}\hat{f}_a\rangle=0 \quad (2.45)$$

$$\langle\hat{f}_a^{\dagger}\hat{b}^{\dagger}\rangle+\langle\hat{a}^{\dagger}\hat{f}_b^{\dagger}\rangle=0 \quad (2.46)$$

2.4CorrelationPropertiesofaCavityModeNoiseOp-erators

TheformalsolutionofEqs.(2.7)and(2.8)canbeputintheform

$$\hat{a}(t)=\hat{a}(0)e^{-\frac{\kappa t}{2}}+\int_0^t e^{-\frac{\kappa(t-t')}{2}}\left[\epsilon-g\hat{\sigma}_a(t')+\hat{f}_a(t')\right]dt' \quad (2.47)$$

$$\hat{b}(t)=\hat{b}(0)e^{-\frac{\kappa t}{2}}+\int_0^t e^{-\frac{\kappa(t-t')}{2}}\left[\epsilon-g\hat{\sigma}_b(t')+\hat{f}_b(t')\right]dt' \quad (2.48)$$

$$\hat{a}^{\dagger}(t)=\hat{a}^{\dagger}(0)e^{-\frac{\kappa t}{2}}+\int_0^t e^{-\frac{\kappa(t-t')}{2}}\left[\epsilon-g\hat{\sigma}_a^{\dagger}(t')+\hat{f}_a^{\dagger}(t')\right]dt' \quad (2.49)$$

$$\hat{b}^{\dagger}(t)=\hat{b}^{\dagger}(0)e^{-\frac{\kappa t}{2}}+\int_0^t e^{-\frac{\kappa(t-t')}{2}}\left[\epsilon-g\hat{\sigma}_b^{\dagger}(t')+\hat{f}_b^{\dagger}(t')\right]dt' \quad (2.50)$$

UponmultiplyingEq.(2.47)by $\hat{f}^{\dagger}$ fromtheleftwhileEq.(2.49)by f

fromtherightandtaking f_a a

expectationvalues,weget

$$\langle \hat{f}_a(t)\hat{a}^\dagger(t)\rangle = \langle \hat{f}_a(t)\hat{a}^\dagger(0)\rangle e^{-\frac{\kappa t}{2}} + \int_0^t e^{-\frac{\kappa(t-t')}{2}} \left[g\langle \hat{f}_a(t)\hat{\sigma}_a^\dagger(t')\rangle + \langle \hat{f}_a(t)\hat{f}_a^\dagger(t')\rangle \right] dt'$$

From the fact that the operator at a time t will not affect the operator at latter time t', thus

$$\langle \hat{a}(0)\hat{f}_a^{\dagger}(t)\rangle=\langle \hat{a}(0)\rangle\langle \hat{f}_a^{\dagger}(t)\rangle=0$$

$$\langle \hat{f}_a^{\dagger}(t')\hat{\sigma}(t)\rangle=\langle \hat{f}_a^{\dagger}(t')\rangle\langle \hat{\sigma}(t)\rangle=0$$

$$\langle \hat{f}_a^{\dagger}(t)\rangle=0$$

$$\langle \hat{f}_a^{\dagger}(t)\hat{a}(t)\rangle=\int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle \hat{f}_a^{\dagger}(t)\hat{f}_a(t')\rangle dt'$$

Similarly

$$\langle \hat{a}^{\dagger}(t)\hat{f}_a(t)\rangle=\int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle \hat{f}_a^{\dagger}(t')\hat{f}_a(t)\rangle dt'$$

Adding these two one can get

$$\langle \hat{f}_a^{\dagger}(t)\hat{a}(t)\rangle+\langle \hat{a}^{\dagger}(t)\hat{f}_a(t)\rangle=\int_0^t e^{-\frac{\kappa(t-t')}{2}}\left(\langle \hat{f}_a^{\dagger}(t)\hat{f}_a(t')\rangle+\langle \hat{f}_a^{\dagger}(t')\hat{f}_a(t)\rangle\right)dt'$$

In view of Eq.(2.35), this reduces to

$$0=\int_0^t e^{-\frac{\kappa(t-t')}{2}}\left(\langle \hat{f}_a^{\dagger}(t')\hat{f}_a(t)\rangle+\langle \hat{f}_a^{\dagger}(t)\hat{f}_a(t')\rangle\right)dt'$$

Setting

$$\langle \hat{f}_a^{\dagger}(t)\hat{a}(t)\rangle=\langle \hat{f}_a(t)\hat{a}(t)\rangle^{\dagger}$$

$$0=2\int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle \hat{f}_a^{\dagger}(t)\hat{f}_a(t')\rangle dt'$$

which implies

$$\langle \hat{f}_a^{\dagger}(t)\hat{a}(t)\rangle=0 \qquad (2.51)$$

Hence, one can also show from Eqs.(2.48) and (2.50) in the same manner that

$$\langle \hat{f}_b^{\dagger}(t)\hat{b}(t)\rangle=0 \qquad (2.52)$$

Upon multiplying Eq.(2.47) by $\hat{f}_a^\dagger$ from the right side while Eq.(2.49) by $\hat{f}_a$ from the left and taking expectation values, we get

$$\langle \hat{a}^\dagger(t)\hat{f}_a(t)\rangle = \langle \hat{a}^\dagger(0)\hat{f}_a\rangle e^{-\frac{\kappa t}{2}} + \int_0^t e^{-\frac{\kappa(t-t')}{2}} \left[g\langle \hat{\sigma}_b^\dagger(t')\hat{f}_a(t)\rangle + \langle \hat{f}_a^\dagger(t')\hat{f}_a(t)\rangle \right] dt'$$

$$\langle \hat{f}_a^\dagger(t)\hat{a}(t)\rangle = \langle \hat{f}_a^\dagger(t)\hat{a}(0)\rangle e^{-\frac{\kappa t}{2}} + \int_0^t e^{-\frac{\kappa(t-t')}{2}} \left[g\langle \hat{f}_a^\dagger(t)\hat{\sigma}_a(t')\rangle + \langle \hat{f}_a^\dagger(t)\hat{f}_a(t')\rangle \right] dt'$$

Since the initial state of a system could not be affected by a noise force at a latter time, we see

$$\langle \hat{f}_a(t)\hat{a}^\dagger(0)\rangle = \langle \hat{a}(0)\rangle\langle \hat{f}_a^\dagger(t)\rangle = 0 \quad and \quad \langle \hat{f}_a(t')\hat{\sigma}^\dagger(t)\rangle = \langle \hat{f}_a(t)\rangle\langle \hat{\sigma}^\dagger(t')\rangle = 0 \tag{2.53}$$

when we sum the two resulting equations, we get

$$\langle \hat{a}^\dagger(t)\hat{f}_a(t)\rangle + \langle \hat{f}_a^\dagger(t)\hat{a}(t)\rangle = \int_0^t e^{-\frac{\kappa(t-t')}{2}} \left[\langle \hat{f}_a^\dagger(t')\hat{f}_a(t)\rangle + \langle \hat{f}_a^\dagger(t)\hat{f}_a(t')\rangle \right] dt'$$

Setting the assumption

$$\langle \hat{f}_a(t')\hat{f}_a^\dagger(t)\rangle = \langle \hat{f}_a(t)\hat{f}_a^\dagger(t')\rangle$$

we arrive at

$$\langle \hat{a}^\dagger(t)\hat{f}_a(t)\rangle + \langle \hat{f}_a^\dagger(t)\hat{a}(t)\rangle = 2\int_0^t e^{-\frac{\kappa(t-t')}{2}} \langle \hat{f}_a^\dagger(t')\hat{f}_a(t)\rangle dt'$$

Employing Eq.(2.36), this reduces to

$$\kappa = 2\int_0^t e^{-\frac{\kappa(t-t')}{2}} \langle \hat{f}_a(t)\hat{f}_a^\dagger(t')\rangle dt'$$

From the fact that if

$$\int_0^t e^{-a(t-t')} F(t)G(t')dt' = D \tag{2.54}$$

then follows

$$F(t)G(t')dt = 2D\delta(t-t') \tag{2.55}$$

Hence, we get

$$\langle \hat{f}_a(t)\hat{f}_a^\dagger(t')\rangle = \kappa\delta(t-t') \tag{2.56}$$

Followingthesamesteponb(t)anditsconjugate,onecanshowthat

$$\langle \hat{f}_b(t)\hat{f}_b^{\dagger}(t')\rangle=\kappa\delta(t-t')$$

(2.57)UponmultiplyingEq.(2.47)byfafromtheleftandtakeexpectvalueonbothsides,weget

$$\langle \hat{f}_a(t)\hat{a}(t)\rangle=\langle \hat{f}_a(t)\hat{a}(0)\rangle e^{\frac{-\kappa t}{2}}+\int_0^t e^{\frac{-\kappa(t-t')}{2}}\left(g\langle \hat{f}_a(t)\hat{\sigma}_a(t')\rangle+\langle \hat{f}_a(t)\hat{f}_a(t')\rangle\right)dt'$$

ExpectationvalueofEq.(2.36)aftermultipliedby$f_a$fromtherightwouldbe

$$\langle \hat{a}(t)\hat{f}_a(t)\rangle=\langle \hat{a}(0)\hat{f}_a(t)\rangle e^{\frac{-\kappa t}{2}}+\int_0^t e^{\frac{-\kappa(t-t')}{2}}\left(g\langle \hat{\sigma}_a(t')\hat{f}_a(t)\rangle+\langle \hat{f}_a(t')\hat{f}_a(t)\rangle\right)dt'$$

Thesumoftheseequationsafterdroppingnon-correlatedoperatorsbythesamereasonas previousonesgiveus

$$\langle \hat{f}_a(t)\hat{a}(t)\rangle+\langle \hat{a}(t)\hat{f}_a(t)\rangle=\int_0^t e^{\frac{-\kappa(t-t')}{2}}\left(\langle \hat{f}_a(t)\hat{f}_a(t')\rangle+\langle \hat{f}_a(t')\hat{f}_a(t)\rangle\right)dt'$$

Withtheassumption,$\langle \hat{f}_a(t)\hat{f}_a(t')\rangle=\langle \hat{f}_a(t')\hat{f}_a(t)\rangle$weget

$$\langle \hat{f}_a(t)\hat{a}(t)\rangle+\langle \hat{a}(t)\hat{f}_a(t)\rangle=2\int_0^t e^{\frac{-\kappa(t-t')}{2}}\left(\langle \hat{f}_a(t')\hat{f}_a(t)\rangle\right)dt'$$

InviewofEq.(2.29),thistakestheform

$$0=2\int_0^t e^{\frac{-\kappa(t-t')}{2}}\langle \hat{f}_a(t')\hat{f}_a(t)\rangle dt'$$

Therefore

$$\langle \hat{f}_a(t)\hat{f}_a(t')\rangle=0 \quad (2.58)$$

$$\langle \hat{f}_b(t)\hat{f}_b(t')\rangle=0 \quad (2.59)$$

OnecanalsoshowfromEqs.(2.29)and(2.30)that

$$\langle \hat{f}_a^{\dagger}(t)\hat{f}_a^{\dagger}(t')\rangle=\langle \hat{f}_b^{\dagger}(t)\hat{f}_b^{\dagger}(t')\rangle=0 \quad (2.60)$$

UponmultiplyingEq.(2.47)by$f_b$fromtherightsideandtakingexpectationvaluestogetherwiththefactthatanoiseoperatoratcertaintimeshouldnothaveanyeffectonalightmode operatorattheearliertime,weget

$$\langle\hat{a}(t)f_b(t)\rangle = \int_0^t e^{\frac{-\kappa(t-t')}{2}} \langle f_a(t) f_b^{\prime}(t)\rangle dt$$

DoingthesameforEq.(2.48)aftermultiplyingitby$f_a$fromtheleft,weget

$$\langle f_a(t)\hat{b}(t)\rangle = \int_0^t e^{\frac{-\kappa(t-t')}{2}} \langle f_a(t) f_b^{\prime}(t)\rangle dt$$

Thesumofthesetwowillproduce

$$\langle\hat{a}(t)f_b(t)\rangle + \langle f_a(t)\hat{b}(t)\rangle = \int_0^t e^{\frac{-\kappa(t-t')}{2}} \left(\langle f_a^{\prime}(t) f_b(t)\rangle + \langle f_a(t^{\prime}) f_b(t)\rangle\right) dt$$

But,employingEq.(2.45),weget

$$\int_0^t e^{\frac{-\kappa(t-t')}{2}} [\langle f_a(t) f_b(t)\rangle^{\prime} + \langle f_a(t) f_b(t)\rangle] dt^{\prime} = 0$$

Setting

$$\langle f_a(t) f_b(t')\rangle = \langle f_a(t) f_b(t')\rangle$$

wearriveat

$$\langle f_b(t) f_b(t')\rangle = 0 \qquad (2.61)$$

Similarly,usingEqs.(2.47)-(2.50)togetherwiththerelationsunderEqs.(2.43-2.46),onecan showthefollowing

$$\langle f_a^{\dagger}(t) f_b(t')\rangle = \langle f_b^{\dagger}(t) f_a(t')\rangle = 0 \qquad (2.62)$$

$$\langle f_a^{\dagger}(t) f_b^{\dagger}(t')\rangle = 0 \qquad (2.63)$$

Finally,we can easily see that the Eqs.(2.51), (2.52), (2.55), (2.56), (2.57), (2.58), (2.59), (2.60),(2.61), (2.62) and (2.63) are the correlation properties of the noise operators associated withcavitymodeoperatorsaandbwhenthecavityiscoupledtotwomodethermalreservoir. Thesecorrelationpropertiesarerearrangedas

$$\langle\hat{f}_a\rangle=\langle\hat{f}_b\rangle=0 \tag{2.64}$$

$$\langle f_a^{\dagger}(t)f_a(t')\rangle=\langle f_b^{\dagger}(t)f_b(t')\rangle=0 \tag{2.65}$$

$$\langle f_a^{\dagger}(t)f_b(t')\rangle=\langle f_b^{\dagger}(t)f_a(t')\rangle=0 \tag{2.66}$$

$$\langle f_a(t)f_a^{\dagger}(t')\rangle=\langle f_b(t)f_b^{\dagger}(t')\rangle=\kappa\delta(t-t') \tag{2.67}$$

$$\langle f_a(t)f_b(t')\rangle=\langle f_a(t)f_a(t')\rangle=\langle f_b(t)f_b(t')\rangle=0 \tag{2.68}$$

$$\langle f_a^{\dagger}(t)f_b^{\dagger}(t')\rangle=\langle f_a^{\dagger}(t)f_a^{\dagger}(t')\rangle=\langle f_b^{\dagger}(t)f_b^{\dagger}(t')\rangle=0 \tag{2.69}$$

From these equations, we can easily observe that the noise operators associated with the twomode thermal reservoir under consideration do not have any effect on the dynamics of cavitymodesprovidedthatthecavitymodeoperatorsareusedinnormalorder.

2.5EquationsofEvolutionoftheAtomicOperators

Hereweseektoderivetheequationsofevolutionoftheexpectationvaluesoftheatomicoperators by applying the master equation.Moreover, we find the steady-state solutions of theequationsofevolutionoftheatomicoperators. Tothisend,employingtherelation

$$\frac{d}{dt}\langle\hat{A}\rangle=Tr(\frac{d\rho}{dt}\hat{A})$$

Along with the master Eq.(2.10), the equation of evolution for the expectation value of theatomicoperator$\hat{\sigma}_a$canbewrittenas

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = Tr\left(\frac{d\hat{\rho}}{dt}\hat{\sigma}_a\right)$$

Wecanshowthat

$$\frac{d}{dt}\langle\hat\sigma_a\rangle=\epsilon(\hat a^\dagger\hat\rho\sigma_a-\hat a\hat\rho\sigma_a+\hat b^\dagger\hat\rho\sigma_a-\hat b\hat\rho\sigma_a-\hat\rho\hat a^\dagger\sigma_a+\hat\rho\hat a\sigma_a-\hat\rho\hat b^\dagger\sigma_a+\hat\rho\hat b\sigma_a)$$

$$+\frac{\Omega}{2}(\sigma_c^\dagger\hat\rho\sigma_a-\sigma_c\hat\rho\sigma_a-\hat\rho\sigma_c^\dagger\sigma_a+\hat\rho\sigma_c\sigma_a)$$

$$+g\left(\sigma_a^\dagger\hat a\hat\rho\sigma_a-\hat a^\dagger\sigma_a\hat\rho\sigma_a+\sigma_b^\dagger\hat b\hat\rho\sigma_a-\hat b^\dagger\sigma_b\hat\rho\sigma_a-\hat\rho\sigma_a^\dagger\hat a\sigma_a+\hat\rho\hat a^\dagger\sigma_a-\hat\rho\sigma_b^\dagger\hat b\sigma_a+\hat\rho\hat b^\dagger\sigma_b\sigma_a\right)$$

$$+\frac{\kappa}{2}(\bar n+1)(2\hat a\hat\rho\hat a^\dagger\sigma_a-\hat a^\dagger\hat a\hat\rho\sigma_a-\hat\rho\hat a^\dagger\hat a\sigma_a)+\frac{\kappa\bar n}{2}(2\hat a^\dagger\hat\rho\hat a\sigma_a-\hat a\hat a^\dagger\hat\rho\sigma_a-\hat\rho\hat a\hat a^\dagger\sigma_a)$$

$$+\frac{\kappa}{2}(\bar n+1)(2\hat b\hat\rho\hat b^\dagger\sigma_a-\hat b^\dagger\hat b\hat\rho\sigma_a-\hat\rho\hat b^\dagger\hat b\sigma_a)+\frac{\kappa\bar n}{2}(2\hat b^\dagger\hat\rho\hat b\sigma_a-\hat b\hat b^\dagger\hat\rho\sigma_a-\hat\rho\hat b\hat b^\dagger\sigma_a)$$

Nowapplyingthecyclicpropertyoftraceoperationto Eq.(2.11)onceobtain

$$\frac{d}{dt}\langle\sigma_a\rangle=g(\langle(\eta_b-\eta_a)\hat a+\hat b^\dagger\sigma_c\rangle)+\frac{\Omega}{2}\langle\sigma_c^\dagger\rangle \quad (2.70)$$

$$\frac{d}{dt}\langle\sigma_b\rangle=g(\langle(\eta_c-\eta_b)\hat b-\hat a^\dagger\sigma_c\rangle)-\frac{\Omega}{2}\langle\sigma_a^\dagger\rangle \quad (2.71)$$

$$\frac{d}{dt}\langle\sigma_c\rangle=g(\langle\sigma_b\hat a-\sigma_a\hat b\rangle)+\frac{\Omega}{2}(\langle\eta_c-\eta_a\rangle) \quad (2.72)$$

$$\frac{d}{dt}\langle\eta_a\rangle=g(\langle\sigma_a^\dagger\hat a\rangle+\langle\hat a^\dagger\sigma_a\rangle)+\frac{\Omega}{2}(\langle\sigma_c^\dagger+\sigma_c\rangle) \quad (2.73)$$

$$\frac{d}{dt}\langle\eta_b\rangle=g(\langle\sigma_b^\dagger\hat b\rangle+\langle\hat b^\dagger\sigma_b\rangle)-\langle\hat a^\dagger\sigma_a\rangle-\langle\sigma_a^\dagger\hat a\rangle) \quad (2.74)$$

$$\frac{d}{dt}\langle\eta_c\rangle=-g(\langle\hat b^\dagger\sigma_b\rangle+\langle\sigma_b^\dagger\hat b\rangle)-\frac{\Omega}{2}(\langle\sigma_c+\sigma_c^\dagger\rangle) \quad (2.75)$$

It can be noted that expressions under Eqs.(2.70)-(2.75) are coupled nonlinear differentialequations and hence it is not possible to find exact time-dependent solutions of such equations.We would try to over come this challenge by applying the large-time approximation [1, 2].Then,employingthelarge-timeapproximationscheme,wegetanapproximatelyvalidrelationsfromequations(2.7)and(2.8),thesewouldbetheexactrelationsatsteadystate.

$$\hat a=-\frac{2g}{\kappa}\sigma_a+\frac{2\epsilon}{\kappa}+\frac{2}{\kappa}\hat f_a(t),\quad \hat a^\dagger=-\frac{2g}{\kappa}\sigma_a^\dagger+\frac{2\epsilon}{\kappa}+\frac{2}{\kappa}\hat f_a^\dagger(t) \quad (2.76)$$

$$\hat b=-\frac{2g}{\kappa}\sigma_b+\frac{2\epsilon}{\kappa}+\frac{2}{\kappa}\hat f_b(t),\quad \hat b^\dagger=-\frac{2g}{\kappa}\sigma_b^\dagger+\frac{2\epsilon}{\kappa}+\frac{2}{\kappa}\hat f_b^\dagger(t) \quad (2.77)$$

EmployingtheresultsunderEqs.(2.76)and(2.77)intoEqs.(2.70)-(2.75),theequationsof evolutionoftheatomicoperatorstaketheform

$$\frac{d}{dt}\langle\sigma_a\rangle=-\gamma\langle\sigma_a\rangle+\frac{\Omega}{2}\langle\sigma_c^\dagger\rangle+\frac{2g\epsilon}{\kappa}\left[\langle\eta_b\rangle-\langle\eta_a\rangle+\langle\sigma_c\rangle\right]+\frac{2g}{\kappa}\left[\langle\eta_b\hat f_a\rangle-\langle\eta_a\hat f_a\rangle+\langle\hat f_b^\dagger\sigma_c\rangle\right] \quad (2.78)$$

$$\frac{d}{dt}\langle\sigma_b\rangle = -\frac{\gamma_c}{2}\langle\sigma_b\rangle - \frac{\Omega}{2}\langle\sigma_a^{\dagger}\rangle + \frac{2g\epsilon}{\kappa}\left[\langle\eta_b\rangle - \langle\eta_a\rangle - \langle\sigma_c\rangle\right] + \frac{2g}{\kappa}\left[\langle\eta_c\hat{f}_b\rangle - \langle\eta_b\hat{f}_c\rangle - \langle\hat{f}_a^{\dagger}\sigma_c\rangle\right] \quad (2.79)$$

$$\frac{d}{dt}\langle\sigma_c\rangle = -\frac{\gamma_c}{2}\langle\sigma_c\rangle + \frac{2g\epsilon}{\kappa}\left[\langle\sigma_b\rangle - \langle\sigma_a\rangle\right] + \frac{2g}{\kappa}\left[\langle\sigma_b\hat{f}_a\rangle - \langle\sigma_a\hat{f}_b\rangle\right] + \frac{\Omega}{2}\left[\langle\eta_c\rangle - \langle\eta_a\rangle\right] \quad (2.80)$$

$$\frac{d}{dt}\langle\eta_a\rangle = -\gamma_c\langle\eta_a\rangle + \frac{2g\epsilon}{\kappa}\left[\langle\sigma_a^{\dagger}\rangle + \langle\sigma_a\rangle\right] + \frac{2g}{\kappa}\left[\langle\sigma_a^{\dagger}\hat{f}_a\rangle + \langle\hat{f}_a^{\dagger}\sigma_a\rangle\right] + \frac{\Omega}{2}\left[\langle\sigma_c^{\dagger}\rangle + \langle\sigma_c\rangle\right] \quad (2.81)$$

$$\frac{d}{dt}\langle\eta_b\rangle = -\gamma_c\left[\langle\eta_b\rangle - \langle\eta_c\rangle\right] + \frac{2g\epsilon}{\kappa}\left[\langle\sigma_b^{\dagger}\rangle + \langle\sigma_b\rangle - \langle\sigma_a^{\dagger}\rangle - \langle\sigma_a\rangle\right] + \frac{2g}{\kappa}\left[\langle\hat{f}_b^{\dagger}\sigma_b\rangle + \langle\sigma_b^{\dagger}\hat{f}_b\rangle - \langle\hat{f}_a^{\dagger}\sigma_a\rangle - \langle\sigma_a^{\dagger}\hat{f}_a\rangle\right] \quad (2.82)$$

$$\frac{d}{dt}\langle\eta_c\rangle = -\gamma_c\left[\langle\eta_c\rangle - 1\right] - \frac{2g\epsilon}{\kappa}\left[\langle\sigma_b\rangle + \langle\sigma_b^{\dagger}\rangle\right] - \frac{2g}{\kappa}\left[\langle\hat{f}_b^{\dagger}\sigma_b\rangle + \langle\sigma_b^{\dagger}\hat{f}_b\rangle\right] - \frac{\Omega}{2}\left[\langle\sigma_c\rangle + \langle\sigma_c^{\dagger}\rangle\right] \quad (2.83)$$

where,

$$\gamma_c = \frac{4g^2}{\kappa} \quad (2.84)$$

isthestimulatedemissiondecayconstant.EquationsunderEqs.(2.78)-

(2.83)couldberewritteninamoremanageableformbyintroducingatomicnoiseoperatorswithavanishingmean.

$$\frac{d}{dt}\sigma_a(t) = -\gamma_c\sigma_a(t) + \frac{\Omega}{2}\sigma_b^{\dagger}(t) + \frac{2g\epsilon}{\kappa}\left[\eta_b(t) - \eta_a(t) + \sigma_c(t)\right] + \frac{2g}{\kappa}\left[\eta_b(t)\hat{f}_a(t) - \eta_a(t)\hat{f}_a(t) + \hat{f}_b^{\dagger}(t)\sigma_c(t)\right] + \hat{F}_a \quad (2.85)$$

$$\frac{d}{dt}\sigma_b(t) = -\frac{\gamma_c}{2}\sigma_b(t) - \frac{\Omega}{2}\sigma_a^{\dagger}(t) + \frac{2g\epsilon}{\kappa}\left[\eta_c(t) - \eta_b(t) - \sigma_c(t)\right] + \frac{2g}{\kappa}\left[\eta_c(t)\hat{f}_b(t) - \eta_b(t)\hat{f}_b(t) - \hat{f}_a^{\dagger}(t)\sigma_c(t)\right] + \hat{F}_b \quad (2.86)$$

$$\frac{d}{dt}\sigma_c(t) = -\frac{\gamma_c}{2}\sigma_c(t) + \frac{2g\epsilon}{\kappa}\left[\sigma_b(t) - \sigma_a(t)\right] + \frac{\Omega}{2}\left[\eta_c(t) - \eta_a(t)\right] + \frac{2g}{\kappa}\left[\sigma_b(t)\hat{f}_a(t) - \sigma_a(t)\hat{f}_b(t)\right] + \hat{F}_c(t) \quad (2.87)$$

$$\frac{d}{dt}\eta_a(t) = -\gamma_c\eta_a(t) + \frac{2g\epsilon}{\kappa}\left[\sigma_a^{\dagger}(t) + \sigma_a(t)\right] + \frac{2g}{\kappa}\left[\sigma_a^{\dagger}(t)\hat{f}_a(t) + \hat{f}_a^{\dagger}(t)\sigma_a(t)\right] + \frac{\Omega}{2}\left[\sigma_c^{\dagger}(t) + \sigma_c(t)\right] + \hat{F}_1 \quad (2.88)$$

$$\frac{d}{dt}\eta_b(t) = -\gamma_c\left[\eta_b(t) - \eta_c(t)\right] + \frac{2g\epsilon}{\kappa}\left[\sigma_b^{\dagger}(t) + \sigma_b(t) - \sigma_a^{\dagger}(t) - \sigma_a(t)\right]$$

$$+\frac{2g}{\kappa}\Big[\hat{f}_b^{\dagger}(t)\sigma(t)+\hat{f}_b(t)\sigma^{\dagger}(t)-\hat{f}_b^{\dagger}(t)\sigma(t)-\sigma^{\dagger}(t)\hat{f}_a(t)\Big]+\hat{F}_a \qquad (2.89)$$

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$$\frac{d}{dt}\eta_c(t) = -\gamma_c\Big[\eta_c(t)+\eta_a(t)-1-\frac{2g\epsilon}{\kappa}\big[\sigma_b(t)+\sigma_b^\dagger(t)\big]-\frac{2g}{\kappa}\big[\sigma_b^\dagger(t)\hat{f}_b(t)+\hat{f}_b^\dagger(t)\sigma_b(t)\big]\Big]-\frac{\Omega}{2}\big[\sigma_c(t)+\sigma_c^\dagger(t)\big]+\hat{F}_3 \quad (2.90)$$

where $\hat{F}_a(t)$, $\hat{F}_b(t)$, $\hat{F}_c(t)$, $\hat{F}_1(t)$, $\hat{F}_2(t)$ and $\hat{F}_3(t)$ areatomicnoiseoperatorswhosecorrelationprop-ertiesremaintobedetermined.Now,weneedtoobtaintheexpectationvalueoftheproductoftheatomicoperatorsandthecavitymodenoiseoperatorsinvolvedinEqs.(2.78)-(2.83).To doso,theformalsolutionofEq.(2.89)couldbewrittenas

$$\eta_b(t) = \bar{\eta}_b(0)e^{-\gamma_c t}+\int_0^t e^{-\gamma_c(t-t')}\Big[-\eta_a(t')+\frac{2g\epsilon}{\kappa}\big[\sigma_b^\dagger(t')+\sigma_b(t')-\sigma_a^\dagger(t')-\sigma_a(t')\big]+\frac{2g}{\kappa}\big[\hat{f}_b^\dagger(t')\sigma_b(t')+\hat{f}_b^\dagger(t')\sigma_b^\dagger(t')-\hat{f}_a^\dagger(t')\sigma_a(t')-\sigma_a^\dagger(t')\hat{f}_a(t')\big]+\hat{F}_2(t')\Big]dt' \quad (2.91)$$

UponmultiplyingEq.(2.91)fromrightby $\hat{f}_a(t)$ andtakingexpectationvalue,wefind

$$\langle\eta_b(t)\hat{f}_a(t)\rangle=\langle\eta_b(0)\hat{f}_a(t)\rangle e^{-\gamma_c t}+\int_0^t e^{-\gamma_c(t-t')}\Big[-\langle\eta_a(t')\hat{f}_a(t)\rangle+\frac{2g\epsilon}{\kappa}\big(\langle\sigma_b^\dagger(t')\hat{f}_a(t)\rangle+\langle\sigma_b(t')\hat{f}_a(t)\rangle\big)-\frac{2g\epsilon}{\kappa}\langle\sigma_a^\dagger(t')\hat{f}_a(t)\rangle-\frac{2g\epsilon}{\kappa}\langle\sigma_a(t')\hat{f}_a(t)\rangle+\frac{2g}{\kappa}\langle\hat{f}_b^\dagger(t')\sigma_b(t')\hat{f}_a(t)\rangle+\frac{2g}{\kappa}\langle\hat{f}_b(t')\sigma_b^\dagger(t')\hat{f}_a(t)\rangle-\frac{2g}{\kappa}\big(\langle\hat{f}_a^\dagger(t')\sigma_a(t')\hat{f}_a(t)\rangle+\langle\sigma_a^\dagger(t')\hat{f}_a(t')\hat{f}_a(t)\rangle\big)+\langle\hat{F}_2(t')\hat{f}_a(t)\rangle\Big]dt' \quad (2.92)$$

Itisnotpossibletoevaluatetheaboveintegralunlessweadoptsomeapproximationscheme.Tothisend,weprimarilyneglectthecorrelationbetweentheatomicoperatorsandthecavity modenoiseoperators

$$\langle\eta_a(t')\hat{f}_a(t)\rangle=\langle\eta_a(t')\rangle\langle\hat{f}_a(t)\rangle=0 \quad (2.93)$$

$$\langle\sigma_b^\dagger(t')\hat{f}_a(t)\rangle+\langle\sigma_b(t')\hat{f}_a(t)\rangle=\langle\sigma_b^\dagger(t')\rangle\langle\hat{f}_a(t)\rangle+\langle\sigma_b(t')\rangle\langle\hat{f}_a(t)\rangle=0 \quad (2.94)$$

$$\langle\sigma_a^\dagger(t')\hat{f}_a(t)\rangle+\langle\sigma_a(t')\hat{f}_a(t)\rangle=\langle\sigma_a^\dagger(t')\rangle\langle\hat{f}_a(t)\rangle+\langle\sigma_a(t')\rangle\langle\hat{f}_a(t)\rangle=0 \quad (2.95)$$

Besides,weignorethenon-commutativityoftheatomicandcavitymodenoiseoperators,we seethat

$$\langle\hat{f}_b^\dagger(t')\sigma_b(t')\hat{f}_a(t)\rangle+\langle\hat{f}_b(t')\sigma_b^\dagger(t')\hat{f}_a(t)\rangle=\langle\sigma_b(t')\hat{f}_b^\dagger(t')\hat{f}_a(t)\rangle+\langle\sigma_b^\dagger(t')\hat{f}_b(t')\hat{f}_a(t)\rangle \quad (2.96)$$

$$\langle \hat{f}^{\dagger}_a(t)\sigma_a(t)\hat{f}_a(t')\rangle+\langle \hat{f}_a(t)\sigma^{\dagger}_a(t)\hat{f}_a(t')\rangle=\langle\sigma_a(t)\hat{f}^{\dagger}_a(t)\hat{f}_a(t')\rangle+\langle\sigma^{\dagger}_a(t)\hat{f}_a(t)\hat{f}_a(t')\rangle \quad (2.97)$$

Moreover,weusethefactthatanoperatoratacertaintimecouldnotaffectanoperatoratan earliertimesothat

$$\langle\eta_b(0)\hat{f}_a(t)\rangle=\langle\eta_b(0)\rangle\langle\hat{f}_a(t)\rangle=0 \quad (2.98)$$

andthesamefactappliedonEqs.(2.96)and(2.97)willleadto

$$\langle\sigma_b(t')\hat{f}^{\dagger}_b(t')\hat{f}_a(t)\rangle+\langle\sigma^{\dagger}_b(t')\hat{f}_b(t')\hat{f}_a(t)\rangle=\langle\sigma_b(t')\rangle\langle\hat{f}^{\dagger}_b(t')\hat{f}_a(t)\rangle+\langle\sigma^{\dagger}_b(t')\rangle\langle\hat{f}_b(t')\hat{f}_a(t)\rangle \quad (2.99)$$

$$\langle\sigma_a(t')\hat{f}^{\dagger}_a(t')\hat{f}_a(t)\rangle+\langle\sigma^{\dagger}_a(t')\hat{f}_a(t')\hat{f}_a(t)\rangle=\langle\sigma_a(t')\rangle\langle\hat{f}^{\dagger}_a(t')\hat{f}_a(t)\rangle+\langle\sigma^{\dagger}_a(t')\rangle\langle\hat{f}_a(t')\hat{f}_a(t)\rangle \quad (2.100)$$

EmployingthefactunderEqs.(2.64-2.69)inEqs.(2.99)and(2.100),wearriveat

$$\langle\sigma_b(t')\rangle\langle\hat{f}^{\dagger}_b(t')\hat{f}_a(t)\rangle+\langle\sigma^{\dagger}_b(t')\rangle\langle\hat{f}_b(t')\hat{f}_a(t)\rangle=\langle\sigma_a(t')\rangle\langle\hat{f}^{\dagger}_a(t')\hat{f}_a(t)\rangle+\langle\sigma^{\dagger}_a(t')\rangle\langle\hat{f}_a(t')\hat{f}_a(t)\rangle=0 \quad (2.101)$$

Then,uponneglectingthecorrelationbetweenthenoiseoperatorsofatomicoperatorsandthe cavitymodeoperators,weget

$$\langle\hat{F}_2(t')\hat{f}_a(t)\rangle=\langle\hat{F}_2(t')\rangle\langle\hat{f}_a(t)\rangle=0 \quad (2.102)$$

Therefore,employingEqs.(2.93-2.95),(2.98),(2.101)and(2.102)intoEq.(2.92),wewrite

$$\langle\eta_b(t)\hat{f}_a(t)\rangle=0 \quad (2.103)$$

$$\langle\hat{f}^{\dagger}_a(t)\eta^{\dagger}_b(t)\rangle=0 \quad (2.104)$$

Followingthesameprocedure,wealsoreadilyobtainthat

$$\langle\eta_b(t)\hat{f}_b(t)\rangle=\langle\hat{f}^{\dagger}_b(t)\eta^{\dagger}_b(t)\rangle=0 \quad (2.105)$$

$$\langle\eta_a(t)\hat{f}_a(t)\rangle=\langle\hat{f}^{\dagger}_a(t)\eta^{\dagger}_a(t)\rangle=\langle\eta_c(t)\hat{f}_b(t)\rangle=\langle\hat{f}^{\dagger}_b(t)\eta^{\dagger}_c(t)\rangle=0 \quad (2.106)$$

$$\langle\hat{f}^{\dagger}_b(t)\sigma_c(t)\rangle=\langle\sigma^{\dagger}_c(t)\hat{f}_b(t)\rangle=\langle\hat{f}_a(t)\sigma_c(t)\rangle=\langle\sigma^{\dagger}_c(t)\hat{f}^{\dagger}_a(t)\rangle=0 \quad (2.107)$$

$$\langle\sigma_a(t)\hat{f}_b(t)\rangle=\langle\hat{f}^{\dagger}_b(t)\sigma^{\dagger}_a(t)\rangle=\langle\sigma_b(t)\hat{f}_a(t)\rangle=\langle\hat{f}^{\dagger}_a(t)\sigma^{\dagger}_b(t)\rangle=0 \quad (2.108)$$

$$\langle\sigma^{\dagger}_a(t)\hat{f}_a(t)\rangle=\langle\hat{f}^{\dagger}_a(t)\sigma_a(t)\rangle=\langle\sigma^{\dagger}_b(t)\hat{f}_a(t)\rangle=\langle\hat{f}^{\dagger}_b(t)\sigma_b(t)\rangle=0 \quad (2.109)$$

UsingEqs.(2.93-2.95),(2.103),(2.104),and(2.105-2.109)inEqs.(2.78)-(2.83),weget

$$\frac{d}{dt}\langle\sigma_b\rangle = -\gamma_a\langle\sigma_b\rangle + \frac{\Omega}{2}\langle\sigma_c^{\dagger}\rangle + \frac{2g\epsilon}{\kappa}\left(\langle\eta_a\rangle - \langle\eta_b\rangle + \langle\sigma_c\rangle\right) \tag{2.110}$$

$$\frac{d}{dt}\langle\sigma_a\rangle = -\frac{\gamma_c}{2}\langle\sigma_b\rangle - \frac{\Omega}{2}\langle\sigma_b^{\dagger}\rangle + \frac{2g\epsilon}{\kappa}\left(\langle\eta_a\rangle - \langle\eta_b\rangle - \langle\sigma_c\rangle\right) \tag{2.111}$$

$$\frac{d}{dt}\langle\sigma_c\rangle = -\frac{\gamma_c}{2}\langle\sigma_c\rangle + \frac{2g\epsilon}{\kappa}\left(\langle\sigma_b\rangle - \langle\sigma_a\rangle\right) + \frac{\Omega}{2}\left(\langle\eta_c\rangle - \langle\eta_a\rangle\right) \tag{2.112}$$

$$\frac{d}{dt}\langle\eta_a\rangle = -\gamma_a\langle\eta_a\rangle + \frac{2g\epsilon}{\kappa}\left(\langle\sigma_a^{\dagger}\rangle + \langle\sigma_a\rangle\right) + \frac{\Omega}{2}\left(\langle\sigma_c^{\dagger}\rangle + \langle\sigma_c\rangle\right) \tag{2.113}$$

$$\frac{d}{dt}\langle\eta_b\rangle = -\gamma_b\left(\langle\eta_b\rangle - \langle\eta_c\rangle\right) + \frac{2g\epsilon}{\kappa}\left(\langle\sigma_b^{\dagger}\rangle + \langle\sigma_a\rangle - \langle\sigma_c^{\dagger}\rangle - \langle\sigma_c\rangle\right) \tag{2.114}$$

$$\frac{d}{dt}\langle\eta_c\rangle = -\gamma_c\left(\langle\eta_c\rangle + \langle\eta\rangle - 1\right) - \frac{2g\epsilon}{\kappa}\left(\langle\sigma_b\rangle + \langle\sigma_a^{\dagger}\rangle\right) - \frac{\Omega}{2}\left(\langle\sigma_c\rangle + \langle\sigma_c^{\dagger}\rangle\right) \tag{2.115}$$

Wenotethattheseequationsrepresenttheequationsofevolutionfortheatomicoperatorsforasinglenon-degeneratethree-levelatomplacedinsideacavitydrivenbytwomodecoherentlight and coupled to thermal reservoir.From these equations one can easily observe that thenoiseoperatorsassociatedwiththermalreservoirdonotaffectthedynamicsoftheatomicoperatorsusedinnormal order.Tomakeitmorevisible,webettertakethesituationwhereNnon-degeneratethree-levelatomsareplacedinthecavitycoupledtothermalreservoir.Consequently, stronger cavity radiations will be produced due to the superposition of radiationmodesradiatedfromeachoftheseatoms.Setting$\hat{m}_a$to betotalamodeand$\hat{m}_b$to betotal bmoderadiationsgenerated,thetotalcavitymoderadiation$\hat{m}$willbesumofthetwocavity modes.Usingthedefinition

$$\hat{m} = \hat{m}_a + \hat{m}_b \tag{2.116}$$

$$\hat{m}^{\dagger} = \hat{m}_a^{\dagger} + \hat{m}_b^{\dagger} \tag{2.117}$$

$$\hat{m}_a = \sum_{k=1}^{N}\sigma_a^{k} = \sum_{k=1}^{N}|\hat{b}\rangle\langle\hat{a}| = N|\hat{b}\rangle\langle\hat{a}| \tag{2.118}$$

$$\hat{m}_b = \sum_{k=1}^{N}\sigma_b^{k} = \sum_{k=1}^{N}|\hat{c}\rangle\langle\hat{b}| = N|\hat{c}\rangle\langle\hat{b}| \tag{2.119}$$

$$\hat{m}_c=\sum_{k=1}\sigma^k=\sum_{k=1}|\hat{c}\rangle\langle\hat{a}_c|=N|\hat{c}\rangle\langle\hat{a}| \qquad (2.120)$$

$$\hat{N}_a=\sum_{k=1}^{N}\eta_a^k=\sum_{k=1}^{N}|\hat{a}\rangle\langle\hat{a}|=N|\hat{a}\rangle\langle\hat{a}| \qquad (2.121)$$

$$\hat{N}_b=\sum_{k=1}^{N}\eta_b^k=\sum_{k=1}^{N}|\hat{b}\rangle\langle\hat{b}|=N|\hat{b}\rangle\langle\hat{b}| \qquad (2.122)$$

$$\hat{N}_c=\sum_{k=1}^{N}\eta_c^k=\sum_{k=1}^{N}|\hat{c}\rangle\langle\hat{c}|=N|\hat{c}\rangle\langle\hat{c}| \qquad (2.123)$$

withtheoperatorsN_a,$N_b$and$N_c$representingthenumberofatomsinthetop,intermediateand bottom levels.let us taking the summation of the above equations(2.121-2.123) of the atomsyields

$$\langle\hat{N}_a\rangle+\langle\hat{N}_b\rangle+\langle\hat{N}_c\rangle=\sum_{k=1}^{N}\eta_a^k+\sum_{k=1}^{N}\eta_b^k+\sum_{k=1}^{N}\eta_c^k=\sum_{k=1}^{N}\langle\eta_a^k\rangle+\langle\eta_b^k\rangle+\langle\eta_c^k\rangle \qquad (2.124)$$

employingthecompletenessrelation

$$\eta_a^k+\eta_b^k+\eta_c^k=I=\sum_{k=1}^{N}\langle I\rangle N=N \qquad (2.125)$$

wheretherespectivetransitionprobabilityamplitudesη_a,$\eta_b$and$\eta_c$forthepossiblestatesorenergylevels$|\hat{a}\rangle$$|\hat{b}\rangle$and$|\hat{c}\rangle$are

$$\eta_a=|\hat{a}\rangle\langle\hat{a}|,\eta_b=|\hat{b}\rangle\langle\hat{b}|,and,\eta_c=|\hat{c}\rangle\langle\hat{c}|$$

weobservethatcombinationofeq(2.116)andeq(2.117)yields

$$\hat{m}^\dagger\hat{m}=(\hat{m}_a^\dagger+\hat{m}_b^\dagger)(\hat{m}_a+\hat{m}_b) \qquad (2.126)$$

$$\hat{m}^\dagger\hat{m}=\hat{m}_a^\dagger\hat{m}_a+\hat{m}_a^\dagger\hat{m}_b+\hat{m}_b^\dagger\hat{m}_a+\hat{m}_b^\dagger\hat{m}_b \qquad (2.127)$$

$$\hat{m}^\dagger\hat{m}=\hat{N}\hat{N}_a+\hat{N}\hat{N}_b=\hat{N}(\hat{N}_a+\hat{N}_b) \qquad (2.128)$$

$$\hat{m}\hat{m}^\dagger=(\hat{m}_a+\hat{m}_b)(\hat{m}_a^\dagger+\hat{m}_b^\dagger) \qquad (2.129)$$

$$\hat{m}\hat{m}^\dagger=\hat{m}_a\hat{m}_a^\dagger+\hat{m}_a\hat{m}_b^\dagger+\hat{m}_b\hat{m}_a^\dagger+\hat{m}_b\hat{m}_b^\dagger \qquad (2.130)$$

$$\hat{m}\hat{m}^\dagger=\hat{N}\hat{N}_b+\hat{N}\hat{N}_c=\hat{N}(\hat{N}_b+\hat{N}_c) \qquad (2.131)$$

wenextproceedtodetermine$\hat{m}^2$usingthedefinitiongivenbyeq(2.116)onecanwrite

$$\hat{m}^2=(\hat{m}_a+\hat{m}_b)(\hat{m}_a+\hat{m}_b) \quad (2.132)$$

$$\hat{m}^2=\hat{m}_a\hat{m}_a+\hat{m}_a\hat{m}_b+\hat{m}_b\hat{m}_a+\hat{m}_b\hat{m}_b \qquad (2.133)$$

$$\hat{m}^2=\hat{N}\hat{m}, \text{ and } \hat{m}_c^{\dagger 2}=\hat{N}\hat{m}_c \qquad (2.134)$$

$$\hat{m}_a^2=\hat{m}_b^2=\hat{m}_c^2=\hat{m}_a^{\dagger 2}=\hat{m}_b^{\dagger 2}=\hat{m}_c^{\dagger 2}=0 \qquad (2.135)$$

$$\hat{m}_a\hat{m}_b^{\dagger}=\hat{m}_a^{\dagger}\hat{m}_b=\hat{m}_b^{\dagger}\hat{m}_a=\hat{m}_b\hat{m}_a^{\dagger}=0 \qquad (2.136)$$

$$\hat{m}_a^{\dagger}\hat{m}_a=\hat{m}_c^{\dagger}\hat{m}_c=N\hat{N}_a \qquad (2.137)$$

$$\hat{m}_a\hat{m}_a^{\dagger}=\hat{m}_b^{\dagger}\hat{m}_b=N\hat{N}_b \qquad (2.138)$$

$$\hat{m}_c\hat{m}_c^{\dagger}=\hat{m}_b\hat{m}_b^{\dagger}=N\hat{N}_c \qquad (2.139)$$

Inordertoincludetheeffectsofthepresenceof Nthreelevelatomsinthecavity,weneedto develop the equations of evolution of the atomic operator for the system having N non-degenerate three-level atoms inside a coherently driven cavity coupled to the two mode thermalreservoir and modify the quantum Langevin equations.The equations of evolution of the atomicoperators could be simply generated by summing Eqs.(2.110)-(2.111) over N atoms placed inthe cavity. Thus equations of evolutions of atomic operators for N non-degenerate three levelatomswillappearas

$$\frac{d}{dt}\langle\hat{m}_a\rangle=-\gamma\langle\hat{m}_a\rangle+\frac{\Omega}{2}\langle\hat{m}_c^{\dagger}\rangle+\frac{2g\epsilon}{\kappa}\left(\langle\hat{N}_b\rangle-\langle\hat{N}_a\rangle+\langle\hat{m}_c\rangle\right) \qquad (2.140)$$

$$\frac{d}{dt}\langle\hat{m}_b\rangle=-\frac{\gamma_c}{2}\langle\hat{m}_b\rangle-\frac{\Omega}{2}\langle\hat{m}_a^{\dagger}\rangle+\frac{2g\epsilon}{\kappa}\left(\langle\hat{N}_c\rangle-\langle\hat{N}_b\rangle-\langle\hat{m}_c\rangle\right) \qquad (2.141)$$

$$\frac{d}{dt}\langle\hat{m}_c\rangle=-\frac{\gamma_c}{2}\langle\hat{m}_c\rangle+\frac{2g\epsilon}{\kappa}\left(\langle\hat{m}_b\rangle-\langle\hat{m}_a\rangle\right)+\frac{\Omega}{2}\left(\langle\hat{N}_c\rangle-\langle\hat{N}_a\rangle\right) \qquad (2.142)$$

$$\frac{d}{dt}\langle\hat{N}_a\rangle=-\gamma\langle\hat{N}_a\rangle+\frac{2g\epsilon}{\kappa}\left(\langle\hat{m}_a^{\dagger}\rangle+\langle\hat{m}_a\rangle\right)+\frac{\Omega}{2}\left(\langle\hat{m}_c^{\dagger}\rangle+\langle\hat{m}_c\rangle\right) \qquad (2.143)$$

$$\frac{d}{dt}\langle\hat{N}_b\rangle=-\gamma\left(\langle\hat{N}_b\rangle-\langle\hat{N}_c\rangle\right)+\frac{2g\epsilon}{\kappa}\left(\langle\hat{m}_b^{\dagger}\rangle+\langle\hat{m}_b\rangle-\langle\hat{m}_a^{\dagger}\rangle-\langle\hat{m}_a\rangle\right) \qquad (2.144)$$

$$\frac{d}{dt}\langle\hat{N}_c\rangle=-\gamma\left(\langle\hat{N}_c\rangle-\langle\hat{N}_c\rangle-N\right)-\frac{2g\epsilon}{\kappa}\left(\langle\hat{m}_b\rangle+\langle\hat{m}_b^{\dagger}\rangle\right)-\frac{\Omega}{2}\left(\langle\hat{m}_c^{\dagger}\rangle+\langle\hat{m}_c\rangle\right) \qquad (2.145)$$

ThesearetheequationsofevolutionforatomicoperatorswhenNnon-degeneratethree-levelatomsareplacedinsideacavitydrivenbytwomodecoherentlightandcoupledtotwomode thermalreservoir.Asweconfineourselvestolinearanalysis,weignoretheg-terms[1,2,7]in Eqs.(2.142),(2.143),(2.144)and(2.145)sothat

$$\frac{d}{dt}\langle\hat{m}_c\rangle = -\frac{\gamma_c}{2}\langle\hat{m}_c\rangle + \frac{\Omega}{2}\left(\langle\hat{N}_c\rangle - \langle\hat{N}_a\rangle\right) \quad (2.146)$$

$$\frac{d}{dt}\langle\hat{N}_a\rangle = -\gamma_c\langle\hat{N}_a\rangle + \frac{\Omega}{2}\left(\langle\hat{m}_c^{\dagger}\rangle + \langle\hat{m}_c\rangle\right) \quad (2.147)$$

$$\frac{d}{dt}\langle\hat{N}_b\rangle = -\gamma_c\left(\langle\hat{N}_b\rangle - \langle\hat{N}_a\rangle\right) \quad (2.148)$$

$$\frac{d}{dt}\langle\hat{N}_c\rangle = -\gamma_c\left(\langle\hat{N}_c\rangle - \langle\hat{N}_a\rangle\right) - \frac{\Omega}{2}\left(\langle\hat{m}_c\rangle + \langle\hat{m}_c^{\dagger}\rangle\right) \quad (2.149)$$

Besides, we impose the good-cavity limit condition($\kappa\gamma$). Under such condition, the cavitymode variables change slowly compared with the atomic variables as the result atomic variablesreach steady state in a relatively short period γ^{-1}. One can take the time derivatives of suchvariablestobezero,whilekeepingthezero-orderatomicandcavitymodevariablesattime t.Thisproceduremaybereferredtoastheadiabaticapproximationscheme.Thus,uponapplyingtheadiabaticapproximationschemeontheequationsunder(2.146),(2.147),(2.148) and(2.149),weget

$$\langle\hat{m}_c\rangle = \frac{\Omega}{\gamma_c}\left(\langle\hat{N}_c\rangle - \langle\hat{N}_a\rangle\right) \quad (2.150)$$

$$\langle\hat{N}_a\rangle = \frac{\Omega}{2\gamma_c}\left(\langle\hat{m}_c^{\dagger}\rangle + \langle\hat{m}_c\rangle\right) \quad (2.151)$$

$$\langle\hat{N}_b\rangle = \langle\hat{N}_a\rangle \quad (2.152)$$

$$\langle\hat{N}_c\rangle = N - 2\langle\hat{N}_a\rangle = N - \frac{\Omega}{\gamma_c}\left(\langle\hat{m}_c\rangle + \langle\hat{m}_c^{\dagger}\rangle\right) \quad (2.153)$$

NowcombinationofEqs.(2.140),(2.150),(2.151),and(2.152)aswellasEqs.(2.141),(2.150),(2.151),and(2.153)leadsto

$$\frac{d}{dt}\langle\hat{m}\rangle = -\gamma_a\langle\hat{m}\rangle + \frac{\Omega}{2}\langle\hat{m}_c^{\dagger}\rangle + \frac{2g\epsilon\Omega}{\kappa\gamma_c}\left(N - 3\langle\hat{N}_a\rangle\right) \quad (2.154)$$

$$\frac{d}{dt}\langle\hat{m}_b\rangle = -\frac{\gamma_c}{2}\langle\hat{m}_b\rangle - \frac{\Omega}{2}\langle\hat{m}^{\dagger}\rangle + \frac{2g\epsilon}{\kappa}\left(N - 3\langle\hat{N}_a\rangle\right) - \frac{\Omega}{\gamma_c}\left(N - 3\langle\hat{N}_a\rangle\right) \quad (2.155)$$

$_{b}\rangle - \quad 2\langle m_a \quad \kappa \quad b \quad \gamma_c \quad a$

Usingoncemoretheadiabaticapproximationscheme,weeasilyfind

$$\langle \hat{m}_a \rangle = \frac{\Omega}{2\gamma_c} \langle \hat{m}_b^{\dagger} \rangle + \frac{2g\epsilon\Omega}{\kappa\gamma_c^2} \left(N - 3\langle \hat{N}_a \rangle \right)$$

$$\langle \hat{m}_b \rangle = \frac{\Omega}{\gamma_c} \langle \hat{m}_a^{\dagger} \rangle + \frac{4g\epsilon}{\kappa\gamma_c} \left(N - 3\langle \hat{N}_b \rangle \right) - \frac{\Omega}{\gamma_c} \left(N - 3\langle \hat{N}_a \rangle \right)$$

Solvingtheseequations,wegetthesteadystatevaluestobe

$$\langle \hat{m}_a \rangle = \frac{4g\epsilon\Omega}{2\gamma_c^2 + \Omega^2} \left(N - 3\langle \hat{N}_b \rangle \right) - \frac{4g\epsilon\Omega^2}{\gamma_c(2\gamma_c^2 + \Omega^2)} \left(N - 3\langle \hat{N}_a \rangle \right) \quad (2.156)$$

$$\langle \hat{m}_b \rangle = \frac{4g\epsilon\gamma_c}{2\gamma_c^2 + \Omega^2} \left(N - 3\langle \hat{N}_b \rangle \right) - \frac{4g\epsilon\Omega^2}{\gamma_c(2\gamma_c^2 + \Omega^2)} \left(N - 3\langle \hat{N}_a \rangle \right) \quad (2.157)$$

Solvingequationsunder(2.150),(2.151),(2.152)(2.153)andemployingtheresultsinequationsunder(2.156),(2.157),wecouldeasilydevelopthefollowingsteadystateequations

$$\langle \hat{m} \rangle_{ss} = \langle \hat{m}_c^{\dagger} \rangle_{ss} = \left[\frac{\Omega\gamma_c}{(\gamma_c^2 + 3\Omega^2)} \right] N \quad (2.158)$$

$$\langle \hat{N}_a \rangle_{ss} = \left[\frac{\Omega^2}{(\gamma_c^2 + 3\Omega^2)} \right] N \quad (2.159)$$

$$\langle \hat{N}_b \rangle_{ss} = \left[\frac{\Omega^2}{(\gamma_c^2 + 3\Omega^2)} \right] N \quad (2.160)$$

$$\langle \hat{N}_c \rangle_{ss} = \left[\frac{\gamma_c^2 \Omega^2}{(\gamma_c^2 + 3\Omega^2)} \right] N \quad (2.161)$$

$$\langle \hat{m}_a \rangle_{ss} = \langle \hat{m}_a^{\dagger} \rangle_{ss} = \left[\frac{4g\epsilon(\Omega\gamma_c - \Omega^2)}{2\gamma_c^2 + \Omega^2} \right] \left[\frac{N\gamma_c}{2\gamma_c^2 + 3\Omega^2} \right] \quad (2.162)$$

$$\langle \hat{m}_b \rangle_{ss} = \langle \hat{m}_b^{\dagger} \rangle_{ss} = \left[\frac{4g\epsilon(\gamma_c^2 - \Omega^2)}{2\gamma_c^2 + \Omega^2} \right] \left[\frac{N\gamma_c}{2\gamma_c^2 + 3\Omega^2} \right] \quad (2.163)$$

We note that Eqs.(2.158)- (2.163) represent the steady-state solutions of the equations of evo-lution of the atomic operators in the adiabatic approximation scheme. From these equations,one could easily observe that the interaction between the two mode coherent light and the twomodecavityradiation($g\epsilon - term$)doesnotaffectthenumbersofatoms(N_a, N_b, N_c)inthethreeenergylevelsorpossibleenergystatesandtheatomiccouplingbetweentopandbottomlevels($\hat{m}_c$)intheadiabaticapproximationscheme.But,thetwomodecoherentlighthasadi-rectimpactonthecavitymoderadiations($\hat{m}_a$,$\hat{m}_b$).Intheabsenceofstimulateddecay(γ_c=0) atsteadystate,alloftheenergylevelswillhaveone-thirdofthetotalatomsinthecavity

$$\langle \hat{N}_a\rangle_{ss}=\langle \hat{N}_b\rangle_{ss}=\langle \hat{N}_c\rangle_{ss}=\frac{N}{3} \quad \gamma_c=0 \tag{2.164}$$

Moreover,uponsettingϵ=0,forthecaseinwhichthereisnodriving coherent light,the steady-statesolutionsdescribedbyEqs.(2.158)-(2.163)taketheform

$$\langle \hat{m}_c\rangle=\langle \hat{m}_c^\dagger\rangle=0 \tag{2.165}$$

$$\langle \hat{N}_a\rangle_{ss}=0 \tag{2.166}$$

$$\langle \hat{N}_b\rangle_{ss}=0 \tag{2.167}$$

$$\langle \hat{N}_c\rangle_{ss}=N \tag{2.168}$$

$$\langle \hat{m}_a\rangle_{ss}=\langle \hat{m}_a^\dagger\rangle_{ss}=0 \tag{2.169}$$

$$\langle \hat{m}_b\rangle_{ss}=\langle \hat{m}_b^\dagger\rangle_{ss}=0 \tag{2.170}$$

The physical interpretation of Eqs.(2.165)- (2.170) is as follows.When there is no drivingcoherentlight(ϵ=0)thatpumptheatomsbackto thetoplevel,noanyatom($\langle \hat{m}_c\rangle$=0) could get a means to come to higher energy level.As the result this there could not be anyatom in the top (N_a= 0) and the middle (N_b= 0) levels. Consequently, all atoms (N_c= N)willpopulate in the lower energy level. Furthermore, if there are no atoms in higher energy levels($|\hat{a}\rangle,|\hat{b}\rangle$),therewouldbenoatomiclasingprocess($\langle \hat{m}_a\rangle=\langle \hat{m}_b\rangle$=0).Here,wenotethatthe coherent light is interacting with the cavity modes (ϵ) and pumping (Ω) the atoms from thebottom level to top level.But, we found that the interaction of coherent light and the cavitymodesdoesnotaffectthenumberofatomsinenergylevelswhichisthecentralideainatomicprocess.Moreover,theeffectofthisinteractionontheradiationmodesisnotclearlyobservedascomparedtothestimulatedemission(γ_c)andtheatomiccoupling(Ω)whichshowclearandstrong effect.so, we better ignore the interaction of coherent light while such light is used forcouplingatthesametime.Consequently,weneedtomodifythequantumLangevinequations ofcavitymodesinacoherentlydriventhreelevellaserpresentinthecavityunderEqs.(2.7)and (2.8)as

$$\frac{d}{dt}\hat{a}(t)=-\frac{\kappa}{2}\hat{a}-g\sigma_a+\hat{f}_a(t) \quad (2.171)$$

$$\frac{d}{dt}\hat{b}(t)=-\frac{\kappa}{2}\hat{b}-g\sigma_b+\hat{f}_b(t) \quad (2.172)$$

Besides,toincludetheeffectofNatoms,wewriteitas

$$\frac{d}{dt}\hat{a}(t)=-\frac{\kappa}{2}\hat{a}+\lambda^{J}\hat{m}_a+\beta^{J}\hat{f}_a(t) \quad (2.173)$$

$$\frac{d}{dt}\hat{b}(t)=-\frac{\kappa}{2}\hat{b}+\lambda^{JJ}\hat{m}_b+\beta^{JJ}\hat{f}_b(t) \quad (2.174)$$

where$\hat{g}(t)$anoiseoperator.inwhichλisaconstantwhosevalueremainstobefixed.Applying the steady-state solution of Eq.(2.7), we get for all the atoms, we can show the commutationrelationofcavitymodeoperatorstobe

$$[\hat{a},\hat{a}^{\dagger}]=\frac{4g^2}{\kappa^2}(N_b-N_a)+\frac{4N}{\kappa^2}[\hat{f}_a,\hat{f}_a^{\dagger}] \quad (2.175)$$

$$[\hat{a},\hat{a}^{\dagger}]=\frac{\gamma_c}{\kappa}(\eta_b-\eta_a) \quad (2.176)$$

andonsummingoverallatoms,wehave

$$[\hat{a},\hat{a}^{\dagger}]=\frac{\gamma_c}{\kappa}(N_b-N_a) \quad (2.177)$$

where,

$$[\hat{a},\hat{a}^{\dagger}]=\sum_{k=1}^{N}[\hat{a},\hat{a}^{\dagger}] \quad (2.178)$$

standsforthecommutatorof$\hat{a}$and$\hat{a}^{\dagger}$whenlightmodeaisinteractingwithalltheNthree-level atoms.ThesteadystatesolutionofEq(2.173)isfoundtobe

$$\hat{a}=\frac{2\lambda\hat{m}_a}{\kappa} \quad (2.179)$$

Ontheotherhand,applyingthesteadystatesolutionofEq.(2.179)andusingthefactthatatomicoperatorcommute withcavitymodenoiseoperator[2,7]wecouldshowthat

$$[\hat{a},\hat{a}^{\dagger}]=\hat{a}\hat{a}^{\dagger}-\hat{a}^{\dagger}\hat{a}$$

$$[\hat{a},\hat{a}^{\dagger}]=\frac{4N\lambda'^2}{\kappa}(N_b-N_a)+\frac{4\beta'^2}{\kappa^2}[\hat{f}_a,\hat{f}_a^{\dagger}] \tag{2.180}$$

ComparingEqs.(2.192)and(2.197),weget

$\lambda'=\lambda''=\pm\frac{g}{\sqrt{N}}$ and $\beta'=\beta''=\sqrt{N}$ inviewofthisresult,thenewquantumLangevinequations

canberewrittenas

$$\frac{d}{dt}\hat{a}(t)=-\frac{\kappa}{2}\hat{a}-\frac{g}{\sqrt{N}}\hat{m}_a+\sqrt{N}\hat{f}_a(t) \tag{2.181}$$

$$\frac{d}{dt}\hat{b}(t)=-\frac{\kappa}{2}\hat{b}-\frac{g}{\sqrt{N}}\hat{m}_b+\sqrt{N}\hat{f}_b(t) \tag{2.182}$$

Uponsetting $\hat{F}(t)=\sqrt{N}\hat{f}(t)$ asacavitymodenoiseoperator whenthecavitymodeis interacting with N three-level atoms with a vanishing mean, the quantum Langevin equationforcavitymodes $\hat{a}$ and $\hat{b}$ canbewrittenas

$$\frac{d}{dt}\hat{a}=-\frac{\kappa}{2}\hat{a}-\frac{g}{\sqrt{N}}\hat{m}_a+F_a(t) \tag{2.183}$$

$$\frac{d}{dt}\hat{b}=-\frac{\kappa}{2}\hat{b}-\frac{g}{\sqrt{N}}\hat{m}_b+F_b(t) \tag{2.184}$$

From Eq.(2.64-2.269), we note that the noise operators associated with two mode thermalreservoirdonothaveanyeffectonthequantumdynamicsofcavitymodeoperatorsinnormalordering. So, one can remove the noise operator in quantum Langevin equations for normalorderoperations.

$$\frac{d}{dt}\hat{a}=-\frac{\kappa}{2}\hat{a}-\frac{g}{\sqrt{N}}\hat{m}_a \tag{2.185}$$

$$\frac{d}{dt}\hat{b}=-\frac{\kappa}{2}\hat{b}-\frac{g}{\sqrt{N}}\hat{m}_b \tag{2.186}$$

2.6CorrelationPropertiesofAtomicNoiseOperators

Furthermore,toincludetheeffectsofNnon-degeneratethreelevelatomplacedinacavitydrivenbytwomodecoherentlightandcoupledtotwomodethermalreservoir,wewrite

$$\frac{d}{dt}\hat{m}_a(t)=-\frac{\mu}{2}\hat{m}_a+\hat{g}_a(t) \tag{2.187}$$

$$\frac{d}{dt}\hat{m}_b(t)=-\frac{\mu}{2}\hat{m}_b+\hat{g}_b(t) \quad (2.188)$$

where μ is a constant whose value remain to be determined and $\hat{g}(t)$ is atomic nose operator with vanishing mean.

$$\langle \hat{g}_a(t)\rangle=\langle \hat{g}_b(t)\rangle=0 \quad (2.189)$$

From the relation

$$\frac{d}{dt}\langle \hat{m}_a^{\dagger}\hat{m}_a\rangle=\langle \frac{d\hat{m}_a^{\dagger}}{dt}\hat{m}_a\rangle+\langle \hat{m}_a^{\dagger}\frac{d\hat{m}_a}{dt}\rangle$$

Using Eqs.(2.137) and (2.187), we find

$$\frac{d}{dt}\langle N\rangle=-\mu\langle N\rangle+\frac{1}{N}\left(\langle \hat{g}_a^{\dagger}\hat{m}_a\rangle+\langle \hat{m}_a^{\dagger}\hat{g}_a\rangle\right) \quad (2.190)$$

But, from Eq.(2.147) and (2.150), we see

$$\frac{d}{dt}\langle N_a\rangle=-\gamma_c\langle N_a\rangle+\frac{\Omega^2}{\gamma_c}\left[\langle N_c\rangle-\langle N_a\rangle\right]$$

Employing Eq.(2.161), one could rewrite this as

$$\frac{d}{dt}\langle N_a\rangle=-\frac{\left[\gamma_c^2+\Omega^2\right]}{\gamma_c}\langle N_a\rangle+\frac{\Omega^2}{\gamma_c}\frac{\gamma_c^2+\Omega^2}{\gamma_c^2+3\Omega^2}N \quad (2.191)$$

Comparing Eqs.(2.190) and (2.191), we see that

$$\mu=\frac{\gamma_c^2+\Omega^2}{\gamma_c} \quad (2.192)$$

$$\langle \hat{g}_a^{\dagger}\hat{m}_a\rangle+\langle \hat{m}_a^{\dagger}\hat{g}_a\rangle=\frac{\Omega^2}{\gamma_c}\frac{\gamma_c^2+\Omega^2}{\gamma_c^2+3\Omega^2}N^2 \quad (2.193)$$

Eq.(2.193) implies that

$$\langle \hat{g}_a^{\dagger}(t)\hat{g}_a(t')\rangle=\mu N\langle N\rangle_{ss}\delta(t-t')$$

Let us proceed using the relation

$$\frac{d}{dt}\langle \hat{m}_a\hat{m}_a^{\dagger}\rangle=\langle \frac{d\hat{m}_a}{dt}\hat{m}_a^{\dagger}\rangle+\langle \hat{m}_a\frac{d\hat{m}_a^{\dagger}}{dt}\rangle$$

Using Eqs.(2.138) and (2.187), we find

$$\frac{d}{dt}\langle N\rangle_b=-\mu\langle N\rangle_b+\frac{1}{N}\left[\langle \hat{g}_a\hat{m}_a^{\dagger}\rangle+\langle \hat{m}_a\hat{g}_a^{\dagger}\rangle\right] \quad (2.194)$$

EmployingEq.(2.148)andthefact$\langle N_b\rangle=\langle N_b\rangle$,wesee

$$\frac{d}{dt}\langle N_b\rangle = -\gamma_c \left[\langle N_b\rangle - \langle N_b\rangle\right] = 0$$

(2.195)ComparingEqs.(2.212)and(2.213-2.218),wefind

$$\langle \hat{g}_a \hat{m}_a^\dagger\rangle + \langle \hat{m}_a \hat{g}_a^\dagger\rangle = 0$$

$\langle \hat{g}_a(t') \hat{g}_a^\dagger(t)\rangle = 0$Moreover,fromtheresultunderEq.(2.136)andtherelation

$$\frac{d}{dt}\langle \hat{m}_b \hat{m}_a^\dagger\rangle = \langle \frac{d\hat{m}_b}{dt}\hat{m}_a^\dagger\rangle + \langle \hat{m}_b \frac{d\hat{m}_a^\dagger}{dt}\rangle$$

wecanshowthat

$$\langle \hat{g}_b \hat{m}_a^\dagger\rangle + \langle \hat{m}_a \hat{g}_b^\dagger\rangle = 0$$

Thisimplies

$$\langle \hat{g}_b(t)\hat{g}_a^\dagger(t')\rangle = \langle \hat{g}_a(t)\hat{g}_b^\dagger(t')\rangle = 0$$

Wenotealsothat$\hat{m}_a\hat{m}_b=0$and$\hat{m}_b^\dagger\hat{m}_a^\dagger=0$.Fromtheseresults,wecanshowthecorrelation betweenthetwonoiseoperatorsas

$$\langle \hat{g}_a(t)\hat{g}_b(t')\rangle = \langle \hat{g}_b(t)\hat{g}_a(t')\rangle = 0$$

$$\langle \hat{g}_a^\dagger(t)\hat{g}_b^\dagger(t')\rangle = \langle \hat{g}_b^\dagger(t')\hat{g}_a(t)\rangle = 0$$

Furthermore,fromEq.(2.135),wecoulddevelop

$\langle \hat{g}_a(t')\hat{g}_a(t)\rangle = \langle \hat{g}_a^\dagger(t')\hat{g}_a^\dagger(t)\rangle = \langle \hat{g}_b(t')\hat{g}_b(t)\rangle = \langle \hat{g}_b^\dagger(t')\hat{g}_b^\dagger(t)\rangle = 0$Thecorrelationpropertiesarerearrangedas

$$\langle \hat{g}_a(t)\rangle = \langle \hat{g}_b(t)\rangle = 0 \qquad (2.196)$$

$$\langle \hat{g}_a^{\dagger}(t)\hat{g}_a(t')\rangle=\langle \hat{g}_b^{\dagger}(t)\hat{g}_b(t')\rangle\mu N_a\langle N\rangle ss\delta(t-t') \quad (2.197)$$

$$\langle \hat{g}_a(t)\hat{g}_a^{\dagger}(t')\rangle=\langle \hat{g}_b(t)\hat{g}_b^{\dagger}(t')\rangle=0 \quad (2.198)$$

$$\langle \hat{g}_a^{\dagger}(t)\hat{g}_b(t')\rangle=\langle \hat{g}_b^{\dagger}(t)\hat{g}_a(t')\rangle=0 \quad (2.199)$$

$$\langle \hat{g}_a(t)\hat{g}_b(t')\rangle=\langle \hat{g}_a(t)\hat{g}_a(t')\rangle=\langle \hat{g}_b(t)\hat{g}_b(t')\rangle=0 \quad (2.200)$$

$$\langle \hat{g}_a^{\dagger}(t)\hat{g}_b^{\dagger}(t')\rangle=\langle \hat{g}_a^{\dagger}(t)\hat{g}_a^{\dagger}(t')\rangle=\langle \hat{g}_b^{\dagger}(t)\hat{g}_b^{\dagger}(t')\rangle=0 \quad (2.201)$$

These are the equations representing the correlation properties of atomic noise operators.Onecaneasilyobservethatalltherelationsvanishexcepttheonewithnormallyorderedcavitymode operators.Thus,the atomic noise operators associated with the two mode thermal reser-voir under consideration do not have any effect on quantum dynamics of cavity mode operatorsprovided that the cavity mode operators are not used in normal order.We note also that thenoiseoperatorsofthecavitymodeandtheatomicoperatorsassociatedwithtwomodether-mal reservoir affect the dynamics of the cavity modes in opposite ordering of the cavity modeoperators,thelateraffectthecavitymodeoperatorsusedinnormalorder.

Chapter 3PhotonStatistics

Thestatisticalpropertiesofalightmodeisdescribedintermsofthemeanandthevarianceofthe photon number,the photon number distribution and the second-order correlation function.Therelationbetweenthemeanandvarianceofthephotonnumberwilldeterminethephotonstatistics as Poissonian,super-Poissonian and sub-Poissonian[1, 2, 22].Where as the probabilityof two photons reaching a detector in pairs (the bunching and anti bunching phenomena) canbe determined from the second-order correlation function.In this chapter we wish to studythestatisticalpropertiesofthecavitylightmodesproducedbythenon-degeneratethreelevellaser driven by two mode coherent light and coupled two mode thermal reservoir.Applying thesolutionsoftheequationsofevolutionoftheexpectationvaluesfortheatomicoperatorsandthe quantum Langevin equations for the cavity mode operators, we obtain the global photonstatisticsforlightmodes**a**and**b**.Inaddition,wedeterminethe,theglobalmeanandvarianceofthephoton numbersforlightmodes**a**and**b**.

3.1Single-ModePhotonStatistics

Inthissectionweseektoobtaintheglobalmeanandvarianceofthephotonnumbersforlightmodes**a**and**b**.

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3.1.1 GlobalMeanPhotonNumber

Hereweseektocalculatethemeanphotonnumberforlightmodes**a**and**b**intheentirefrequencyinterval.Tothisend,usingtherelation

$$\frac{d}{dt}\langle \hat{a}^{\dagger}(t)\hat{a}(t)\rangle=\langle \frac{d\hat{a}^{\dagger}(t)}{dt}\hat{a}(t)\rangle+\langle \hat{a}^{\dagger}(t)\frac{d\hat{a}(t)}{dt}\rangle \tag{3.1}$$

alongwithEq.(2.183),wereadilyfind

$$\frac{d}{dt}\langle \hat{a}^{\dagger}(t)\hat{a}(t)\rangle=-\kappa\langle \hat{a}^{\dagger}(t)\hat{a}(t)\rangle-\frac{g}{\sqrt{N}}\left(\langle \hat{a}^{\dagger}(t)\hat{m}_a(t)\rangle+\langle \hat{m}_a^{\dagger}(t)\hat{a}(t)\rangle\right)$$
$$+\sqrt{N}\left(\langle \hat{F}_a^{\dagger}(t)\hat{a}(t)\rangle+\langle \hat{a}^{\dagger}(t)\hat{F}_a(t)\rangle\right) \tag{3.2}$$

FromthefactunderEq.(2.64-2.69)

$$\langle \hat{F}_a^{\dagger}(t)\hat{a}(t)\rangle+\langle \hat{a}^{\dagger}(t)\hat{F}_a(t)\rangle=0 \tag{3.3}$$

Nextweseek to evaluate$\langle \hat{a}^{\dagger}(t)\hat{m}_a(t)\rangle$.Tothisend,applyingthelarge-timeapproximation scheme toEq.(2.183),weget

$$\hat{a}=-\frac{2g}{\kappa\sqrt{N}}+\frac{2F_a(t)}{\kappa} \tag{3.4}$$

MultiplyingtheadjointofEq.(3.4)ontherightby$\hat{m}_a(t)$andtakingtheexpectationvalueof theresultingexpression,wehave

$$\langle \hat{a}^{\dagger}(t)\hat{m}_a(t)\rangle=-\frac{2gN}{\kappa\sqrt{N}}\langle \hat{N}_a(t)\rangle+\frac{2}{\kappa}\langle \hat{F}_a^{\dagger}(t)\hat{m}_a(t)\rangle \tag{3.5}$$

WenowproceedtofindtheexpectationvalueoftheproductinvolvinganoiseoperatorandanatomicoperatorthatappearsinEq.(3.5).Tothisend,aformalsolutionofEq.(2.140),withthefactthatthemeannumberofatomsinaandinbequal,canbewrittenas

$$\hat{m}_a(t)=\hat{m}_a(0)e^{-\gamma_c t}+\int_0^t e^{-\gamma_c(t-t')}\left[\frac{\Omega}{2}\hat{m}_b^{\dagger}(t')+\frac{2g\epsilon}{\kappa}\hat{m}_c(t')+F_a(t')\right]dt' \tag{3.6}$$

Upon multiplying Eq. (3.6) from by $\hat{F}_a^\dagger(t)$ left and taking expectation values on both sides, we get

$$\langle \hat{F}_a^\dagger(t)\hat{m}_a(t)\rangle = \langle \hat{F}_a^\dagger(t)\hat{m}_a(0)\rangle e^{-\gamma_c t} + \int_0^t e^{-\gamma_c(t-t')}\Big[\frac{\Omega}{2}\langle \hat{F}_a^\dagger(t)\hat{m}_b(t')\rangle + \frac{2g\epsilon}{\kappa}\langle \hat{F}_a^\dagger(t)\hat{m}_c(t')\rangle + \langle \hat{F}_a^\dagger(t)\hat{F}_a(t')\rangle\Big]dt'$$

But, as the noise operator at certain time should not affect the dynamics of an operator in the earlier time together with the assumption that the noise operator one mode light will not affect the atomic operator of the other mode, we set

$$\langle \hat{F}_a^\dagger(t)\hat{m}(0)\rangle = \langle \hat{F}_a^\dagger(t)\rangle\langle \hat{m}(0)\rangle = 0$$

$$\langle \hat{F}_a^\dagger(t)\hat{m}_b(t')\rangle = \langle \hat{F}_a^\dagger(t)\rangle\langle \hat{m}_b(t')\rangle = 0$$

$$\langle \hat{F}_a^\dagger(t')\rangle\langle \hat{m}(t)\rangle = \langle \hat{F}_a^\dagger(t)\rangle\langle \hat{m}(t')\rangle = 0$$

In view of these two results and the fact under Eq. (2.64-2.69), we get

$$\langle \hat{F}_a^\dagger(t)\hat{m}_a(t)\rangle = 0 \qquad (3.7)$$

Therefore

$$\langle \hat{a}^\dagger(t)\hat{m}_a(t)\rangle = -\frac{2g\sqrt{N}}{\kappa}\langle N_a(t)\rangle \qquad (3.8)$$

Then Eq. (3.2) reduces to

$$\frac{d}{dt}\langle \hat{a}^\dagger\hat{a}\rangle = -\kappa\langle \hat{a}^\dagger\hat{a}\rangle + \frac{4g^2}{\kappa}\langle N_a\rangle \qquad (3.9)$$

The steady-state value of this equation has the form

$$\langle \hat{a}^\dagger\hat{a}\rangle_{ss} = \frac{4g^2}{\kappa^2}\langle N_a\rangle_{ss} = \frac{\gamma_c}{\kappa}\langle N_a\rangle_{ss} \qquad (3.10)$$

In the same way, one could show that

$$\langle \hat{b}^{\dagger}\hat{b}\rangle_{ss}=\frac{\gamma_c}{\kappa_b}\langle N\rangle_{ss} \tag{3.11}$$

Wecontinuetofind$\langle \hat{a}\hat{a}^{\dagger}\rangle$usingtherelation

$$\frac{d}{dt}\langle \hat{a}\hat{a}^{\dagger}\rangle=\langle\frac{d\hat{a}}{dt}\hat{a}^{\dagger}\rangle+\langle\hat{a}\frac{d\hat{a}^{\dagger}}{dt}\rangle$$

(3.12)alongwithEq.(2.185),wereadilyfind

$$\frac{d}{dt}\langle\hat{a}(t)\hat{a}^{\dagger}(t)\rangle=-\kappa\langle\hat{a}(t)\hat{a}^{\dagger}(t)\rangle-\frac{g}{\sqrt{N}}\left[\langle\hat{a}(t)\hat{m}_a^{\dagger}(t)\rangle+\langle\hat{m}_a(t)\hat{a}^{\dagger}(t)\rangle\right]$$
$$+\sqrt{N}\left[\langle\hat{F}_a(t)\hat{a}^{\dagger}(t)\rangle+\langle\hat{a}(t)\hat{F}_a^{\dagger}(t)\rangle\right] \quad (3.13)$$

Sincethecavitymodeoperatorsareinantinormalorder,theeffectofnoiseoperatorcouldnotbeignoredsimplyandweneedto evaluate$\langle\hat{m}_a(t)\hat{a}^{\dagger}(t)\rangle$.MultiplyingtheadjointofEq.(3.4) ontheleftby$\hat{m}_a(t)$andtakingtheexpectationvalueoftheresultingexpression,weget

$$\langle\hat{m}_a(t)\hat{a}^{\dagger}(t)\rangle=\frac{-2gN}{\kappa\sqrt{N}}\langle N_b(t)rangle+\frac{2}{\kappa}\langle\hat{m}_a(t)\hat{F}_a^{\dagger}(t)\rangle \quad (3.14)$$

Now,uponmultiplyingEq.(3.6)ontherightby$F_a^{\dagger}(t)$andtakingtheexpectationvalueofthe resultingexpression,wehave

$$\langle\hat{m}_a(t)\hat{F}_a^{\dagger}(t)\rangle=\langle\hat{m}_a(0)\hat{F}_a^{\dagger}(t)\rangle e^{-\gamma_c t}+\int_0^t e^{-\gamma_c(t-t')}\left[\frac{\Omega}{2}\langle\hat{m}_b(t')\hat{F}_a^{\dagger}(t)\rangle+\frac{2g\epsilon}{\kappa}\langle\hat{m}_c(t')\hat{F}_a^{\dagger}(t)\rangle+\langle\hat{F}_a(t')\hat{F}_a^{\dagger}(t)\rangle\right]dt'$$

Using the time difference between noise operator and cavity mode operators together with theassumption that the noise operator one mode light will not affect, we do expectation valuesseparatelytoget

$$\langle\hat{m}_a(0)\hat{F}_a^{\dagger}(t)\rangle=\langle\hat{m}_a(0)\rangle\langle\hat{F}_a^{\dagger}(t)\rangle=0$$

$$\langle\hat{m}_b(t')\hat{F}_a^{\dagger}(t)\rangle=\langle\hat{m}_b(t')\rangle\langle\hat{F}_a^{\dagger}(t)\rangle=0$$

$$\langle \hat{F}_a(t') \hat{m}_c^\dagger(t)\rangle = \langle \hat{m}_c(t') \hat{F}_a^\dagger(t)\rangle = 0$$

Then,wefind

$$\langle \hat{m}_a(t)\hat{F}^\dagger_a(t)\rangle = \int_0^t e^{-\gamma_c(t-t')}\langle \hat{F}_a(t)\hat{F}^\dagger_a(t')\rangle dt'$$

$$\langle \hat{m}_a(t)\rangle\langle \hat{F}^\dagger_a(t)\rangle = \int_0^t e^{-\gamma_c(t-t')}\langle \hat{F}^\dagger_a(t')\rangle\langle \hat{F}_a(t)\rangle dt'$$

InviewofEq.(2.67),weget

$$\langle \hat{m}_a(t)\hat{F}^\dagger_a(t)\rangle = \kappa\sqrt{N}\int_0^t e^{-\gamma_c(t-t')}\delta(t-t')dt'$$

NotingthepropertiesofKroneckerdeltafunction,weget

$$\langle \hat{m}_a(t)\hat{F}^\dagger_a(t)\rangle = \kappa\sqrt{N} \tag{3.15}$$

EmployingEq.(3.15)inEq.(3.14),wefind

$$\langle \hat{m}_a(t)\hat{a}^\dagger(t)\rangle = \frac{2gN}{\kappa\sqrt{N}}\langle \hat{N}_b(t)\rangle \pm \sqrt{\frac{N}{2}} \tag{3.16}$$

Onecanalsoshowthat

$$\langle \hat{a}(t)\hat{F}^\dagger_a(t)\rangle = \frac{-2g}{\kappa}\sqrt{N}\langle \hat{N}_b(t)\rangle \pm \sqrt{\frac{N}{2}} \tag{3.17}$$

Toevaluate$\langle \hat{a}(t)\hat{F}^\dagger_a(t)\rangle$,weusetheformalsolutionofEq.(2.183)givenby

$$\hat{a}(t) = \hat{a}(0)e^{\frac{-kt}{2}} + \int_0^t e^{\frac{-k(t-t')}{2}}\left[\hat{F}_a(t') - \frac{g}{\sqrt{N}}\hat{m}_a(t')\right]dt' \tag{3.18}$$

UponmultiplyingEq.(3.18)by $\hat{F}^\dagger_a(t)$fromtherightandtakingexpectationvalueonboth sides,weget

$$\langle \hat{a}(t)\hat{F}^\dagger_a(t)\rangle = \langle \hat{a}(0)\hat{F}^\dagger_a(t)\rangle e^{\frac{-kt}{2}} + \int_0^t e^{\frac{-k(t-t')}{2}}\left[\langle \hat{F}_a(t')\hat{F}^\dagger_a(t)\rangle - \frac{g}{\sqrt{N}}\langle \hat{m}_a(t')\hat{F}^\dagger_a(t)\rangle\right]dt'$$

Bythesamereasonasbefore,thisreducesto

$$\langle \hat{a}(t)\hat{F}^\dagger_a(t)\rangle = \int_0^t e^{\frac{-k(t-t')}{2}}\langle \hat{F}_a(t')\hat{F}^\dagger_a(t)\rangle dt'$$

Fromthecorrelationpropertiesofcavitymodenoiseoperatorsunder Eq.(2.67)weget

$$F_a \langle \hat{a}(t) \hat{a}^\dagger(t) \rangle = \frac{\kappa \sqrt{N}}{2} \qquad (3.19)$$

$$\langle \hat{F}_a^\dagger(t)\hat{a}(t)\rangle = \kappa\frac{\sqrt{N}}{2} \quad (3.20)$$

Employing Eqs. (3.16), (3.17), (3.19) and (3.20) in Eq. (3.13), we arrive at

$$\frac{d}{dt}\langle \hat{a}\hat{a}^\dagger\rangle = -\kappa\langle \hat{a}\hat{a}^\dagger\rangle + \frac{4g^2}{\kappa}\langle \overline{N_b(t)}\rangle - 2g + \kappa N \quad (3.21)$$

The steady-state solution of this equation can be written as

$$\langle \hat{a}\hat{a}^\dagger\rangle ss = \frac{\gamma_c}{\kappa}\langle \hat{N}\rangle ss + N - \frac{2g}{\kappa}{}_b \quad (3.22)$$

It can also be established in a similar manner that

$$\langle \hat{b}\hat{b}^\dagger\rangle = \frac{\gamma_c}{\kappa}\langle \hat{N}\rangle + N - \frac{2g}{\kappa}{}_c \quad (3.23)$$

On account of Eqs. (2.159) and (3.10), we see that the steady-state mean photon number of light mode a has the form

$$\bar{n}_a = \langle \hat{a}^\dagger \hat{a}\rangle = \frac{N\gamma_c}{\kappa}\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2} \quad (3.24)$$

We also find from eqs (2.160) and eq (3.11) the steady-state mean photon number of light mode b to be

$$\bar{n}_b = \langle \hat{b}^\dagger \hat{b}\rangle = \frac{N\gamma_c}{\kappa}\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2} \quad (3.25)$$

From equations (3.24) and (3.25), we see that the mean photon number radiated as mode **a** and as mode **b** is the same.

$$\bar{n}_a = \bar{n}_b = \frac{N\gamma_c}{\kappa}\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}$$

For $\Omega \ll \gamma_c$ or $\frac{\Omega}{\gamma_c} \approx 0$, when the amplitude of the driving light small as compared to stimulated emission rate, the mean photon number goes to zero, $\bar{n}_a = \bar{n}_b = 0$. But when the amplitude of the driving light is much much greater as compared to the rate of stimulated emission ($\frac{\gamma_c}{\Omega} \approx 0$), the mean photon number reduces to

$$\bar{n}_{a(b)} = \bar{n}_{max} = \frac{N\gamma_c}{\kappa}\frac{\Omega^2}{\Omega^2\left(\frac{\gamma_c^2}{\Omega^2} + 3\right)} = \frac{N\gamma_c}{3\kappa}$$

The graph under Fig. (3.1), shows that the global mean photon for mode **a(b)** increases with the amplitude of the driving coherent light (Ω), and with the rate of stimulated emission (γ_c)

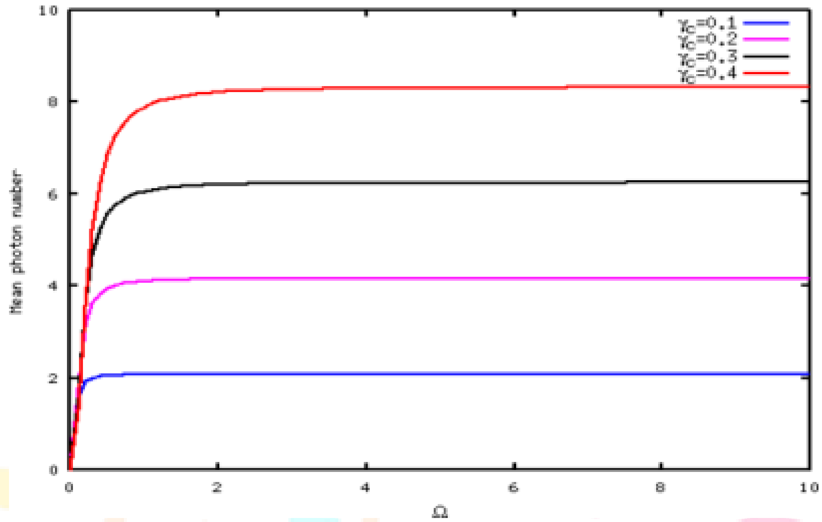

Figure3.1:Globalmeanphotonnumberformode**a(b)**versusΩfordifferent$\gamma_c$atκ=0.8andN=50

upitsthemaximum$\bar{n}_{max}=\frac{N\gamma_c}{3\kappa}=8$,whichoccurat κ =0.8and γ_c=0.4.Furthermore, employingtherelation

$$\frac{d}{dt}\langle\hat{a}(t)\hat{a}(t)\rangle=\langle\frac{d\hat{a}(t)}{dt}\hat{a}(t)\rangle+\langle\hat{a}(t)\frac{d\hat{a}(t)}{dt}\rangle$$

alongwithEq.(2.183),wereadilyfind

$$\frac{d}{dt}\langle\hat{a}(t)\hat{a}(t)\rangle=-\kappa\langle\hat{a}(t)\hat{a}(t)\rangle-\frac{g}{\sqrt{N}}\langle\hat{a}(t)\hat{m}_a(t)\rangle+\langle\hat{m}_a(t)\hat{a}(t)\rangle$$
$$+\sqrt{N}\langle\hat{F}_a(t)\hat{a}(t)\rangle+\langle\hat{a}(t)\hat{F}_a(t)\rangle N\kappa$$

We note that since the noise operator associated with the thermal reservoir has not effect oncavitymodeoperatorswhicharenotintheanti-normalorder,Eqs.(2.64-2.69),weset.

$$\langle\hat{F}_a(t)\hat{a}(t)\rangle+\langle\hat{a}(t)\hat{F}_a(t)\rangle=0$$

Applyingthisfact,theaboverelationreducesto

$$\frac{d}{dt}\langle\hat{a}^2\rangle=-\kappa\langle\hat{a}^2\rangle-\frac{g}{\sqrt{N}}\langle\hat{a}(t)\hat{m}_a(t)\rangle+\langle\hat{m}_a(t)\hat{a}(t)\rangle \qquad (3.26)$$

To evaluate $\langle \hat{m}_a(t)\hat{a}(t)\rangle$, we multiply Eq. (3.18) by $\hat{m}_a(t)$ from the right and taking expectation value on both sides, we get

$$\langle \hat{a}(t)\hat{m}_a(t)\rangle = \langle \hat{a}(0)\hat{m}_a(t)\rangle e^{\frac{-\kappa t}{2}} + \int_0^t e^{\frac{-\kappa(t-t')}{2}} \left[\langle \hat{F}_a(t')\hat{m}_a(t)\rangle - \frac{g}{\sqrt{N}} \langle \hat{m}_a(t')\hat{m}_a(t)\rangle \right] dt'$$

Using the fact that operator at a certain time should not affect an operator in the earlier time, we find

$$\langle \hat{a}(t)\hat{m}_a(t)\rangle = 0 \tag{3.27}$$

In the same way

$$\langle \hat{m}_a(t)\hat{a}(t)\rangle = 0 \tag{3.28}$$

Upon applying the results under Eqs. (3.27) and (3.28) in Eqs. (3.26), we find that

$$\frac{d}{dt}\langle \hat{a}^2\rangle = -\kappa\langle \hat{a}^2\rangle$$

At steady-state this equation turns out to be

$$\langle \hat{a}^2\rangle = 0 \tag{3.29}$$

Similarly, one can show that

$$\langle \hat{a}^{\dagger 2}\rangle = 0 \tag{3.30}$$

On the other hand, the expectation value of Eq. (3.6) will be calculated as

$$\langle \hat{m}_a(t)\rangle = \langle \hat{m}_a(0)\rangle e^{-\gamma_c t} + \left\langle \int_0^t e^{-\gamma_c(t-t')} \left[\frac{\Omega}{2}\hat{m}_b^{\dagger}(t') + \hat{m}_c(t') + \hat{F}_a(t') \right] dt' \right\rangle$$

We expect the operators $\hat{m}_b^{\dagger}(t')$, $\hat{m}_c(t')$ and $\hat{F}_a(t')$ to vary slowly compared with the exponential function $e^{-\kappa(t-t')}$. Thus, we can take these operators outside the integral, so that

$$\langle \hat{m}_a\rangle = \langle \hat{m}_a(0)\rangle e^{-\gamma_c t} + \left\langle \left[\frac{\Omega}{2}\hat{m}_b^{\dagger}(t) + \hat{m}_c(t) + \hat{F}_a(t) \right] \int_0^t e^{-\gamma_c(t-t')} dt' \right\rangle$$

Doing the integration, we find

Butatt=0,weget

$\langle \hat{m}_a \rangle = \langle \hat{m}_a(0) \rangle$Assumingtheatomstobeinitiallyinthebottomlevel,$\langle \hat{m}_a(0) \rangle = \langle \hat{m}_b(0) \rangle = 0$

$$\langle \hat{m}_a \rangle = 0 \tag{3.31}$$

Furthermore,usingEq.(3.31)alongwithEq.(2.64)andtheassumptionthatthecavitylightisinitiallyinavacuumstat e,theexpectationvalueofEq.(3.18)takestheform

$$\langle \hat{a} \rangle = 0, \langle \hat{a}^{\dagger} \rangle = 0 \tag{3.32}$$

InviewofEqs.(2.183)and(3.32),weclaimthat$\hat{a}$isaGaussianvariablewithzeromean. Onecanalsoeasilyverifythat

$$\langle \hat{b}^2 \rangle = 0, \langle \hat{b}^{\dagger 2} \rangle = 0, \langle \hat{b} \rangle = 0 \tag{3.33}$$

ThenonaccountofEqs.(2.184)and(3.33),werealizethat$\hat{b}$isaGaussianvariablewithzero mean.

3.1.2 GlobalPhotonNumberVariance

Thevarianceofphotonnumberforamoderadiationproducedbythenon- degeneratethreelevellasercoupledtothermalreservoirwillbe

$$(\Delta n_a)^2 = \langle \bar{n}^2 \rangle - \langle \bar{n}_a \rangle^2 = \langle (\hat{a}^{\dagger}\hat{a})^2 \rangle - \langle \hat{a}^{\dagger}\hat{a} \rangle^2$$

Since$\hat{a}$isaGaussianvariablewithzeromean,itsphotonnumbervariancewouldbegivenby [1,3]

$$(\Delta n)^2 = \langle \hat{a}^{\dagger}\hat{a} \rangle_a \langle \hat{a}\hat{a}^{\dagger} \rangle + \langle \hat{a}^{\dagger 2} \rangle \langle \hat{a}^2 \rangle$$

Usingthefactthat$\langle \hat{N}_a \rangle = \langle \hat{N}_b \rangle$togetherwithEqs.(3.10),(3.22),(3.29)and(3.30),we

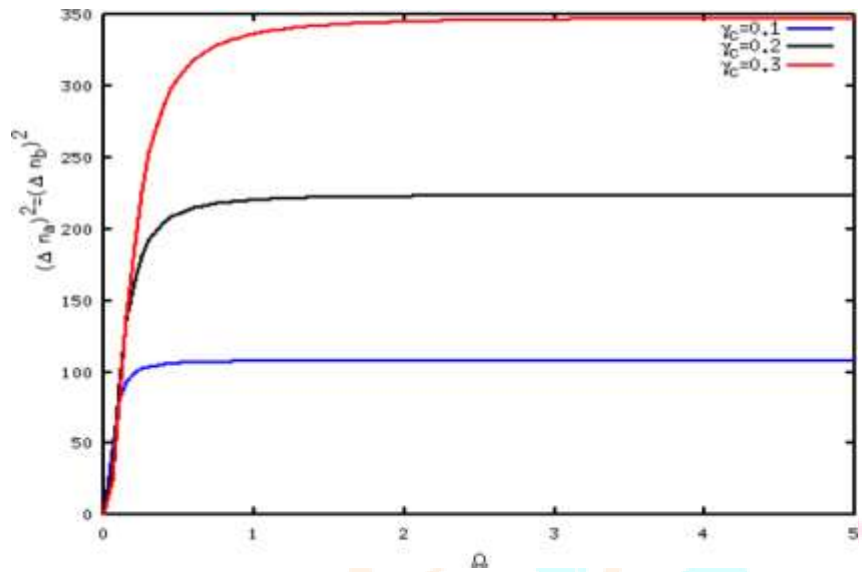

Figure3.2:Globalphotonnumbervariance($\Delta n)^2$versusΩfordifferentvaluesof$\gamma_c$atκ=0.8andN=50

$$(\Delta n_b)^2=\bar{n}_b^2+\bar{n}_b\left(N-\frac{2g}{\kappa}\right)$$

Toseetheimpactoftherateofstimulatedemission,weusethe$\gamma_c=\frac{4g^2}{\kappa}$ whichcouldreduce to$g=0.45\sqrt{\gamma_c}$.Therefore,theseequationswillappearas

$$(\Delta n_a)^2=\bar{n}_a^2+\bar{n}_a(N-1.125\sqrt{\gamma_c}) \qquad (3.34)$$

inasimilarmannerthatthephotonnumbervarianceforlightmode**b**

$$(\Delta n_b)^2=\bar{n}_b^2+\bar{n}_b(N-1.125\sqrt{\gamma_c}) \qquad (3.35)$$

Eqs.(3.34) and (3.35) represents a super-Poisonian photon statistics. In other words, each ofthe two radiation modes are in a chaotic state of light . In view of Eqs.(3.24), (3.25), Eqs.(3.34)and (3.35), we found that the variance of photon number has maximum value of 480 whichoccur at γ_c= 0and it will decrease with increasing γ_c.But variance of single mode increaseswith the amplitude of the coherent light and it terminate atΩ = 1.5for any γ_c. This is clearlydisplayedinfigure3.2.

3.2Two-ModePhotonStatistics

In this section we seek to determine the global mean photon number and variance of the two-mode cavity light.In addition, we obtain the local mean photon number and variance of thetwo-modecavitylight.

3.2.1 GlobalMeanPhotonNumber

To learn about the brightness of the generated light,it is necessary to study the mean number ofphoton pairs describing the two-mode cavity radiation.So,we wish to calculate the steady-statemean photon number for the two-mode cavity light in the entire frequency interval. The twomode Mean photon number for the radiation produced by the coherently driven non-degeneratethree level laser coupled to two mode thermal reservoir is determined using a hermitian numberoperator$\bar{n}=\langle\hat{c}^{\dagger}\hat{c}\rangle$. Here$\hat{c}$istheannihilationoperatorforthetwomodecavitylightradiated whichisgivenby$\hat{c}=\hat{a}+\hat{b}$.From Eqs(3.32)and(3.33)andtherelation$\langle\hat{c}\rangle=\langle\hat{a}\rangle+\langle\hat{b}\rangle$,we seethat

$$\langle\hat{c}\rangle=\langle\hat{c}^{\dagger}\rangle=0 \tag{3.36}$$

Thetwomodemeanphotonnumberisexpressedas

$$\bar{n}=\langle\hat{c}^{\dagger}\hat{c}\rangle=\langle(\hat{a}^{\dagger}+\hat{b}^{\dagger})(\hat{a}+\hat{b})\rangle$$

$$=\langle\hat{a}^{\dagger}\hat{a}\rangle+\langle\hat{b}^{\dagger}\hat{b}\rangle+\hat{a}^{\dagger}\hat{b}\rangle)+\langle\hat{b}^{\dagger}\hat{a}\rangle$$

But, using the formal solution of Eqs.(2.183) and (2.184) or their conjugates together with therespectivefactsunderEqs.(2.136),(2.66),(3.32)and(3.33),onecouldshowthat

$$\langle\hat{a}^{\dagger}\hat{b}\rangle+\langle\hat{b}^{\dagger}\hat{a}\rangle=0$$

Then,theequationoftwomodemeanphotonnumberforanon-degeneratethreelevellaserdrivenbytwomodecoherentlightandcoupledtotwomodethermalreservoirreducesto

$$\bar{n}=\langle\hat{a}^{\dagger}\hat{a}\rangle+\langle\hat{b}^{\dagger}\hat{b}\rangle=\hat{n}_a+\hat{n}_b \tag{3.37}$$

EmployingEqs.(3.24),(3.25)inEq.(3.37),wefind

$$\bar{n}=\langle \hat{c}^{\dagger}\hat{c}\rangle=\frac{N\gamma_c}{\kappa}\frac{2\Omega^2}{\gamma_c^2+3\Omega^2} \tag{3.38}$$

Thisshowsthatthemeanphotonnumberforanon-degeneratethreelevellaserdrivenbytwomode coherent light and coupled to two mode thermal reservoir increases with amplitude of thedriving coherent light(Ω). The mean photon number is zero if there is no stimulated emission(γ_c=0),nodrivingcoherentlight.Besides,itwillvanishwhenthecoherentcouplingismuch muchsmallerthantherateofstimulatedemission($\frac{\Omega}{\gamma_c}\approx 0$). But,itwillhaveamaximum valuewhenthecoherentcouplingismuchmuchgreaterthantherateofstimulatedemission ($\frac{\gamma_c}{\Omega}\approx 0$)givenby

$$\bar{n}_{max}=\frac{N\gamma_c}{\kappa}\frac{2\Omega^2}{\Omega^2\left(\frac{\gamma_c^2}{\Omega^2}+3\right)}=\frac{2N\gamma_c}{3\kappa}$$

Thisshowsthatthemeanphotonnumberforanon-degeneratethreelevellaserdrivenbytwo modecoherentlightandcoupledtotwomodethermalreservoiristwiceoftheglobalmeanphotonnumberofindividualr adiationmodes.Wecanalsoevaluate$\langle \hat{c}\hat{c}^{\dagger}\rangle$as

$$\langle \hat{c}\hat{c}^{\dagger}\rangle=\langle \hat{a}\hat{a}^{\dagger}\rangle+\langle \hat{b}\hat{b}^{\dagger}\rangle+\langle \hat{b}\hat{a}^{\dagger}\rangle+\langle \hat{b}\hat{a}^{\dagger}\rangle$$

Bythesamereasonabove,wecouldshowthat
$\langle \hat{b}\hat{a}^{\dagger}\rangle+\langle \hat{b}\hat{a}^{\dagger}\rangle=0$

$$\langle \hat{c}\hat{c}^{\dagger}\rangle=\langle \hat{a}\hat{a}^{\dagger}\rangle+\langle \hat{b}\hat{b}^{\dagger}\rangle$$

FromEqs.(3.22)and(3.23),weget

$$\langle \hat{c}\hat{c}^{\dagger}\rangle=\frac{\gamma_c}{\kappa}\left[\langle N\rangle_c+\langle N\rangle_b+2N-\frac{4g}{\kappa}\right]$$

Inotherwords,thesameresultcouldbedevelopedusingtheQuantumLangevinequationfor

$\hat{c}$.Todoso,weseektofind$\langle \hat{c}\hat{c}^{\dagger} \rangle$usingtherelation

$$\frac{d}{dt}\langle \hat{c}\hat{c}^{\dagger} \rangle = \langle \frac{d\hat{c}}{dt}\hat{c}^{\dagger} \rangle + \langle \hat{c}\frac{d\hat{c}^{\dagger}}{dt} \rangle \tag{3.39}$$

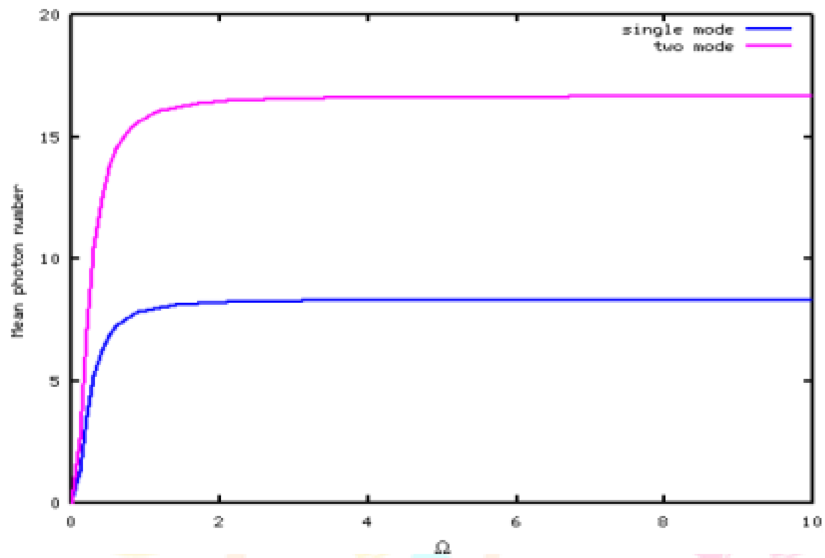

Figure3.3:Comparisonofsinglemode(**blue**)andtwomodelight(**magneta**)versusΩat γ_c=0.4,κ=0.8andN=50

LookingEqs.(2.116),(2.183),(2.185),itisnothardtoseethat

$$\frac{d}{dt}\hat{c}(t)=-\frac{\kappa}{2}\hat{c}-\frac{g}{\sqrt{N}}\hat{m}+\hat{F}_c(t) \qquad (3.40)$$

where,$\hat{F}_c(t)=\hat{F}_a(t)+\hat{F}_b(t)$.Thus,wereadilyfind

$$\frac{d}{dt}\langle\hat{c}^\dagger(t)\hat{c}(t)\rangle=-\kappa\langle\hat{c}^\dagger(t)\hat{c}(t)\rangle-\frac{g}{\sqrt{N}}\left[\langle\hat{c}^\dagger(t)\hat{m}(t)\rangle+\langle\hat{m}^\dagger(t)\hat{c}(t)\rangle\right]+\left[\langle\hat{F}_c^\dagger(t)\hat{c}(t)\rangle+\langle\hat{c}^\dagger(t)\hat{F}_c(t)\rangle\right] \qquad (3.41)$$

Sincethecavitymodeoperatorsareinanti-normalorder,theeffectofnoiseoperatorcouldnotbeignoredsimply.So,toevaluate$\langle\hat{c}^\dagger(t)\hat{F}_c(t)\rangle$,wemultiplytheformalsolutionofEq,(3.40)by $\hat{F}_c^\dagger(t)$fromtherightandtakingexpectationvalueonbothsidesas

$$\langle\hat{c}^\dagger(t)\hat{F}_c(t)\rangle=\langle\hat{c}^\dagger(0)\hat{F}_c(t)\rangle e^{-\frac{\kappa t}{2}}+\int_0^t e^{-\frac{\kappa(t-t')}{2}}\left[-\frac{g}{\sqrt{N}}\langle\hat{m}^\dagger(t')\hat{F}_c(t)\rangle+\langle\hat{F}_c^\dagger(t')\hat{F}_c(t)\rangle\right]dt'$$

Fromthesamefactasbeforeabouttimedifferenceandimpactofnoiseforce,weget

$$\langle\hat{c}^\dagger(t)\hat{F}_c(t)\rangle=\int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle\hat{F}_c^\dagger(t')\hat{F}_c(t)\rangle dt'$$

From Eq. (2.66-2.69) and $\hat{F}_c(t) = \sqrt{N}\hat{f}_a(t) + \sqrt{N}\hat{f}_b(t)$, we see

$$\langle \hat{F}_c(t)\hat{F}_c^\dagger(t)\rangle = \kappa N$$

$$\langle \hat{F}_c^\dagger(t)\hat{F}_c(t)\rangle = \kappa N$$

Furthermore, we need to evaluate $\langle \hat{m}(t)\hat{c}^\dagger(t)\rangle$. Multiplying the adjoint of the steady state solution of Eq. (3.40) on the left by $\hat{m}(t)$ and taking the expectation value of the resulting expression, we get

$$\langle \hat{m}(t)\hat{c}^\dagger(t)\rangle = \frac{-2g\sqrt{N}}{\kappa}\langle \hat{N}_c(t)\rangle + \langle \hat{N}_b(t)\rangle + \frac{2}{\kappa}\langle \hat{m}(t)\hat{F}_c^\dagger(t)\rangle \tag{3.42}$$

As the interaction of cavity modes and driving light is ignored, the sum of Eqs. (2.140) and (2.141) after removing the square brackets give as the equation of evolution for $\hat{m}(t)$ to be

$$\frac{d}{dt}\hat{m} = -\frac{\gamma_c}{2}\hat{m} + \frac{\Omega}{2}(\hat{m}_b^\dagger - \hat{m}_a^\dagger) - \frac{\gamma_c}{2}\hat{m}_a + \hat{F}_c(t)$$

with its formal solution

$$\hat{m}(t) = \hat{m}(0)e^{\frac{-\gamma_c t}{2}} + \int_0^t e^{\frac{-\gamma_c}{2}(t-t')}\left[\frac{\Omega}{2}(\hat{m}_b^\dagger(t') - \hat{m}_a^\dagger(t')) - \frac{\gamma_c}{2}\hat{m}_a(t') + \hat{F}_c(t')\right]dt' \tag{3.43}$$

Now, upon multiplying Eq. (3.43) on the right by $\hat{F}_c^\dagger(t)$ and taking the expectation value of the resulting expression, we have

$$\langle \hat{m}(t)\hat{F}_c^\dagger(t)\rangle = \langle \hat{m}(0)\hat{F}_c^\dagger(t)\rangle e^{\frac{-\gamma_c t}{2}} + \int_0^t e^{\frac{-\gamma_c}{2}(t-t')}\left[\frac{\Omega}{2}\left(\langle \hat{m}_b^\dagger(t')\hat{F}_c^\dagger(t)\rangle - \langle \hat{m}_a^\dagger(t')\hat{F}_c^\dagger(t)\rangle\right) - \frac{\gamma_c}{2}\langle \hat{m}_a(t')\hat{F}_c^\dagger(t)\rangle + \langle \hat{F}_c(t')\hat{F}_c^\dagger(t)\rangle\right]dt'$$

Using the time difference between noise operator and cavity mode operators together with the assumption that the noise operator on e mode light will not affect, the only term that survive will be

$$\langle \hat{m}(t)\hat{F}_c^\dagger(t)\rangle = \int_0^t e^{\frac{-\gamma_c}{2}(t-t')}\langle \hat{F}_c(t')\hat{F}_c^\dagger(t)\rangle dt'$$

In view of Eq. (2.68), this reduces to

$$\langle \hat{m}(t)\hat{F}_c^\dagger(t)\rangle = \frac{\sqrt{N}\kappa}{2} \tag{3.44}$$

$$\langle \hat{m}(t)\hat{c}^{\dagger}(t)\rangle = \frac{-2g}{\sqrt{N}}\left\langle \frac{N(t)\rangle + \langle N(t)\rangle}{2}\right._{c} + \sqrt{N}_{b} \quad (3.45)$$

$$\langle \hat{c}(t)\hat{m}^{\dagger}(t)\rangle = \frac{-2g}{\sqrt{N}}\left\langle \frac{N(t)\rangle + \langle N(t)\rangle}{2}\right._{c} + \sqrt{N}_{b} \quad (3.46)$$

UponusingEqs.(3.45)and(3.46)inEq.(3.41),weget
$\frac{d}{dt}\langle \hat{c}\hat{c}^{\dagger}\rangle = \kappa\langle \hat{c}\hat{c}^{\dagger}\rangle + \frac{4g^2}{\kappa}\langle N(t)\rangle_c + \langle N(t)\rangle_b - 2g + 2N\kappa$Atsteadystate,theaboveequ.isreduces to

$\langle \hat{c}(t)\hat{c}^{\dagger}(t)\rangle = \frac{\gamma_c}{\kappa}\langle N_c(t)\rangle + N_b(t)\rangle + 2N - \frac{4g}{\kappa}$Fromthefactthat$\langle N_a(t)\rangle + \langle N_b(t)\rangle + \langle N_c(t)\rangle = N$

and$\langle N_a(t)\rangle = \langle N_b(t)\rangle$

$\langle \hat{c}\hat{c}^{\dagger}\rangle = \frac{\gamma_c}{\kappa}N - \langle N_b(t)\rangle + 2N - \frac{4g}{\kappa}$

EmployingEq.(2.160),thisreducesto

$$\langle \hat{c}(t)\hat{c}^{\dagger}(t)\rangle = 2N - \frac{4g}{\kappa}\frac{N\gamma_c}{\kappa}\frac{\gamma^2_c + 2\Omega^2}{\gamma^2_c + 3\Omega^2} \quad (3.47)$$

3.2.2 GlobalVariancePhotonNumber

Inthissectionweseektocalculatethevarianceofthephotonnumberforthetwomodecavitylight.Tothisend,thevariance ofthephotonnumberfortwo-modecavitylightisexpressible as

$$(\Delta n)^2 = \langle (\hat{c}^{\dagger}\hat{c})^2\rangle - \langle \hat{c}^{\dagger}\hat{c}\rangle^2$$

Since$\hat{c}$aGaussianvariablewithzeromean,itsphotonnumbervariancewouldbegivenby [1,2].

$$(\Delta n)^2 = \langle \hat{c}^{\dagger}\hat{c}\rangle\langle \hat{c}\hat{c}^{\dagger}\rangle + \langle \hat{c}^{\dagger 2}\rangle\langle \hat{c}^2\rangle \quad (3.48)$$

Furthermore,employingtherelation

$$\frac{d}{dt}\langle \hat{c}(t)\hat{c}(t)\rangle = \left\langle \frac{d\hat{c}(t)}{dt}\hat{c}(t)\right\rangle + \left\langle \hat{c}(t)\frac{d\hat{c}(t)}{dt}\right\rangle$$

alongwithEq.(3.60),weeasilyobtain

$$\frac{d}{dt}\langle \hat{c}(t)\hat{c}(t)\rangle = -\kappa\langle \hat{c}(t)\hat{c}(t)\rangle - \frac{g}{\sqrt{N}}\left(\langle \hat{c}(t)\hat{m}(t)\rangle + \langle \hat{m}(t)\hat{c}(t)\rangle\right) + \sqrt{N}\left(\langle \hat{F}_c(t)\hat{c}(t)\rangle + \langle \hat{c}(t)\hat{F}_c(t)\rangle\right)$$

couldbeignoredsimplysothat

$\langle \hat{F}_c(t)\hat{c}(t)\rangle+\langle \hat{c}(t)\hat{F}_c(t)\rangle=0$.Thus

$$\frac{d}{dt}\langle \hat{c}^2\rangle=-\kappa\langle \hat{c}^2\rangle-\frac{g}{\sqrt{N}}\left(\langle \hat{c}(t)\hat{m}(t)\rangle+\langle \hat{m}(t)\hat{c}(t)\rangle\right) \tag{3.49}$$

Weseektoevaluate$\langle \hat{c}(t)\hat{m}(t)\rangle$.BymultiplyingthesteadystatesolutionofEq.(3.40)by$\hat{m}(t)$ fromtherightandtakingtheexpectationvaluesonbothsides,weget

$$\langle \hat{c}(t)\hat{m}(t)\rangle=-\frac{2g\sqrt{N}}{\kappa}\langle \bar{N}_c(t)\rangle+\frac{2}{\kappa}\langle \hat{F}_c(t)\hat{m}(t)\rangle$$

UponmultiplyingEq.(3.43)by$\hat{F}_c(t)$fromtheleftandtakingtheexpectationvalue,weget

$$\langle \hat{F}_c(t)\hat{m}(t)\rangle=\langle \hat{F}_c(t)\hat{m}(0)\rangle e^{\frac{-\gamma_c t}{2}}+\int_0^t e^{\frac{-\gamma_c}{2}(t-t')}\left[\frac{\Omega}{2}\left(\langle \hat{F}_c(t)\hat{m}_b^{\dagger}(t')\rangle-\langle \hat{F}_c^{\dagger}(t)\hat{m}_a(t')\rangle\right)+\langle \hat{F}_c(t)\hat{F}_c(t')\rangle\right]dt'$$

Forthenoiseoperatoratacertaintimeshouldnotaffecttheoperatorsintheearliertime,all thetermsinthisequationvanish.hence.$\langle \hat{F}_c(t)\hat{m}(t)\rangle=0$Sothat

$$\langle \hat{c}(t)\hat{m}(t)\rangle=-\frac{2g\sqrt{N}}{\kappa}\langle \bar{N}_c(t)\rangle \tag{3.50}$$

$$\langle \hat{m}(t)\hat{c}(t)\rangle=-\frac{2g\sqrt{N}}{\kappa}\langle \bar{N}_c(t)\rangle \tag{3.51}$$

EmployingEqs.(3.50)and(3.51)inEq.(3.49),wefind

$$\frac{d}{dt}\langle \hat{c}^2\rangle=-\kappa\langle \hat{c}^2\rangle+\frac{4g^2}{\kappa}\langle N_c(t)\rangle$$

withsteadystatevalue

$$\langle \hat{c}^2\rangle=\frac{\gamma_c}{\kappa}\langle N_c(t)\rangle$$

inviewofeqs(2.161),weget

$$\langle \hat{c}^2\rangle=\frac{N\gamma_c}{\kappa}\frac{\gamma_c^2+\Omega^2}{\gamma_c^2+3\Omega^2} \tag{3.52}$$

$$\langle \hat{c}^{\dagger 2}\rangle=\frac{\gamma_c}{\kappa}\langle \hat{m}_c^{\dagger}(t)\rangle=\frac{N\gamma_c}{\kappa}\frac{\gamma_c\Omega}{\gamma_c^2+3\Omega^2} \tag{3.53}$$

EmployingEqs.(3.38),(3.47),(3.52)and(3.53)inEq.(3.48),onecouldget

$$(\Delta n)^2=\frac{N\gamma_c}{\kappa}\frac{2\Omega^2}{\gamma_c^2+3\Omega^2}\left[2N-\frac{4g}{\kappa}\frac{N\gamma_c}{\kappa}\frac{\gamma_c^2+2\Omega^2}{\gamma_c^2+3\Omega^2}\right]$$

$$= \bar{n}\left[2N - \frac{4g}{\kappa} + \bar{n} + \frac{N\gamma_c}{\kappa}\frac{\gamma_c^2}{\gamma_c^2+3\Omega^2}\right]$$ 49

Onecanfurtherrewritethisequationas

$$(\Delta n)^2=\bar{n}^2+2\bar{n}N-\frac{4g\bar{n}}{\kappa}-\frac{\bar{n}N\gamma_c}{\kappa}\frac{\gamma_c^2}{\gamma_c^2+3\Omega^2}$$

Usingthevalue$g$=0.45$\sqrt{\gamma_c}$,weget

$$(\Delta n)^2=\bar{n}^2+2\bar{n}N-\frac{2.25\bar{n}\sqrt{\gamma_c}}{\kappa}-\frac{\bar{n}N\gamma_c}{\kappa}\frac{\gamma_c^2}{\gamma_c^2+3\Omega^2} \tag{3.54}$$

Thisequationindicatesthatfornon-zeromeanphotonnumber,thephotonnumbervarianceofthe light generated by coherently driven non-degenerate three level laser coupled to two modethermal reservoir is greater than its mean photon number. Consequently,the two mode lightproduced by coherently driven non-degenerate three level laser coupled to two mode thermalreservoirownsasuper-poisonianstatisticsanditisachaoticlight.

Chapter4

QuadratureVarianceandSqueezing

In this chapter we seek to study the quadrature variance of single-mode light beams and thequadrature squeezing of the two-mode cavity light, produced by the system under consideration.Applyingthesteady-statesolutionsoftheequationsofevolutionoftheexpectationvaluesofthe atomic operators and the quantum Langevin equations for the cavity mode operators, weobtain the global quadrature variance for light modes a and b.In addition, we determine theglobalquadraturesqueezingofthetwo-modecavitylight.

4.1Single-ModeQuadratureVariance

Inthissectionwewishtoobtaintheglobalquadraturevarianceoflightmodes**a**and**b**.

4.1.1 GlobalQuadratureVariance

Here we seek to calculate the quadrature variance of light modes a and b in the entire frequencyinterval.Thesqueezingpropertiesofthecavitymodelightaaredescribedbytwoquadrature operatorsdefinedas

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a} \tag{4.1}$$

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}) \tag{4.2}$$

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where $\hat{a}_+$ and $\hat{a}_-$ are Hermitian operators representing physical quantities called the plus and minus quadratures of cavity mode light a. From the definition of the commutator of two operators

$$[\hat{a},\hat{a}^\dagger]=\hat{a}\hat{a}^\dagger-\hat{a}^\dagger\hat{a}$$

and on taking the expectation value of the operators on the right hand side of this equation, we see that

$$[\hat{a},\hat{a}^\dagger]=\langle\hat{a}\hat{a}^\dagger\rangle-\langle\hat{a}^\dagger\hat{a}\rangle$$

Thus in view of Eqs.(3.10) and (3.22), we find

$$[\hat{a},\hat{a}^\dagger]=\frac{\gamma_c}{\kappa}\langle N_b\rangle ss-\frac{2g}{\kappa}+N-\frac{\gamma_c}{\kappa}\langle N_a\rangle ss$$

As $\langle N_a\rangle=\langle N_b\rangle$, this reduces to

$$[\hat{a},\hat{a}^\dagger]=N\frac{2g}{\kappa}$$

The commutation relation of such plus and minus quadrature operators will be

$$[\hat{a}_-,\hat{a}_+]=-2i[\hat{a},\hat{a}^\dagger]$$

$$[\hat{a}_-,\hat{a}_+]=-2i\left(N-\frac{2g}{\kappa}\right) \quad (4.3)$$

The quadrature variances are defined by

$$(\Delta a\pm)^2=\langle \hat{a}_\pm^2\rangle-\langle\hat{a}_\pm\rangle^2$$

This can also be put in the form

$$(\Delta a\pm)^2=\pm\langle[\hat{a}^\dagger\pm\hat{a}]^2\rangle\pm(\langle\hat{a}^\dagger\rangle\pm\langle\hat{a}\rangle)^2$$

In view of Eqs.(3.29),(3.30), and (3.32), one can show easily that

$$(\Delta a_\pm)^2=\langle\hat{a}^\dagger\hat{a}\rangle+\langle\hat{a}^\dagger\hat{a}\rangle \quad (4.4)$$

EmployingEqs.(3.22)and(3.24)inEq.(4.4),wearriveat

$$(\Delta a_\pm)^2 = \frac{\gamma_c}{\kappa}\langle N_a\rangle + \frac{\gamma_c}{\kappa}\langle N_b\rangle_{ss} - \frac{2g}{\kappa} + N$$

Usingthefact$\langle N_a\rangle=\langle N_b\rangle$,thiscouldberewrittenas

$$(\Delta a_\pm)^2 = 2\bar{n}_a + N - \frac{2g}{\kappa} \tag{4.5}$$

which is the quadrature variance of a light mode in a chaotic state.In line with the photonstatistics,thequadraturevariancesoftheplusandminusquadraturesindicatethatlightmodea is in a chaoticstate.In the same manner,the squeezing propertiesof the other cavity modelightbaredescribedbytwoquadratureoperatorsdefinedas

$$\hat{b}_+ = \hat{b}^\dagger + \hat{b} \tag{4.6}$$

$$\hat{b}_- = i(\hat{b}^\dagger - \hat{b}) \tag{4.7}$$

where$\hat{a}_+$and$\hat{a}_-$areHermitianoperatorsrepresentingphysicalquantitiescalledtheplusandminusquadraturesofcavitymodelightb. Fromthedefinitionofthecommutatoroftwo

operators

$$[\hat{b}_-,\hat{b}_+] = -2i[\hat{b},\hat{b}^\dagger]$$

$$[\hat{b}_-,\hat{b}_+] = -2i(\langle\hat{b}\hat{b}^\dagger\rangle - \langle\hat{b}^\dagger\hat{b}\rangle)$$

FromEqs.(3.11)and(3.23),weget

$$[\hat{b}_-,\hat{b}_+] = -2i\left[\frac{\gamma_c}{\kappa}\langle N_c\rangle - \langle N_b\rangle + \frac{2g}{\kappa} + N\right]$$

Fromeqs.(2.159)andeqs.(2.160),thisexpressionreducesto

$$[\hat{b}_-,\hat{b}_+] = -2i\left[\frac{N\gamma_c}{\kappa}\frac{\gamma_c^2}{\gamma_c^2+3\Omega^2} - \frac{2g}{\kappa} + N\right] \tag{4.8}$$

Nextweproceedtocalculatethequadraturevarianceoflightmodeb.Thevarianceoftheplusandminusquadratureoperatorsaredefinedby

$$(\Delta\hat{b}_\pm)^2 = \langle\hat{b}_\pm^2\rangle - \langle\hat{b}_\pm\rangle^2$$

Thiscanalsobeputintheform

$$(\Delta\hat{a}_{\pm})^2=\pm\langle[\hat{b}^{\dagger}\pm\hat{b}]^2\rangle\pm\langle[\hat{b}^{\dagger}\rangle\pm\langle\hat{b}\rangle]^2$$

InviewofEq.(3.33),onecanshoweasilythat

$$(\Delta\hat{b}_{\pm})^2=\langle\hat{b}^{\dagger}\hat{b}\rangle\pm+\langle\hat{b}\hat{b}^{\dagger}\rangle \quad (4.9)$$

EmployingEqs.(3.23)and(3.25)inEq.(4.9),wearriveat

$$(\Delta b\pm)^2_{\bar{b}}=\frac{\gamma_c}{\kappa}\langle N\rangle+\frac{\gamma_c}{\kappa_c}\langle N\rangle ss-\frac{2g}{\kappa}+N$$

Thiscouldberewrittenas

$$(\Delta b_{\pm})^2=\bar{n}_{\pm}+\frac{\gamma_c}{\kappa_c}\langle N\rangle_{\bar{b}}-\frac{2g}{\kappa}+N \quad (4.10)$$

which is the quadrature variance of **b** light mode in a chaotic state.In line with the photonstatistics still, the quadrature variances of the plus and minus quadratures indicate that lightmode**b**isinachaoticstate.Ontheotherhand,iftwohermitianoperatorsAandBdonot commute , [A,B] = iC, the uncertainties in their measurements in a given state satisfy thegeneralizedHeisenberguncertaintyrelationgivenby$\Delta\hat{A}\Delta\hat{B}\geq\frac{1}{2}|C|$.

Then,wecouldseethat

$$\Delta\hat{a}_{+}\Delta\hat{a}_{-}\geq\frac{1}{2}|[\hat{a}_{-},\hat{a}_{+}^{\dagger}]|$$

EmployingEq.(4.3)inthisrelation,weseethat

$$\Delta\hat{a}_{+}\Delta\hat{a}_{-}\geq N-\frac{2g}{\kappa}$$

Formodeb,wecouldseethat

$$\Delta\hat{b}_{+}\Delta\hat{b}_{-}\geq\frac{1}{2}|[\hat{b}_{-},\hat{b}_{+}^{\dagger}]|$$

EmployingEq.(4.8)inthisrelation,weseethat

$$\Delta\hat{b}_{+}\Delta\hat{b}_{-}\geq\frac{N\gamma_c}{\kappa_c}\frac{\gamma_c^2}{\gamma^2+3\Omega^2}-\frac{2g}{\kappa}+N \quad (4.11)$$

4.2Two-ModeQuadratureSqueezing

Inthissectionweseektostudytheglobalquadraturevarianceofthetwo-modecavitylight.

4.2.1 GlobalQuadratureSqueezing

We now proceed to calculate the variance of the plus and minus quadrature of the the two-modecavitylightproducedbyacoherentlydrivennon-degeneratethreelevellaserinacavitycoupledtothermalreservoir.Thesqueezingpropertiesofthetwo-modecavitylightaredescribedby twoquadratureoperatorsdefinedby

$$\hat{c}_{+}=\hat{c}^{\dagger}+\hat{c} \qquad (4.12)$$

$$\hat{c}_{-}=i(\hat{c}^{\dagger}-\hat{c}) \qquad (4.13)$$

where$\hat{c}_{+}$and$\hat{c}_{-}$.areHermitianoperatorsrepresentingphysicalquantitiescalledtheplusandminusquadraturesoftwo modecavitylight.

$$[\hat{c}_{-},\hat{c}_{+}]=-2i[\hat{c},\hat{c}^{\dagger}]$$

where

$$[\hat{c},\hat{c}^{\dagger}]=\hat{c}\hat{c}^{\dagger}-\hat{c}^{\dagger}\hat{c}$$

Therefore,takingexpectationvalues

$$[\hat{c},\hat{c}^{\dagger}]=\langle\hat{c}\hat{c}^{\dagger}\rangle-\langle\hat{c}^{\dagger}\hat{c}\rangle$$

But,weremember

$$\langle\hat{c}\hat{c}^{\dagger}\rangle=\frac{\gamma_c}{}\langle N(t)\rangle+\langle N(t)\rangle+2N-\frac{4g}{}$$

$$\kappa \quad c \quad b \quad \kappa$$

$$\hat{c}^{\dagger}\hat{c} = 2\frac{\gamma_c}{\kappa}\langle N(t)\rangle \quad a$$

Besides,usingthefactthat$\langle \hat{N}_a(t)\rangle = \langle \hat{N}_b(t)\rangle$,weget

$$[\hat{c},\hat{c}^{\dagger}] = \frac{\gamma_c}{\kappa}\langle \hat{N}(t)\rangle - \langle \hat{N}(t)\rangle + 2N - \frac{4g}{\kappa} \quad b \quad c \quad \kappa \tag{4.14}$$

WiththeaidofEqs.(4.22),(4.23),and(4.24),wecanshowthattheplusandminusquadrature operatorssatisfythecommutationrelation

$$[\hat{c}_-, \hat{c}_+^\dagger] = 2i\frac{\gamma_c}{\kappa}\left(\langle \hat{N}(t)\rangle - \langle \hat{N}_d(t)\rangle - 4iN + 8i\frac{g}{\kappa}\right)_c$$

Itthenfollowsthat

$$\Delta\hat{c}_+\Delta\hat{c}_- \geq \frac{\gamma_c}{\kappa}\left(\langle N\rangle_c - \langle N\rangle_b + 2N - 4\frac{g}{\kappa}\right) \tag{4.15}$$

where Δc_+ and Δc_- are the uncertainties in the plus and minus quadratures. Next we proceedto calculate the quadrature variance of the two-mode cavity light. To this end, the variance oftheplusandminusquadratureoperatorsofthetwo-modecavitylightaredefinedby

$$(\Delta c_+)^2_+ = \langle \hat{c}^2_+\rangle - \langle \hat{c}_+\rangle^2$$

$$(\Delta c_-)^2_- = \langle \hat{c}^2_-\rangle - \langle \hat{c}_-\rangle^2$$

OnaccountofEqs.(4.12)and(4.13),theequationtaketheform

$$(\Delta c_+)^2 = \langle \hat{c}^\dagger\hat{c}\rangle + \langle \hat{c}\hat{c}^\dagger\rangle + \langle \hat{c}^{\dagger 2}\rangle + \langle \hat{c}^2\rangle - \langle \hat{c}\rangle^2 - \langle \hat{c}^\dagger\rangle^2 - 2\langle \hat{c}^\dagger\rangle\langle \hat{c}\rangle \tag{4.16}$$

$$(\Delta c_-)^2 = \langle \hat{c}^\dagger\hat{c}\rangle + \langle \hat{c}\hat{c}^\dagger\rangle + \langle \hat{c}^{\dagger 2}\rangle + \langle \hat{c}^2\rangle - \langle \hat{c}\rangle^2 - \langle \hat{c}^\dagger\rangle^2 - 2\langle \hat{c}^\dagger\rangle\langle \hat{c}\rangle \tag{4.17}$$

$$(\Delta c_+)^2 = \langle \hat{c}^\dagger\hat{c}\rangle + \langle \hat{c}\hat{c}^\dagger\rangle + \langle \hat{c}^{\dagger 2}\rangle + \langle \hat{c}^2\rangle$$

$$(\Delta c_-)^2 = \langle \hat{c}^\dagger\hat{c}\rangle + \langle \hat{c}\hat{c}^\dagger\rangle - \langle \hat{c}^{\dagger 2}\rangle - \langle \hat{c}^2\rangle$$

EmployingEqs.(3.36),(3.38),(3.47),(3.52)and(3.53)inEqs.(4.16)and(4.17),wefind

$$(\Delta c_+)^2 = \frac{N\gamma_c}{\kappa}\frac{2\Omega^2}{\gamma_c^2+3\Omega^2} + 2N - \frac{4g}{\kappa} + \frac{N\gamma_c}{\kappa}\frac{\gamma_c^2+2\Omega^2}{\gamma_c^2+3\Omega^2} + \frac{N\gamma_c}{\kappa}\frac{\gamma_c^2+\Omega^2}{\gamma^2+3\Omega^2} + \frac{N\gamma_c}{\kappa}\frac{\Omega\gamma_c}{\gamma^2+3\Omega^2}$$

$$+\frac{N\gamma_c}{\kappa}\frac{2\gamma_c^2+5\Omega^2+\Omega\gamma_c}{\gamma_c^2+3\Omega^2} \quad (4.18)$$

$$(\Delta c_-)^2 = 2N-\frac{4g}{\kappa}-\frac{N\gamma_c}{\kappa}\frac{\Omega\gamma_c-3\Omega^2}{\gamma_c^2+3\Omega^2} \quad (4.19)$$

Moreover,forthecaseinwhichthederivingcoherentlightisabsent(Ω=0),onecanseethat

$$(\Delta c_+)_v^2=2N-\frac{4g}{\kappa}+\frac{2N\gamma_c}{\kappa} \quad (4.20)$$

$$(\Delta c_-)_v^2=2N-\frac{4g}{\kappa} \quad (4.21)$$

which are the quadrature variance of the two-mode cavity light in vacuum state. ComparingEqs.(4.18)and(4.19),weexpectthesqueezingtooccurintheminusquadrature.

4.3QuadratureSqueezing

Inthissectionweseektostudythesqueezingpropertiesofthelightgeneratedbynon-degeneratethreelevellasersystem.Tothisend,wecalculatethequadraturesqueezingofthetwo-modecavitylightintheentirefrequencyintervalrelativetothequadraturevarianceof the two-mode cavity vacuum state.The quadrature squeezing of the two-mode cavity lightrelativetothequadraturevarianceofthetwo-modevacuumlightcanbedefinedas[2,7]

$$S_-=\frac{(\Delta c_-)_v^2-(\Delta c_-)^2}{(\Delta c_-)_v^2} \quad (4.22)$$

where$(\Delta c_-)_v^2$isquadraturevarianceofthetwo-modevacuumcavitylight,S_-isthesqueezingof minus quadrature.Now employing quadrature variance of the two-mode vacuum cavity lightgivenbyEq.(4.19)andthequadraturevarianceofthecavitylightinavacuumstateexpressedinEq.(4.21),Eq.(4.22)reducesto

$$S_-=\frac{2N-\frac{4g}{\kappa}-\left(2N-\frac{4g}{\kappa}-\frac{N\gamma_c}{\kappa}\frac{\Omega\gamma_c-3\Omega^2}{\gamma_c^2+3\Omega^2}\right)}{2N-\frac{4g}{\kappa}}$$

AscomparedtoN,$\frac{4g}{\kappa}$,wecouldset $\frac{4g}{\kappa}\approx 0$.Then,aftersomealgebra,weget

$$S_-=\frac{\gamma_c}{2\kappa}\frac{\Omega\gamma_c-3\Omega^2}{\gamma_c^2+3\Omega^2} \quad (4.23)$$

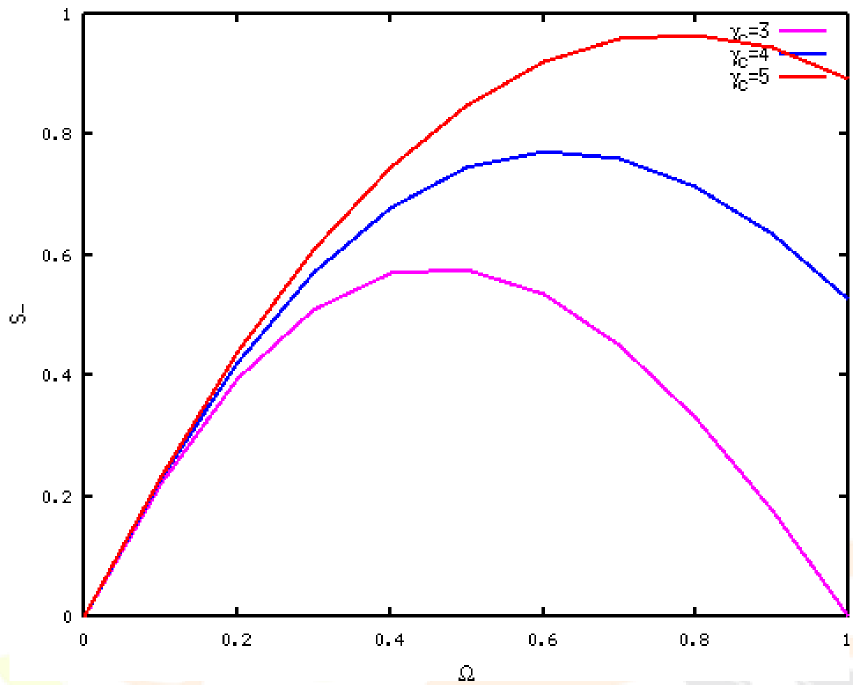

Figure4.1:PlotsofthequadraturesqueezingS_-versusΩforκ=0.2andfordifferentγ_c

We note that, unlike to the mean photon number and variance of the photon number, thequadraturesqueezingdoesnotdependonthenumberofatoms.Thisimpliesthatthequadra-turesqueezingofthecavitylightisindependentofthenumberofphotons.FromEq.(4.23),onecan easily observe that the degree of squeezing depends highly on both rate of stimulatedemission(γ_c)andtheamplitudeofthedrivingcoherentlight(Ω). Itincreaseswithincreasingboththestimulateddecayrateandtheamplitudeofthedrivingcoherent light.Finally, we found that the light generated by a coherently driven non-degeneratethree level laser placed inside a cavity coupled to two mode thermal reservoir produces asqueezed state light with a maximum squeezing of (96.22%) $\gamma_c$below the quantum standardlimitorthecoherentstatelevelwhichoccuratγ_c=5andΩ=0.78.

Chapter5Conclusion

In this thesis, we have investigated the statistical and squeezing properties of coherently drivennon-degeneratethreelevellasersinacavitycoupledtotwomodethermalreservoir.Inthecaseofinteractingdrivingcoherentlightwithacavitymodeandnondegeneratethreelevelatom,we obtain the Hamiltonian equation.After that, we develop the master equation and quantumlangiven equations for the system under consideration and then, we derive the equations of evo-lutionoftheexpectationvaluesofthecavitymodeandatomicoperators.wethen,determinethesteady-statesolutionsoftheresultingequationsintheadiabatic(largetime)approxima-tion scheme.By the help of this equation and solutions, we obtained the photon statistics andquadrature squeezing.We find that the global mean photon number for mode**a(b)** increaseswiththeamplitudeofthedrivingcoherentlight(Ω)andwiththerateofstimulatedemission(γ_c).Wealsoobtainedthatthephotonnumberstatisticsofsinglelightmodeissuper-Poiso-nianphotonstatisticswhichshowsthateachofthetworadiationmodesareinachaoticstateoflight.Weverifythattheglobalmeanphotonnumberfortwomodelightincreaseswiththeamplitudeofthedrivingcoherentlight(Ω)andwiththerateofstimulatedemission(γ_c).Then, we show that the light generated by coherently driven non-degenerate three level lasercoupledtotwomodethermalreservoirownsa super-poisonianstatisticsanditisa chaoticlight.Finally,wefindthatthelightmodegeneratedbysuchlightisinasqueezedstatewithamaximumdegreeofsqueezing(96.22%)belowthecoherentlevel.

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